Lessons from Bohemia

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Abstract: This paper presents a "true life adventure" into asteroid photometry methods, which hopefully will be encouraging and useful to other amateur astronomers who are doing asteroid photometry. The key themes are: (a) a densely-sampled light curve, and fine sampling of the Fourier analysis errors, are very valuable in establishing the asteroid's rotation period; (b) the reliability of the "Hardie method" of determining atmospheric extinction is improved by taking account of the colors of standard stars; and (c) determination of asteroid absolute magnitude and "slope parameter" requires special efforts to achieve photometric accuracy of .03 magnitude or better.

References:

1. Riccioli, D., et al, "Lightcurves and rotational periods of main belt asteroids, III", *Astronomy and Astrophysics Supplement Series*, v 111, p 297-303 (1995).
2. Mohamed, R. A., et al, "Light curves and rotation periods of asteroids 371 Bohemia, 426 Hippo, 480 Hansa, and 735 Marghanna", *Astronomical Journal*, Vol 109, No 4, 1995.
3. Behrend, Raoul, Observatoire de Genève, http://obswww.unige.ch/~behrend/page_cou.html
4. Warner, Brian D, A Practical Guide to Lightcurve Photometry and Analysis, (Bdw Publishing, Colorado Springs, CO, 2003)
5. Hardie, R. H., "Photoelectric Reductions" in Hiltner W. A, ed *Astronomical Techniques*, Univ. of Chicago Press, 1962.
6. Hardie, R. H. "An Improved Method for Measuring Extinction", Astrophysical Journal, v. 130, p. 663, Sept. 1959.
7. Henden, Arne A and Kaitchuck, Ronald H, Astronomical Photometry, (William-Bell, Richmond, Virginia, 1990)
8. Bowell, E. and Lumme, K., "Colorimetry and Magnitudes", in Gehrels, et al (ed), *Asteroids I*, University of Arizona Press, Tucson, 1982
9. Bowell, E. et al, "Application of Photometric Models to Asteroids", in Binzel, R.P. et al (ed), *Asteroids II*, University of Arizona Press, Tucson, 1989.
10. Planetary Data System Small Bodies Node at http://pdssbn.astro.umd.edu
11. Tedesco, E. F. "Asteroid Magnitudes, UBV colors, and IRAS albedos and diameters", in Binzel, R.P et al (ed), *Asteroids II*, University of Arizona Press, Tucson, 1989.

1. Introduction

Several of last year's IAPPP speakers pointed us to projects that take advantage of the key advantage that amateur astronomers have over professionals: our access to telescope time is limited only by our interest and our endurance. Long runs of densely-packed data points can bring out features in a light curve that may be skipped over by the necessarily sparser data sets that typically come from professional studies. The speakers also emphasized the skills that amateur astronomers must strive for in order to do reliable science:

- ✓ know your instrument and the details of what it can (and cannot) do;
- ✓ review the professional literature on the topic you're interested in;
- ✓ learn from other's experiences;
- ✓ meticulously check both your methods and your results; and
- ✓ collaborate with other amateurs and with professional astronomers.

After a couple of years experience in gathering unfiltered asteroid light curves, I used a study of 371 Bohemia for my self-education on techniques for multi-color (BVR) photometry of asteroids. I set out to determine the asteroid's rotation period, color index, absolute magnitude and slope parameter, and to search for possible color variations as it rotated. The project provided a "hands-on" lesson in the value of doing all the things that last year's speakers recommended.

2. Light curve period determination

Previous professional studies apparently had trouble with this object. Two studies done during the 1993 apparition reported rotational periods of 3.8 hours and 12.48 hours, respectively. In retrospect, both of these studies probably suffered from insufficient data. Riccioli's [1] data is shown in Figure 1. Note that it is a very sparse data set, and that only one maximum was observed, so that they had to use quite a bit of inference to conclude that the rotational period is 12.48 hours.

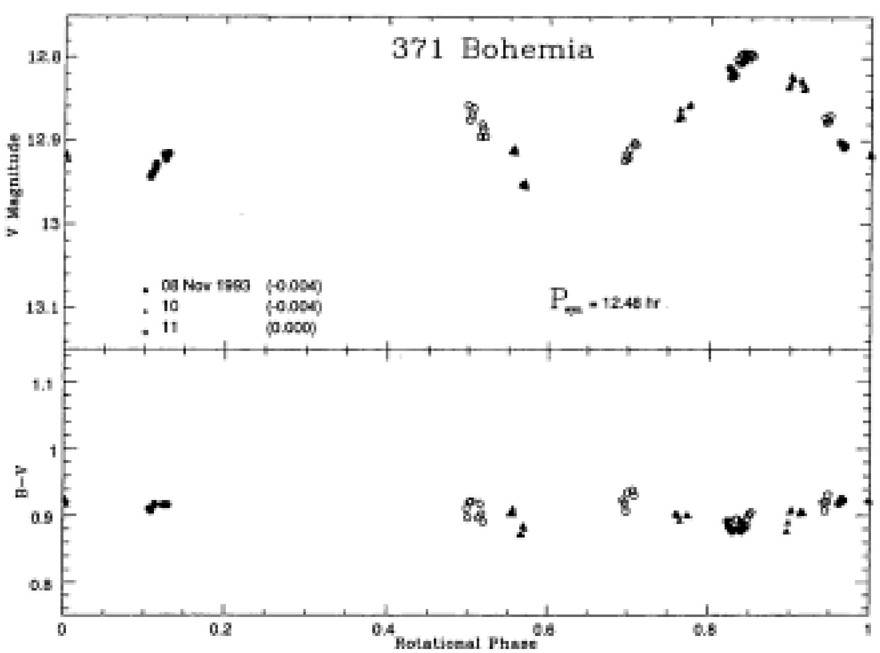

Fig. 3. Composite V lightcurve and $B-V$ color of the asteroid 371 Bohemia vs. rotational phase. The 0° phase corresponds to 1993 November 8 at $12^{h}00$ UT. The mean error of the single measurements is 0.005 mag. The V magnitude shifts are reported, in brackets, near the date

Fig 1: Riccioli's sparse data set indicated P= 12.48 hours

Figure 2 shows Mohamed's [2] data. The derived rotation period of 3.792 hrs relies almost totally on the data from a single night (10/20/93), and in retrospect, the fit between the four nights isn't very good.

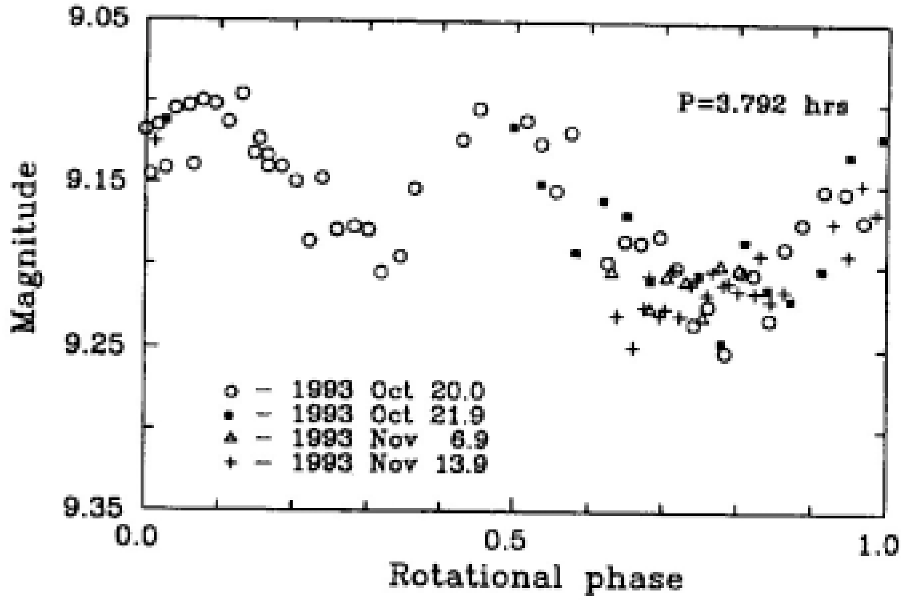

FIG. 1. Composite light curve of 371 Bohemia. Zero rotation phase is at 1993 October 19.9283 corrected for light time.

