Universidad de Puerto Rico
Recinto Universitario de Mayagüez
Mecánica de Materiales II - INGE 4012 Sección: 156
Profesor: Orlando Cundumi S.

Asignación No 8

Se deben resolver los siguientes problemas del libro seguido en la clase: **Mechanics of Materials**, James M. Gere.

- 10.3-1
- 10.3-5
- 10.3-7
- 10.4-1
- 10.4-2
- 10.4-3
- 10.4-5
- 11.2-1
- 11.2-2
- 11.3-1
- 11.3-4
- 11.3-8
- 11.3-16
- 11.4-2
- 11.4-3
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- 11.6-1
- 11.6-4

Fecha de entrega: Antes del examen parcial No 3.

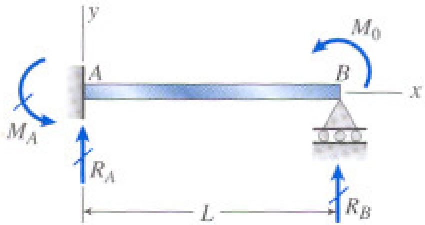
PROBLEMS CHAPTER 10

Differential Equations of the Deflection Curve

The problems for Section 10.3 are to be solved by integrating the differential equations of the deflection curve. All beams have constant flexural rigidity EI. When drawing shear-force and bending-moment diagrams, be sure to label all critical ordinates, including maximum and minimum values.

10.3-1 A propped cantilever beam AB of length L is loaded by a counterclockwise moment M_0 acting at support B (see figure).

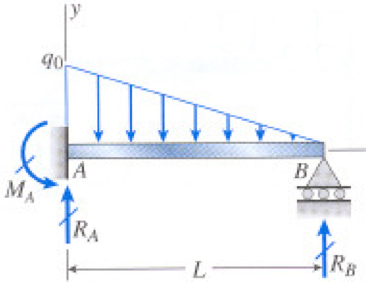
Beginning with the second-order differential equation of the deflection curve (the bending-moment equation), obtain the reactions, shear forces, bending moments, slopes, and deflections of the beam. Construct the shear-force and bending-moment diagrams, labeling all critical ordinates.



PROB. 10.3-1

10.3-5 A propped cantilever beam AB of length L supports a triangularly distributed load of maximum intensity q_0 (see figure).

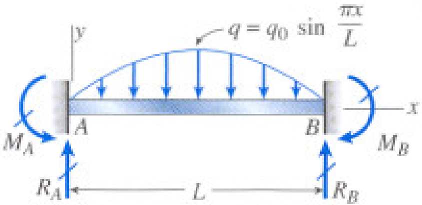
Beginning with the fourth-order differential equation of the deflection curve (the load equation), obtain the reactions of the beam and the equation of the deflection curve.



PROB. 10.3-5

10.3-7 The load on a fixed-end beam AB of length L is distributed in the form of a sine curve (see figure). The intensity of the distributed load is given by the equation $q = q_0 \sin \pi x/L$.

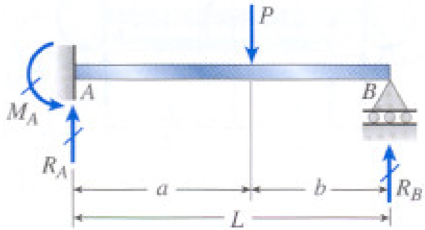
Beginning with the fourth-order differential equation of the deflection curve (the load equation), obtain the reactions of the beam and the equation of the deflection curve.



PROB. 10.3-7

10.4-1 A propped cantilever beam *AB* of length *L* carries a concentrated load *P* acting at the position shown in the figure.

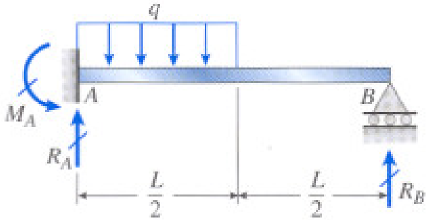
Determine the reactions R_A, R_B, and M_A for this beam. Also, draw the shear-force and bending-moment diagrams, labeling all critical ordinates.



PROB. 10.4-1

10.4-2 The propped cantilever beam shown in the figure supports a uniform load of intensity q on the left-hand half of the beam.

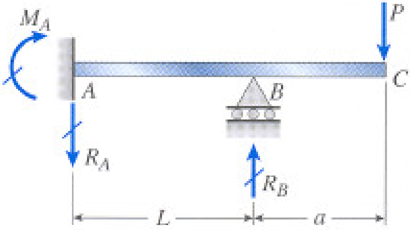
Find the reactions R_A, R_B, and M_A, and then draw the shear-force and bending-moment diagrams, labeling all critical ordinates.



PROB. 10.4-2

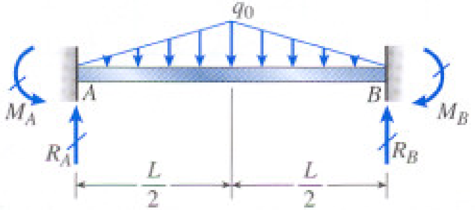
10.4-3 The figure shows a propped cantilever beam *ABC* having span length *L* and an overhang of length *a*. A concentrated load *P* acts at the end of the overhang.

Determine the reactions R_A, R_B, and M_A for this beam. Also, draw the shear-force and bending-moment diagrams, labeling all critical ordinates.



PROB. 10.4-3

10.4-5 Determine the fixed-end moments (M_A and M_B) and fixed-end forces (R_A and R_B) for a beam of length L supporting a triangular load of maximum intensity q_0 (see figure). Then draw the shear-force and bending-moment diagrams, labeling all critical ordinates.

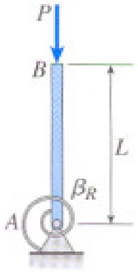


PROB. 10.4-5

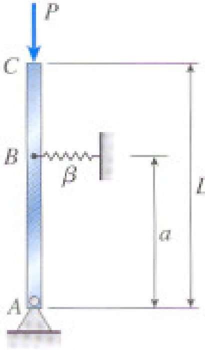
Idealized Buckling Models

11.2-1 through 11.2-4 The figure shows an idealized structure consisting of one or more **rigid bars** with pinned connections and linearly elastic springs. Rotational stiffness is denoted β_R and translational stiffness is denoted β.

Determine the critical load P_{cr} for the structure.



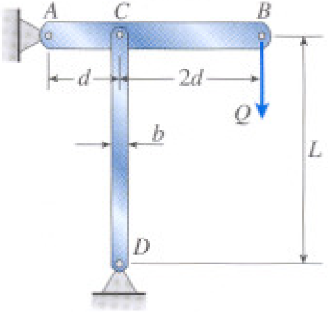
PROB. 11.2-1



PROB. 11.2-2

11.3-4 A horizontal beam *AB* is pin-supported at end *A* and carries a load *Q* at end *B*, as shown in the figure. The beam is supported at *C* by a pinned-end column. The column is a solid steel bar ($E = 200$ GPa) of square cross section having length $L = 1.8$ m and side dimensions $b = 60$ mm.

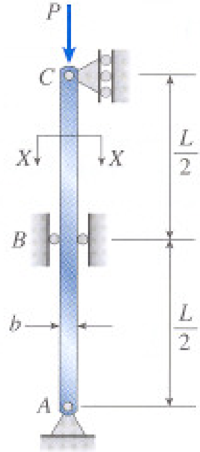
Based upon the critical load of the column, determine the allowable load *Q* if the factor of safety with respect to buckling is $n = 2.0$.

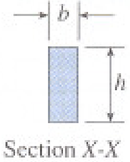


PROBS. 11.3-4 and 11.3-5

11.3-8 A rectangular column with cross-sectional dimensions b and h is pin-supported at ends A and C (see figure). At midheight, the column is restrained in the plane of the figure but is free to deflect perpendicular to the plane of the figure.

Determine the ratio h/b such that the critical load is the same for buckling in the two principal planes of the column.





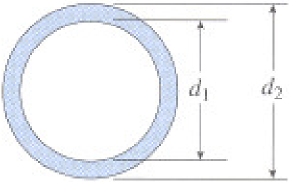
PROB. 11.3-8

Columns with Other Support Conditions

The problems for Section 11.4 are to be solved using the assumptions of ideal, slender, prismatic, linearly elastic columns (Euler buckling). Buckling occurs in the plane of the figure unless stated otherwise.

11.4-1 An aluminum pipe column (E = 10,400 ksi) with length L = 10.0 ft has inside and outside diameters d_1 = 5.0 in. and d_2 = 6.0 in., respectively (see figure). The column is supported only at the ends and may buckle in any direction.

Calculate the critical load P_{cr} for the following end conditions: (1) pinned-pinned, (2) fixed-free, (3) fixed-pinned, and (4) fixed-fixed.

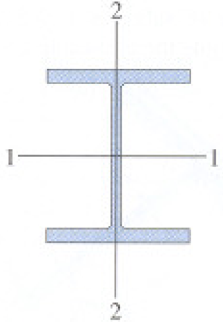


PROBS. 11.4-1 and 11.4-2

11.4-2 Solve the preceding problem for a steel pipe column (E = 210 GPa) with length L = 1.2 m, inner diameter d_1 = 36 mm, and outer diameter d_2 = 40 mm.

11.4-3 A wide-flange steel column ($E = 30 \times 10^6$ psi) of W 12×87 shape (see figure) has length $L = 28$ ft. It is supported only at the ends and may buckle in any direction.

Calculate the allowable load P_{allow} based upon the critical load with a factor of safety $n = 2.5$. Consider the following end conditions: (1) pinned-pinned, (2) fixed-free, (3) fixed-pinned, and (4) fixed-fixed.