Fig 2. Mohamed's data suggested P= 3.792 hours

Meanwhile, the Geneva Observatory web site [3] had reported preliminary observations from 2001-2 which appeared to match an 8.7 hour period.

Clearly, these disparate results couldn't all be correct!

I gathered V-band data on three nights, which seemed to yield a good-quality light curve and a good estimate of the asteroid's rotation period. This subset of data is shown in Figure 3, wrapped with an apparently well-defined rotation period of 4.37 hours – tolerably close to Mohamed's estimate, and a vaguely similar shape.

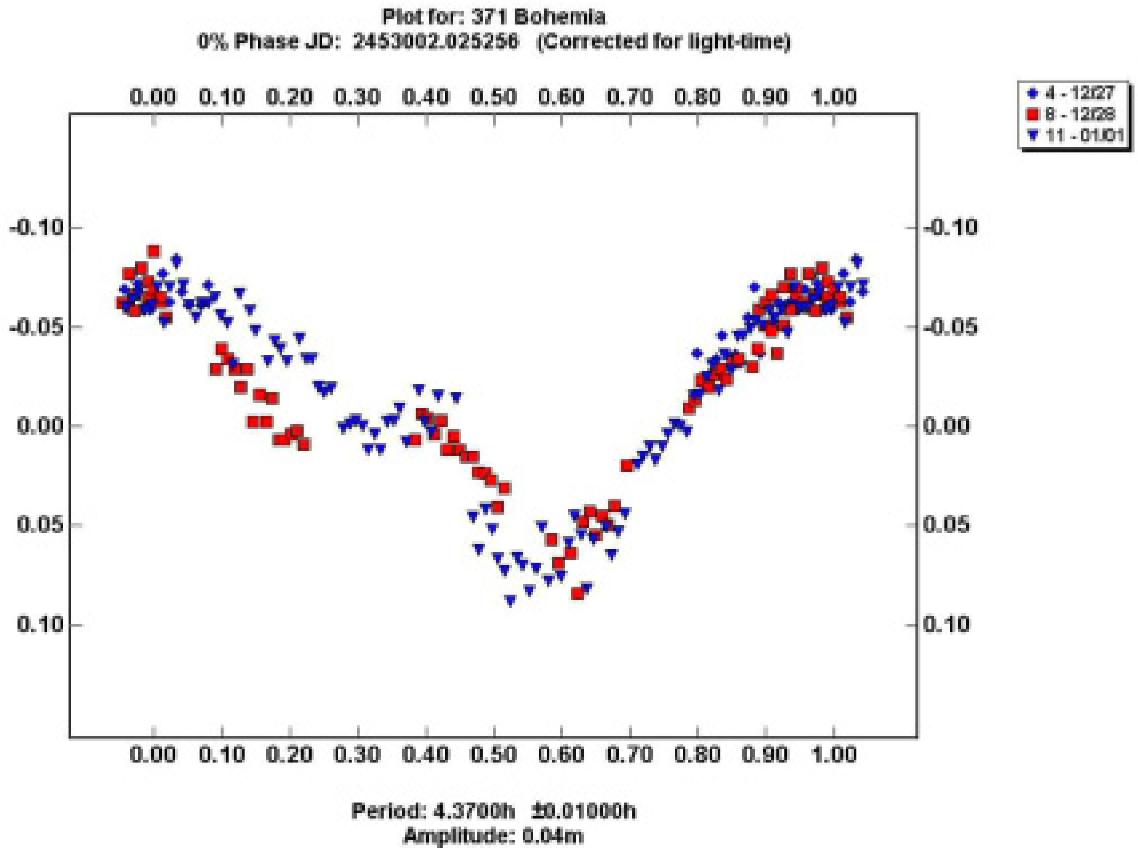

Fig 3. A tolerable fit when wrapped at P= 4.37 hours?

However, note that this solution depends on treating the "little hump" near phase = 0.4 as the secondary maximum.

Since I needed additional data for other parts of this project, I updated the light curve estimate after night 4 was complete. Yikes! The apparent rotation period had slowed to P= 8.76 hours, as shown in Figure 4. This result seemed to be a good match to the report from Behrends [3] (whose provisional estimate was P= 8.77 hours). The "little hump" (now appearing at phase = 0.6) is revealed to be a manifestation of the asteroid's shape, not one of the major maxima. That "little hump" is evidence that this object isn't a "tumbling football" (i.e. it isn't a triaxial ellipsoid: maybe there is some sort of a ridge or canyon that distorts the light curve).

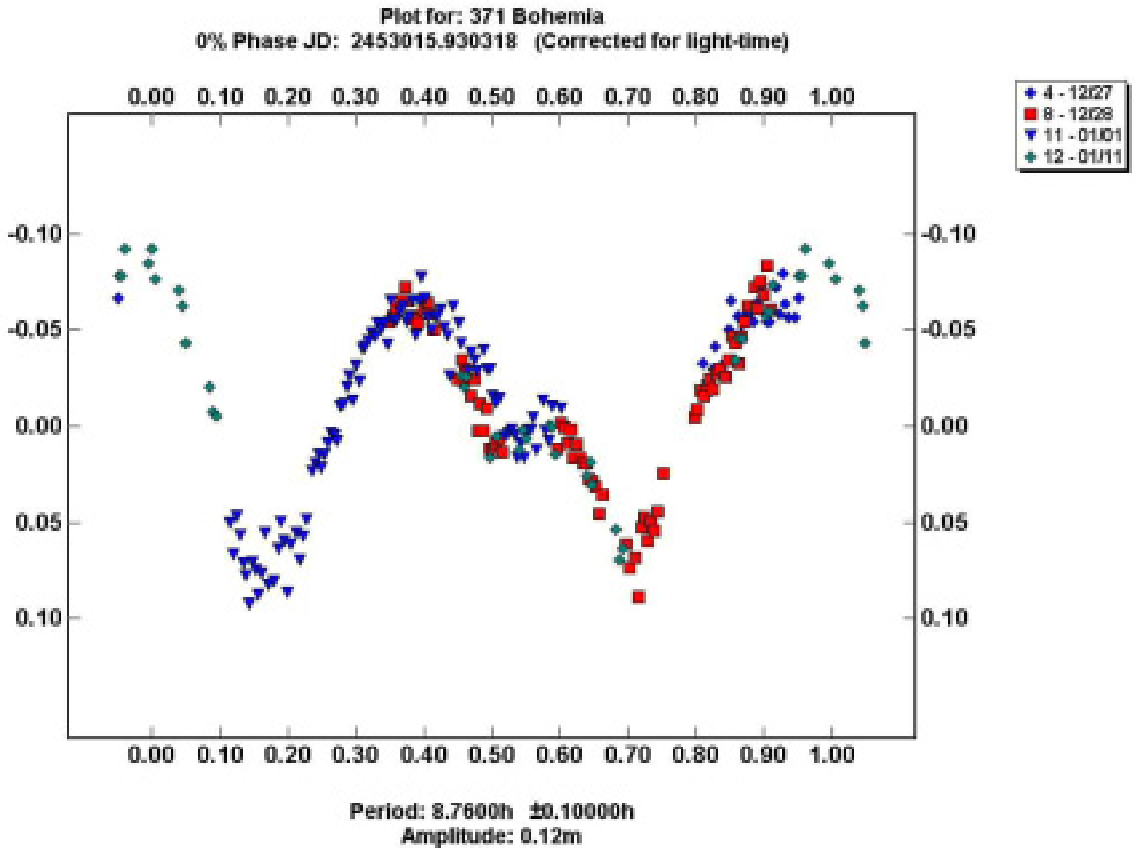

Fig 4. More data, and a better fit when wrapped at P= 8.76 hours?

Then, three weeks later, I gathered another night's worth of V-band data, and discovered that it didn't fit in with the 8.76 hour period at all – but that it did give a good fit using P= 10.76 hours, as shown in Figure 5.

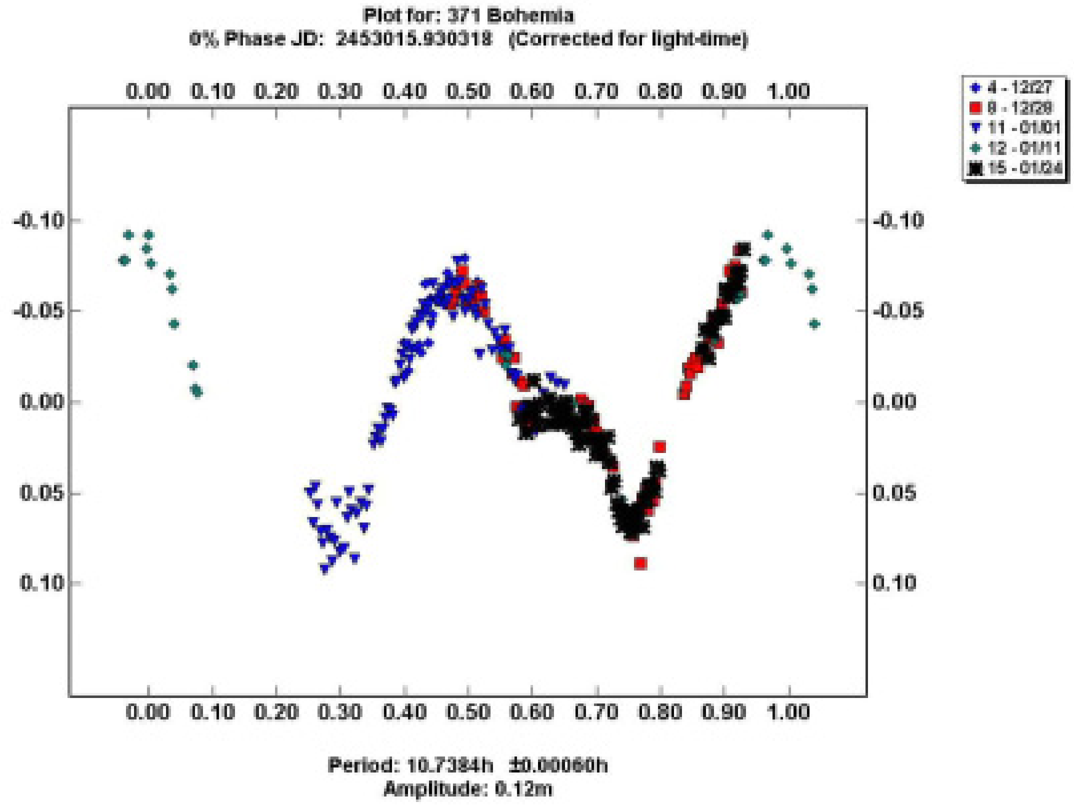

Fig 5. Yet more data, and yet another estimate, this time P= 10.738 hours

Finally, I was circling in on the correct rotation period. Adding in night #6 confirmed the asteroid's rotation rate at P= 10.737 hrs, as shown in Figure 6.

The complex shape of the light curve may explain how the other studies were misled by their sparse data sets and/or short intervals of observation.

This is a real lesson in the importance of re-evaluating asteroids with previously-reported light curves. It also shows the great value of the amateur's freedom to make profligate use of telescope time. Full coverage and dense data clarified an otherwise ambiguous light curve.

Dr. Behrends at Geneva Observatory was gracious in responding to my request (on the CALL website) for additional data. Ultimately he and I shared our data (including his 2001 and 2003 runs), and in response to my result his team gathered additional runs in 2004. This collaboration turned out to be wonderful. His data reduction validated my period estimate (his result, using all of our data, is P= 0.447463±0.000007 day= 10.739 hours). Better, he is hopeful that the wealth of data and a now-certain light curve can be used to determine other parameters of the asteroid (such as whether its rotation is prograde or retrograde).

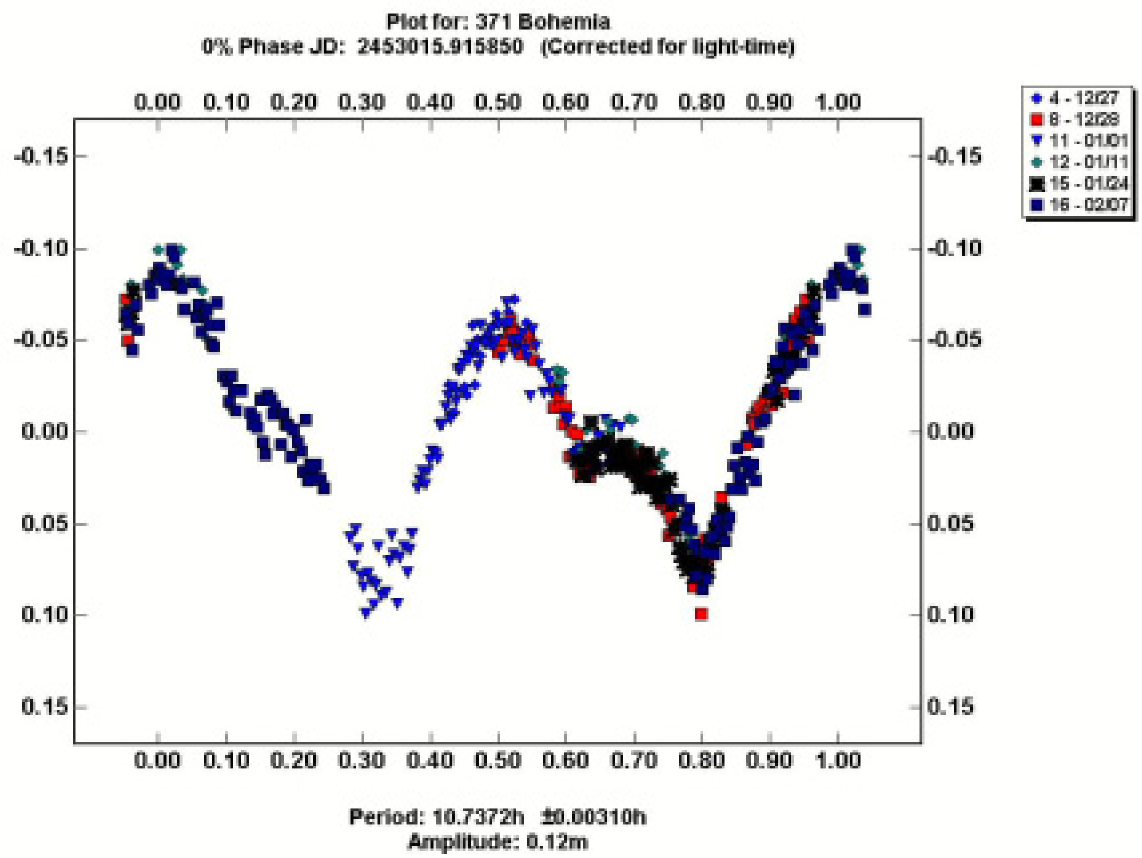

Fig 6. The final estimate: rotation period P= 10.737 hours

This project also showed me the value of a topic that Warner [4] discusses, and uses in the Canopus software: complete evaluation of the Fourier analysis error over the range of candidate periods. The Fourier analysis routine in Canopus tries a sequence of candidate periods, wraps the data at each candidate period, and calculates the resulting error. The period with the smallest error is usually the best fit. Since this method samples a series of candidate periods, the candidates need to be closely spaced in order to avoid the risk of "jumping over" the truly best fit (e.g. the best fit might fall between two candidates). Warner advises using an interval between candidate periods of no more than