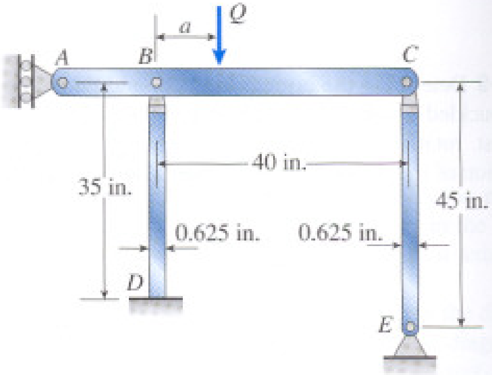


PROBS. 11.4-3 and 11.4-4

11.4-7 The horizontal beam *ABC* shown in the figure is supported by columns *BD* and *CE*. The beam is prevented from moving horizontally by the roller support at end *A*, but vertical displacement at end *A* is free to occur. Each column is pinned at its upper end to the beam, but at the lower ends, support *D* is fixed and support *E* is pinned. Both columns are solid steel bars ($E = 30 \times 10^6$ psi) of square cross section with width equal to 0.625 in. A load *Q* acts at distance *a* from column *BD*.

(a) If the distance $a = 12$ in., what is the critical value Q_{cr} of the load?

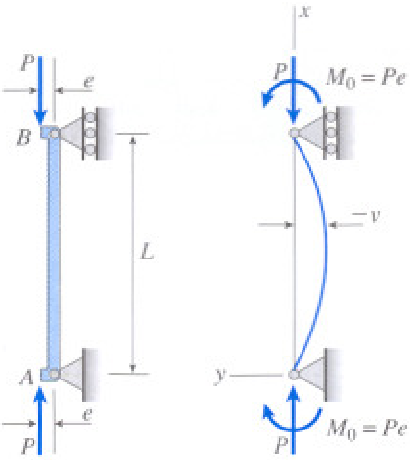
(b) If the distance *a* can be varied between 0 and 40 in., what is the maximum possible value of Q_{cr}? What is the corresponding value of the distance *a*?



PROB. 11.4-7

11.5-3 Determine the bending moment M in the pinned-end column with eccentric axial loads shown in the figure. Then plot the bending-moment diagram for an axial load $P = 0.3P_{cr}$.

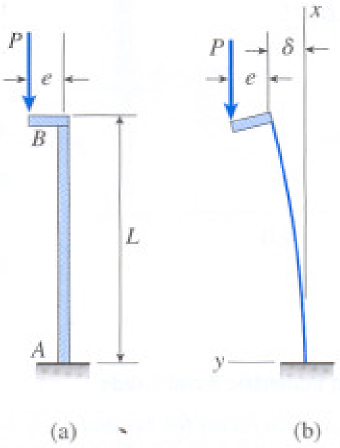
Note: Express the moment as a function of the distance x from the end of the column, and plot the diagram in nondimensional form with M/Pe as ordinate and x/L as abscissa.



PROBS. 11.5-3 through 11.5-5

11.5-9 The column shown in the figure is fixed at the base and free at the upper end. A compressive load P acts at the top of the column with an eccentricity e from the axis of the column.

Beginning with the differential equation of the deflection curve, derive formulas for the maximum deflection δ of the column and the maximum bending moment M_{max} in the column.



PROB. 11.5-9

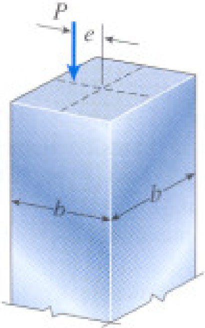
The Secant Formula

When solving the problems for Section 11.6, assume that bending occurs in the principal plane containing the eccentric axial load.

11.6-1 A steel bar has a square cross section of width $b =$ 2.0 in. (see figure). The bar has pinned supports at the ends and is 3.0 ft long. The axial forces acting at the end of the bar have a resultant $P = 20$ k located at distance $e =$ 0.75 in. from the center of the cross section. Also, the modulus of elasticity of the steel is 29,000 ksi.

(a) Determine the maximum compressive stress σ_{max} in the bar.

(b) If the allowable stress in the steel is 18,000 psi, what is the maximum permissible length L_{max} of the bar?

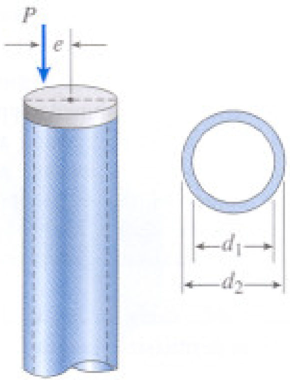


PROBS. 11.6-1 through 11.6-3

11.6-4 A pinned-end column of length $L = 2.1$ m is constructed of steel pipe ($E = 210$ GPa) having inside diameter $d_1 = 60$ mm and outside diameter $d_2 = 68$ mm (see figure). A compressive load $P = 10$ kN acts with eccentricity $e = 30$ mm.

(a) What is the maximum compressive stress σ_{max} in the column?

(b) If the allowable stress in the steel is 50 MPa, what is the maximum permissible length L_{max} of the column?



PROBS. 11.6-4 through 11.6-6