$$\delta = 0.1\ P^2/T$$

where P is the assumed period, and T is the total time over which observations are made (i.e. the time from the first observation to the last). In the case shown in Figure 6, T is nearly 30 days, so

$$\delta \approx 0.1\cdot 11^2/\ [24\cdot 30]\ \approx 0.017 \text{ hours.}$$

Yes, I really did run Canopus through the whole range from P= 2 hours to P= 15 hours, checking every .01 hour interval (i.e. P= 2.00 hrs, P= 2.01 hrs, etc.) to be sure that I wasn't jumping over a potentially good period estimate!

3. Extinction coefficients

The first step in reducing instrumental magnitudes to standard magnitudes is determination of the atmospheric extinction. Hardie's [5, 6] method is quite efficient in terms of telescope time: you take images of a Landolt standard field near the zenith, another near the horizon, and from that information you can calculate the extinction coefficient. Most writers advise that the standard stars used in the two fields should have similar color indices, but this advice is not quantified – e.g. how similar is "similar-enough"? My attempts to use this method prompted a re-examination of the relevant equations in Hardie's original papers. I evaluated the effects of star-color mismatch, and found a simple method of improving the confidence in Hardie-derived extinction coefficients. This revised Hardie method has been incorporated into the latest version of Brian Warner's PhotoRed program.

3.1 Color index effects in the Hardie method of determining extinction: The "Hardie method" of determining extinction is convenient because by imaging two standard fields, you can determine the extinction values and "zero points" for the night. This information (plus the color index transforms for your system) enables you to reduce your instrumental magnitudes to standard, exo-atmospheric magnitudes.

The concept is simplicity itself. Find a field with Landolt standard stars that is near the zenith, and make images in each filter that you'll be using during the night. Call this "field 1" (or, the "zenith field"). Find another Landolt field that is near the horizon, at about 30 to 40 degrees elevation angle, and make images of it. Call this "field 2" (the "horizon field"). Pick a standard star from field 1, whose standard magnitude is V_1, and whose instrumental magnitude is v_1; and pick a standard star from field 2 whose standard magnitude is V_2 and whose instrumental magnitude is v_2.

The fundamental equation for observed magnitude in the presence of atmospheric extinction (neglecting second-order extinction) is

$$v_{0,1} = v_1 - k'_v \cdot X_1$$

(with a similar equation for v_2). Using the definition of the nightly zero point,

$$V_1 = v_{0,1} + ZP$$

it isn't hard to derive the "Hardie equation", which is most often written [4]:

$$\frac{(V_1 - V_2) - (v_1 - v_2)}{(X_2 - X_1)} = k'_v \qquad \text{Eq. (1)}$$

In this equation, I'm using the standard nomenclature:

V_i is the standard, exo-atmospheric magnitude of the i^{th} star
v_i is the measured instrumental magnitude of the i^{th} star

X_i is the air mass for the measurement of the i^{th} star
ZP is the "zero point" that relates camera ADUs to magnitudes

and

k'_v is the extinction coefficient (in magnitudes per air mass)

I've implicitly assumed that we're working v-band, but the same form of equations will be used for all other bands as well.

In order to take advantage of the fact that there will be several standard stars in each field of view, it is common practice to form "Hardie pairs", matching each star in field 1 with several stars in field 2, using Equation (1) to calculate an estimated extinction coefficient for each pair, and then averaging all of the individual extinction estimates to get a best estimate of the actual extinction value, k'_v.

It is not unusual to see quite a bit of scatter in the k'_v values calculated by individual Hardie-pairs. Figure 7 shows a typical result:

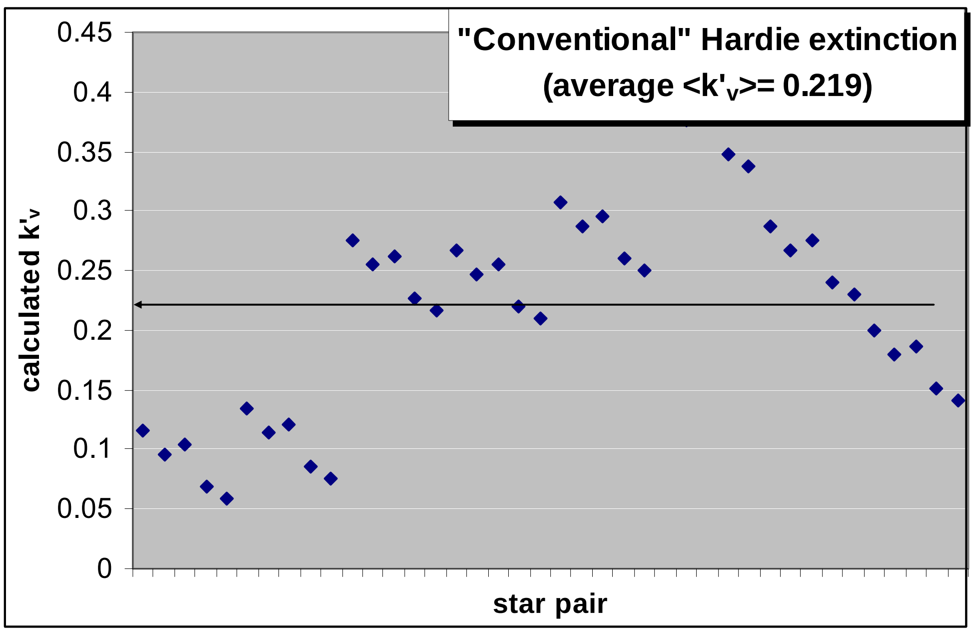

Fig 7. Typical example of "scatter" in calculated k′v from many Hardie pairs formed by stars in field 1 (zenith field) and field 2 (horizon field). The best estimate is k'_v = average ($k'_{v,i}$).

I was taken aback by the wide scatter in the individual estimates of extinction when using this method – s.d. = 40% in this example. We should be able to do better than that!

Figure 8 shows the spreadsheet analysis that led to Figure 7.

						V-R=	0.009	0.08	0.575	0.723	0.683	1.082	0.366	0.29
						B-V=	-0.004	0.157	0.968	1.356	1.146	1.909	0.632	0.463
										Field 2				
						star	98-653	98-650	98-671	98-670	98-676	98-675	98-682	98-685
						V	9.539	12.271	13.385	11.93	13.068	13.398	13.749	11.954
		Field 1				air mas	1.656	1.656	1.656	1.656	1.656	1.656	1.656	1.656
			Std Mag			IM	-10.317	-7.578	-6.408	-7.866	-6.712	-6.347	-6.039	-7.869
V-R	B-V	star	V	air mass	IM									
0.399	0.699	92-249	14.325	1.259	-5.577		0.12	0.13	0.27	0.27	0.31	0.40	0.29	0.20
0.446	0.814	92-250	13.178	1.259	-6.716		0.10	0.11	0.25	0.25	0.29	0.38	0.27	0.18
0.326	0.517	92-252	14.932	1.259	-4.965		0.10	0.12	0.26	0.25	0.29	0.38	0.27	0.19
0.719	1.131	92-253	14.085	1.259	-5.798		0.07	0.09	0.23	0.22	0.26	0.35	0.24	0.15
0.929	1.418	92-245	13.818	1.259	-6.061		0.06	0.08	0.22	0.21	0.25	0.34	0.23	0.14
										calculated extinction using i, j pair				

Fig 8. The spreadsheet that led to Figure 7.

Each cell in the outlined box is the extinction value derived from a single pair of stars. If I average all of the individual values in the box, I get $k'_v = 0.219$ (a plausible value for my observatory, which is at nearly sea level), with std dev = 0.09. But look closely at the individual values – one pair of stars gives a calculated extinction coefficient of $k'_v = 0.07$, while another pair of stars gives $k'_v = 0.40$. In other cases, I've found situations where some pairs of stars will give a calculated extinction that is *negative* – a physical impossibility! The color index data for the standard stars explains what's going on. The most out-of-the-norm extinction values occur when the two standard stars have wildly different color indices. For example, Field 1 star 5 is pretty red, with B-V = 1.418, and when it's paired with the much bluer Field 2 star 1 whose B-V = -.004, the calculated extinction coefficient is only 0.06 – hard to believe from my location.

The conventional wisdom is that when using the Hardie method, you should create "Hardie pairs" of stars that are approximately the same color. Henden and Kaitchuck [7] recommend picking only spectral type A0 stars, to ensure against color mismatch effects. The example in Figures 7 and 8 illustrates what can happen if you don't closely match star's colors. Alas, the conventional wisdom also says that you should use quite a few pairs (in order to get the benefit of averaging). But it's not likely that you can do both at the same time. And besides, how close do two stars have to be, to have "about the same color"?

In Hardie's original papers (for example reference [5]), he implicitly assumed that the standard stars had been calibrated for the instrument's spectral band. My system isn't too far from the standard system, but it isn't perfect. Let's go back to the Hardie equation, and modify it to recognize that instrumental magnitudes must be transformed into standard bands:

$$V_1 = v_{0,1} + T_v (B_1 - R_1) + ZP$$

where T_v is the (possibly not-yet-known) transform that will turn instrumental magnitude into standard V-band magnitude.

If I measure star 1 at air mass X_1, and star 2 at air mass X_2, then the difference will be:

$$(V_1 - V_2) = (v_1 - v_2) + T_v[(V_1 - R_1) - (V_2 - R_2)] - k'_v (X_1 - X_2)$$

and the Hardie equation becomes:

$$\frac{(V_1 - V_2) - (v_1 - v_2) - T_v[(B_1 - R_1) - (B_2 - R_2)]}{(X_2 - X_1)} = k'_v \qquad \text{Eq. (3)}$$

Equation (3) shows what's going on. If the two stars are exactly the same color, then the term $T_v[(B_1 - R_1) - (B_2 - R_2)]$ will drop out, because the term in square braces equals zero. Equation (3) then reduces to the "conventional" Hardie equation. But if the two stars aren't the same color, this color-mismatch term can have a noticeable effect.

This equation also suggests plotting the calculated extinction versus the B-R "color mismatch" of the two stars, as shown in Figure 9:

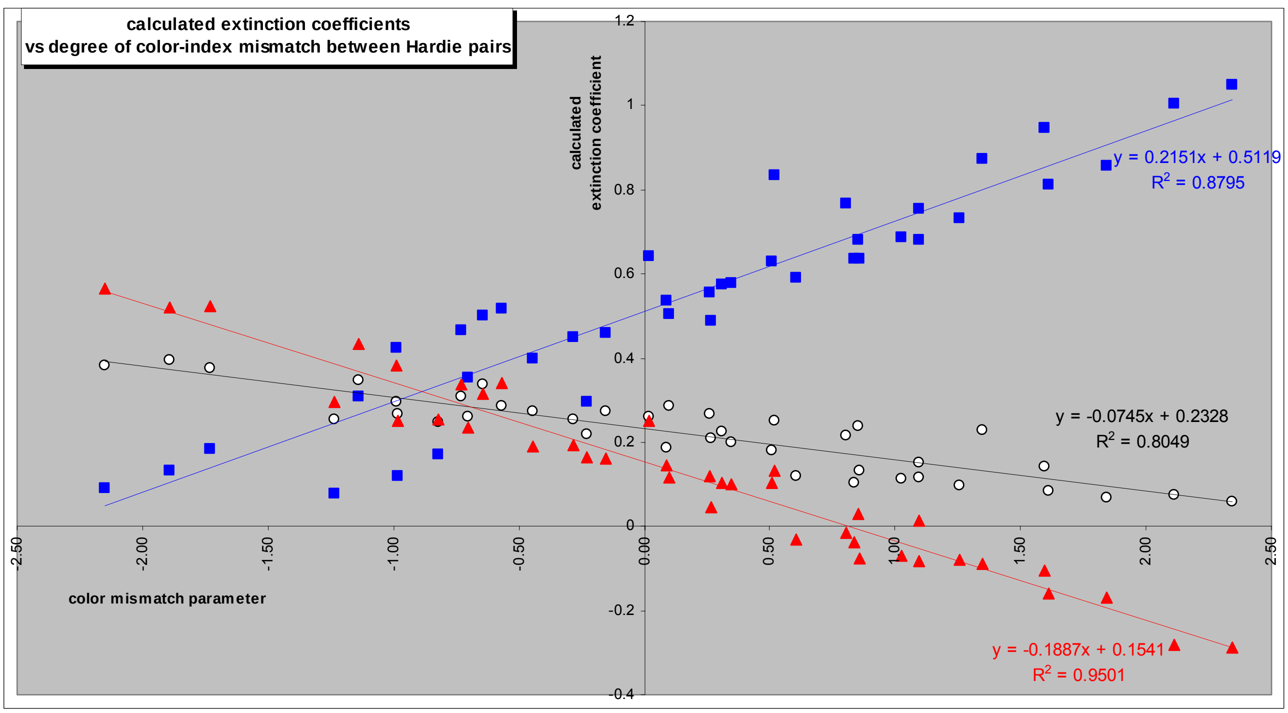

Fig 9. Calculated extinction vs. color mismatch (B-R)

(The circles are the v-band data from the spreadsheet shown in Figure 8. The squares are data from the same night in the b-band, and the triangles are from r-band data.) Note that the data fall along straight lines, as expected from Equation (3). The y-intercept of the best-fit line represents the extinction that would have been calculated if I had two stars with no color mismatch.

In the case of the v-band data, the y-intercept is $k'_v = 0.2328$, which is very close to the value calculated by averaging without regard to color mismatch. In this example, there are about as many mismatched pairs in one direction as in the other, so the average worked out OK. But there are cases where you have mostly-bluish stars in one field, and mostly-reddish stars in the other, and hence the "average" extinction coefficient will be noticeably different than the "zero color mismatch" value.

By plotting $k'_{v,i}$ versus (B-R), and using the y-intercept to define k'_v, there is no need to address the question "how close in color is close-enough".

In this analysis, I used B-R as the "color mismatch parameter" instead of B-V or V-R because it seemed to capture the full range of variation better. There's no deep science in this choice – it just seemed to work best with my instrument and my data. The key is to take account of color-mismatch effects when using the Hardie method.

3.2 Signal-to-Noise requirement for Hardie-Method extinction: There is another piece of advice regarding Hardie-method extinction that bears some quantification: "be sure that you have a sufficient signal-to-noise ratio" (SNR) in the standard star images that you use. How much is sufficient? This question came up when I tried to determine B-band extinction coefficients using star images with SNR≈ 20:1, and simply could not get consistent results. (This is a particular problem for CCD imagers using B-band, where the sensitivity of the CCD falls off pretty severely, compared to V- or R-band). The ultimate solution was, of course, longer exposure times and higher signal-to-noise ratio, but it was useful to discover how SNR affects the extinction calculation.

There are two ways to understand the effect of SNR on the accuracy of k′ determination: the first is heuristic, the second is a more rigorous statistical analysis.

3.2.1 Heuristic approach: Using the Hardie method is roughly equivalent to trying to detect the change in a star's brightness as it moves from air mass X_1 to air mass X_2. We can use some "typical" numbers and "rules of thumb" to get an idea of how accurate that measurement must be.

For my location, typical R-band extinction is about $k'_r = .15$; for the V-band a typical value is $k'_v = .25$.

The Landolt standard stars are on the celestial equator. For my latitude (33º N), the celestial equator never rises higher than 57 degrees elevation, so the smallest possible air mass for the "zenith field" is $X_1 = \sec(57^o) = 1.19$. Realistically, $X_1 \approx 1.25$ is what I usually achieve. The "horizon field" should be at the greatest possible air mass, but not greater than $X_2 = 2.0$ (elevation angle = 30 degrees), because at lower elevation angles effects such as differential refraction become serious. As a practical matter, I've found that elevation angles of ≈ 35 to 40 degrees are typical for my "horizon fields", because it doesn't usually work out that I'm ready at the same time that the standard field is optimally placed; and star-rise waits for no man. Hence, $X_2 = \sec(35^o \text{ to } 40^o) \approx 1.55$ to 1.74. Figure 10 illustrates the situation.

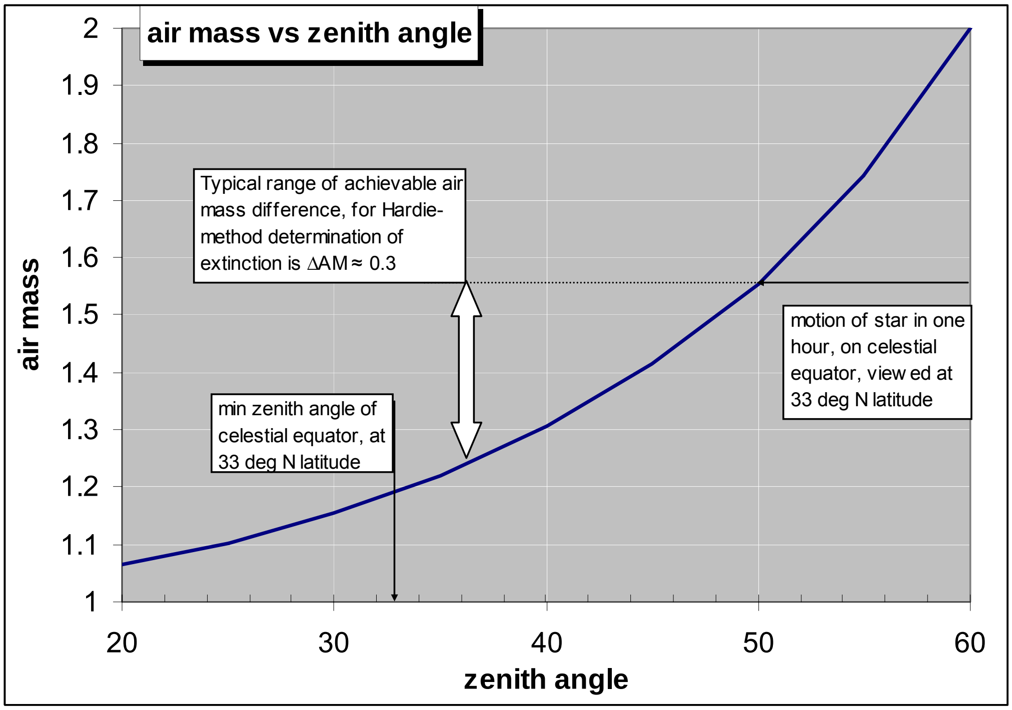

Fig 10. Air mass vs. Zenith angle, and typical situation for Hardie-method extinctions

Thus, we're trying to measure the change in a star's apparent brightness that is caused by moving from $X_1 \approx 1.25$ to $X_2 \approx 1.65$, i.e. $\Delta X \approx 0.4$ air mass. Using the "typical" $k'_v = .25$, that amounts to a brightness change of $\Delta m = 0.4\, k'_v = 0.1$ magnitude.

A well-respected rule of thumb in metrology is that your measuring error should be about 10 times *smaller* than the effect you're attempting to measure. In this example, the measurement accuracy should be about 0.01 magnitudes. The standard deviation of stellar photometric measurement – the accuracy – is usually taken to be $\sigma_M = 1/SNR$, where SNR is the signal-to-noise ratio of the stellar image.

Combining all of this implies that reliable Hardie-method determination of atmospheric extinction requires a signal-to-noise ratio of $SNR \geq 100$.

The exact numbers aren't central to this discussion. The key points in this line of reasoning are that it's very important to get $SNR \approx 100$ or higher, and equally important to get that "horizon field" as close to 30 degrees elevation as possible, to maximize the air-mass difference $|X_2 - X_1|$.

3.2.2 Statistical Analysis: A more rigorous statistical analysis of the Hardie equation (assuming one star in each field), which I won't reproduce here, yields the standard deviation of k'_v:

$$\sigma_k = \frac{[(1/SNR_1)^2 + (1/SNR_2)^2]^{1/2}}{|X_2 - X_1|} \qquad \text{Eq. (4)}$$

where SNR_1 and SNR_2 are the signal-to-noise ratios of star #1 and star #2 respectively. If the stars are of equal SNR, and we expand the analysis to use n stars in field 1 and m stars in field 2, then the benefit of averaging the results from the n·m unique "Hardie pairs" yields a calculated extinction coefficient with an accuracy of:

$$\sigma_k = \frac{2^{1/2}}{SNR \; |X_2 - X_1| \; [n \cdot m]^{1/2}} \qquad \text{Eq. (5)}$$

Re-arranging this equation, the required signal-to-noise ratio to achieve a desired accuracy ($\sigma_{k,\, des}$) is:

$$SNR_{reqd} \geq \frac{2^{1/2}}{\sigma_{k,\, des} \; |X_2 - X_1| \; [n \cdot m]^{1/2}} \qquad \text{Eq. (6)}$$

As a practical example, suppose that we want to estimate the extinction to an accuracy of about 5%, i.e. to about $\sigma_{k,\, des} = .01$ mag/air mass; and that there are 3 standard stars in each field (n = m = 3). With

$$|X_2 - X_1| \approx 0.4$$

the required signal-to-noise ratio is

$$SNR_{reqd} = 118$$

Equation (6) can be used to estimate the required SNR for other cases. It quantifies the general rules: get high signal-to-noise ratio, get as large an air-mass difference as practical, and use as many standard stars in each field as are available.

4. Absolute Magnitude

I have seen plots of asteroid brightness vs. phase angle in some professional studies. It would be neat to document the opposition-effect of my chosen target: the rapid brightness increase as the asteroid approaches phase angle α=0 (fully illuminated, analogous to a "full moon"). Beyond the inherent "neatness" of witnessing it, the shape of the asteroid's brightness-versus-phase curve turns out to have several important uses. First, the brightness at zero phase angle defines the absolute magnitude, H. Second, within some broad limits, the shape of the curve (the so-called "slope parameter", G) sets constraints on the albedo; and if you know the albedo and the absolute magnitude, you can determine the physical size of the asteroid (its projected area). Third, the slope parameter also offers some clues about the surface texture of the asteroid [8].

The relevant equations are found in reference [9]. First, all of the brightness measurements are transformed to standard V-magnitude. Then, the V-magnitudes are converted to "reduced" magnitudes, by:

$$V_R = V - 5 \cdot \log(r \cdot d)$$

where

V = the measured V-magnitude
d = the distance of the asteroid from the Sun (in AU)
r = the distance of the asteroid from the Earth (in AU)

This takes out the effect of the constantly changing distance from the Sun and Earth, so that the only remaining reasons for brightness change are the rotational light curve, and the changing phase angle.

The asteroid's "absolute magnitude" is defined as the brightness that the asteroid would have when fully illuminated (phase angle $\alpha=0$), at a distance of 1 AU from the Sun, and observed at 1 AU from the Earth. The "absolute magnitude" can be found by plotting V_R vs. α, and extrapolating the curve to $\alpha=0$. The curve has a specific shape, defined by the "slope parameter" G, and phase functions Φ_1 and Φ_2, which are described in reference [9].

I calculated the "reduced magnitude" and determined the Sun and Earth distances to the asteroid for each good observing run. Since the asteroid's brightness is changing as it rotates, I also used the light curve to compensate for the rotational phase at the time of the observation.

4.1 Results: My results are shown in Figure 11. They show the expected trend, but aren't sufficient to distinguish between two previously-reported G, H values (references [10] and [11]).

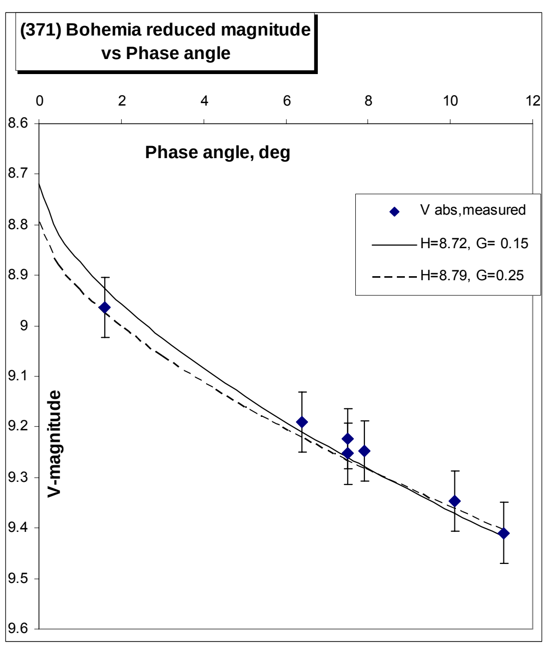

Fig 11: Reduced magnitude vs. phase angle, compared to reported G, H parameters

My data suffer from two weaknesses. First, the accuracy required for G, H determination is .03 magnitude or so. The need for this level of accuracy can be seen by plotting the predicted V_R for various values of G and H, as in Figure 12. The curves are only a few tenths of a magnitude apart in most places.

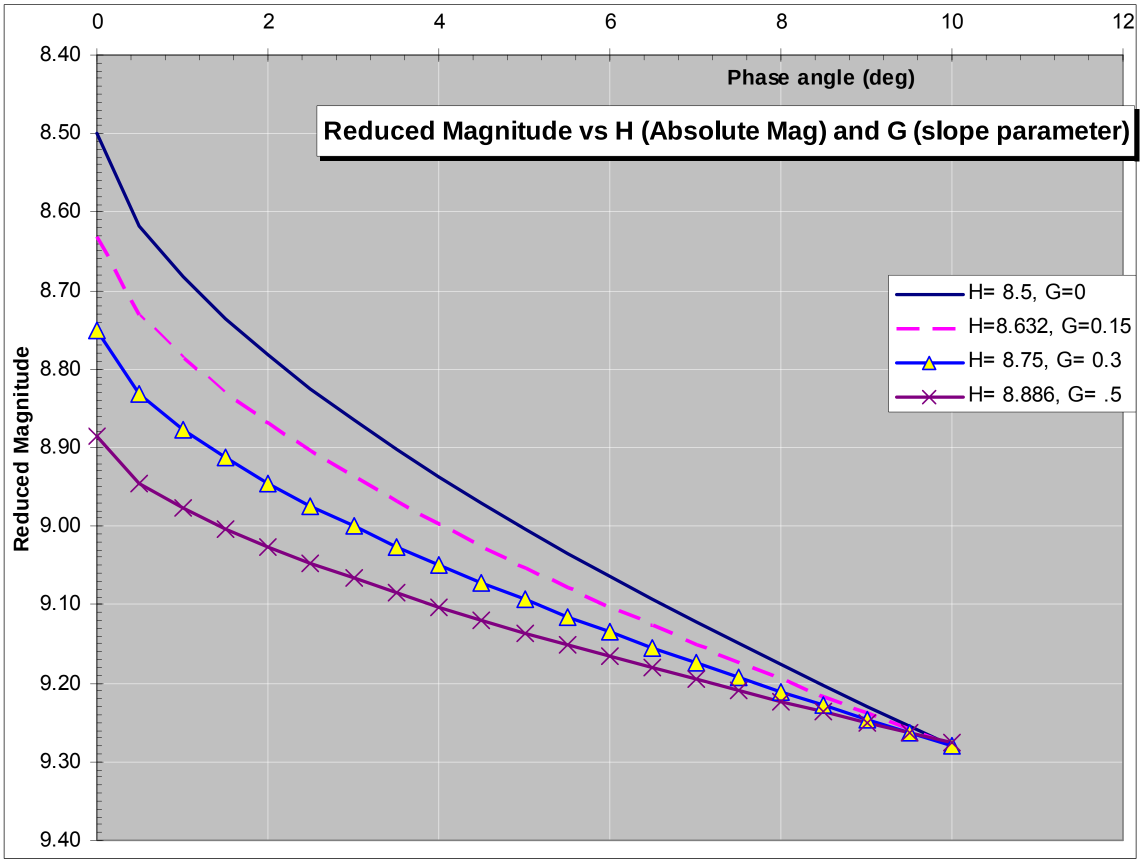

Fig 12: Photometry must be very accurate to distinguish between different G, H values

Second, I wasn't able to observe the asteroid at its minimum phase angle of α= 1.4 degree (it was raining that week). Observations at both large and very small phase angle are needed to reliably determine G. That requires some advanced planning. In order to get large-phase-angle data, you have to observe the asteroid long before it reaches opposition (or long after it passes opposition). For most asteroids, the maximum observable phase angle occurs when the asteroid is in quadrature, i.e. when it rises at about local midnight. Since it will take a couple of hours to rise to >30º altitude, that means getting your initial observations in the wee hours of the morning! For the 2003/4 apparition of Bohemia, the maximum achievable phase angle was 20 degrees, in late October 2003. As it worked out, my first good measurement wasn't made until late December 2003, at α= 12 degrees.

This is a project worth doing. Most of us have heard Brian Warner's lesson that there are thousands of asteroids in need of accurate lightcurves. That is even more true for slope parameter. Scan down the ASTORB or similar database, and you'll find that the majority of asteroids are listed as G=0.15. Most of those aren't well-measured values; G=0.15 is the "default" value that is assumed when there's insufficient data available.

4.2 Accuracy requirements for G, H Determination: Realizing that future projects to determine H and G are going to require greater emphasis on accuracy and repeatability of V magnitudes, I got to thinking about all of the factors that can confound this goal. One rainy night, I re-read the "propagation of errors" section in Henden and Kaitchuck (7) and dredged up some of the statistics that I learned in college. For those who have blissfully forgotten that class, here's the summary version.

The determination of V-magnitude is a function of many different parameters:

$$V = f(a, b, c \ldots n)$$

The general statistical theorem is that the variance of V is

$$\sigma^2_V = \sigma^2_a\, [\,\partial f/\partial a]^2 + \sigma^2_b\, [\,\partial f/\partial b]^2 + \ldots + \sigma^2_n\, [\,\partial f/\partial n]^2$$

where σ^2_a is the variance of parameter a, etc. The form of the function f is found as follows:

The exo-atmospheric instrumental magnitudes are:

$$v_0 = v - k'_v \cdot X \qquad r_0 = r - k'_r \cdot X$$

We determine the transforms and zero-points by measurement of standard stars, and then calculate the color index

$$(V\text{-}R) = T_{VR}(v_0 - r_0) + Z_{VR}$$

and the standard V-magnitude

$$V = v_o + T_{V,VR}(V\text{-}R) + Z_V$$

Substituting all of this into a single equation gives:

$$V = v - k'_v\, X + T_{V,VR}\, T_{VR}\, (v\text{-}r) + T_{V,VR}\, T_{VR}\, (k'_r - k'_v)X + T_{V,VR}\, Z_{VR} + Z_V$$

Of all the factors going into V, only the air mass (X) can be considered as a reliably-known value. The other factors are random variables affected by a variety of noise and other random errors.

The result of taking the partial derivatives, and evaluating the probable standard deviations of the various parameters, is summarized in Figure 13:

Parameter	Partial Derivative		Typ value $\lvert\partial V/\partial x\rvert \approx$		Standard Deviation	
instr mag	$\partial V/\partial v =$	$1 + T_{V,VR}\,T_{VR}$	1.05		$\sigma_v =$	$1/SNR + \sigma_S$
	$\partial V/\partial r =$	$-\,T_{V,VR}\,T_{VR}$	.05		$\sigma_r =$	$1/SNR + \sigma_S$
extinction	$\partial V/\partial k'_v =$	$-\,(1+T_{V,VR}\,T_{VR})X$	1.3		$\sigma_{k'v} =$	$2^{3/2}\,\sigma_v/(nm)^{1/2}$
	$\partial V/\partial k'_r =$	$T_{V,VR}\,T_{VR}\,X$	.07		$\sigma_{k'r} =$	$2^{3/2}\,\sigma_v/(nm)^{1/2}$
zero point	$\partial V/\partial Z_{VR} =$	$T_{V,VR}$	.05		$\sigma_{ZVR} =$	$\sigma_v/(n)^{1/2}$
	$\partial V/\partial Z_V =$	1	1		$\sigma_{ZV} =$	$\sigma_v/(n)^{1/2}$
transforms	$\partial V/\partial T_{V,VR} =$	$T_{VR}\,(v-r) +$ $T_{VR}\,(k'_r - k'_v)X + Z_{VR}$	1.2		$\sigma_{TV,VR} =$	see below
	$\partial V/\partial\, T_{VR} =$	$T_{V,VR}(v-r) +$ $T_{V,VR}(k'_r - k'_v)X$	.06		$\sigma_{TVR} =$	see below

Fig 13: Partial Derivatives and typical values for photometric accuracy

The partials show that the most important parameters are v-instrumental magnitude, v-extinction, v-zero point, and V-transform. These are the measurements that deserve the most attention, toward increasing their accuracy and reducing their variance. The partials associated with r-band instrumental magnitude, r-band extinction and (V-R) transform are negligible – they don't have much effect on the absolute accuracy.

The equations for Standard Deviation quantify the usual advice about how to improve the accuracy, e.g. get high SNR, and use multiple stars to determine extinction coefficients.

As noted above, determining H and G requires photometric accuracy of $\sigma_V \approx 0.03$ or better. My limited experience in this area is that when you're striving for accuracy of a few hundredths of a magnitude, everything about your instrument, imaging, and data reduction is a potentially significant factor! This raised several questions about the repeatability of my instrumentation (e.g. are flat-fields repeatable after motion of the filter wheel? Do transform coefficients change from night to night?).

<u>4.2.1 Scintillation-type errors:</u> The photometric accuracy of instrumental magnitude is often stated as $\sigma_v = 1/SNR$. This equation assumes that photon-counting statistics are the only error source, which isn't quite true. There is such a thing as atmospheric scintillation (probably not a factor for typical imaging conditions) and at least three other effects that can cause a scintillation-like noise: imperfect flat-fielding, imperfect tracking or guiding, and imperfect measuring-aperture placement. This "scintillation-like" noise is labeled σ_S on the table above. I've made some measurements indicating that "scintillation-like" noise in my set-up can get as large as $\sigma_S \approx .02$ magnitude.

Where is this fluctuation coming from? My best guess is that it's driven by a slight non-repeatability of the position of the filter wheel as I cycle it from v to b and r, and then back to v. My system has the inevitable dust-donuts, and (since I use an F/6.3 focal reducer) some vignetting. A typical flat-frame shows overall shading of about 7% peak-to-peak (due to vignetting), and the deepest dust donut has a depth of about 3%. These can be effectively compensated by normal flat-fielding methods (I use a light box, that I've tested against twilight-flats). However, if the filter wheel doesn't register to *exactly* the same position each time it is rotated, then the vignetting will be slightly different, and this difference will cause slight variations in the instrumental magnitude even after flat-fielding (since the filter position – and hence the vignetting – in the flat-field won't be *exactly* the same as the conditions during imaging).

I tested this theory by making a series of v-band flats, then r-band, then b-band, then v-band again. Call the first set of v-band flats "V-flat-1" and the final set "V-flat-2". There were a dozen images in the "V-flat-1" series. Taking the first 6 of these, median-combining, and then using them to "flat field" the second 6, gave a nearly-perfectly-uniform frame, as desired. Then, I took the median-combined "V-flat-1" series and used it to flat-field the "V-flat-2" series. The result was not-quite-perfect: there were subtle remnants of vignetting and hints of portions of the dust donuts. The resulting overall non-uniformity was about 2% (peak-to-peak) in this test. Since this correlates pretty well with the "scintillation-type" noise I've seen in some standard-star runs, I've assumed σ_S = .02 in the error analysis for my absolute magnitude results. The exact value isn't as important as the recognition that it exists. If it's critical to achieve 0.01 magnitude or better accuracy, then the filter wheel should be left untouched during the imaging session and flat-field exposures. For situations where the filter wheel must be moved during a night's session (e.g. monitoring for color changes as the asteroid rotates or binary star revolves), then the practical accuracy is going to be ≈ .02 - .03 magnitude, regardless of how high the signal-to-noise ratio is.

4.2.2 Transform and Zero Point accuracy: I guess unless you own a spectrophotometer, you'll never know exactly what the transforms of your system are. The best I could do was measure them frequently, and pay attention to the repeatability of the measurements. I measured the transform $T_{V,VR}$ on five separate occasions during the study of Bohemia, and the results were quite consistent, with a standard deviation of $\sigma_{TV,VR}$ = .02, which (using Figure 13) contributes about .02 magnitude standard deviation to the absolute magnitudes.

5. Conclusion

The advice given by several speakers at last year's IAPPP conference was well founded. The availability of nearly unlimited telescope time permits the amateur astronomer to conduct projects that would be difficult for a professional to justify, and today's high-quality and modestly-priced imaging equipment puts interesting projects within the reach of dedicated amateurs. However, in order to achieve the accuracy required, and be able to have confidence in your results, the amateur must spend a fair amount of time getting a quantitative understanding of the capabilities of his/her equipment. I hope that my experiences described here will encourage some other beginners, and be useful examples of the sorts of things to watch out for. There are a lot of asteroids and variable stars out there that need our attention!