Role of the nuclear surface on the reaction cross section for spherical nuclei

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Abstract-The Glauber model approach in the optical limit is used in a global analysis of proton-nucleus reaction cross sections. It has been extended to a large body of data with target mass $6 \leq A_T \leq 208$ and energies $30 \leq E_L \leq 800$ MeV. The density of the target is considered in Symmetrized-Fermi form. The finite range of the nucleon-nucleon amplitude is considered. The calculations are performed for modified Glauber model I (*i.e.* taking into account Coulomb field) and modified Glauber model II (*i.e.* taking into account Coulomb and nuclear fields) . The results are compared with the available experimental data.

Key words: *proton-nucleus scattering, reaction cross section, Glauber optical model.*

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1. Introduction:

The reaction cross section is one of the most important observables required to fully describe strong interactions. It's very useful for extracting fundamental information about the nuclear size and the density distributions of neutrons and protons in the nucleus. In particular, neutron halo nuclei have been found by measuring this type of cross section which is induced by radioactive beams. It also finds applications in various fields of research including shielding against heavy ions originating from, for example, space radiation and also against radiobiological effects resulting from clinical exposure. It finds application in another research area such as, radiobiology and space sciences. For carcinogenesis, terrestrial radiation therapy and radiobiological research, knowledge of beam compositions and interactions is necessary to properly evaluate the effects on human and animal tissues. For proper assessment of radiation exposures both reliable transport codes and accurate input parameter are needed. Therefore, it's not surprising to find the reaction cross section being a centre of attraction of experimental and theoretical studies for more than 50 years.

High energy proton- nucleus reactions are successfully treated in the framework of the Glauber model [1]. Even in the simplest cases in which some physical effects such as Coulomb effects, Fermi motion, Pauli blocking, *etc*, are ignored, yet, the Glauber approach provides reasonable agreements with the experimental data at high and intermediate energies.

provides reasonable agreements with the experimental data at high and intermediate energies. Microscopic calculations, based on the optical limit of the Glauber theory, have been done by Peng et al [2] to calculate both the total and the reaction cross sections for P, d, ^{3}He, α and ^{12}C on ^{12}C. They have suggested that effects such as the internal Fermi motion of the nucleons, Pauli-blocking and nuclear potential should be taken into consideration at low energy levels. Also they have found that the values of σ_E / σ_T (σ_E and σ_T are the elastic and total cross section) for P-^{12}C at low energies were anomaly high and could not be described by this style of calculations.

A simple expression, based on Glauber multiple scattering theory in its optical limit, has been presented by Abul-Magd *et al* [3] to calculate the total reaction cross section for both nucleon-nucleus and nucleus-nucleus scattering. Their formula satisfied a satisfactory description to the experimental data of both nucleon-nucleus and nucleus-nucleus scattering at energies above 100 MeV. They referred to the discrepancy, through the low energy range, as due to the physical effects which were not included in their calculations such as the internal Fermi-motion of the nucleons in the nuclei and the Pauli- blocking effects.

Most of the previous analyses using Glauber's theory assumed Gaussian density distributions for both the target and projectile. This leads to a considerable simplification in the calculations of the nuclear transparency. However, while the Gaussian distribution may provide a reasonable description for the density of light nuclei, certainly it will be invalid for intermediate mass and heavy nuclei. Concerning this point, Lukyanov *et al* [4] have derived an expression to calculate the eikonal phase shift function using the Symmetrized-Fermi function in the field of nucleus-nucleus collisions. This function has many advantages since it is even analytic function, it can be expanded in even powers of r. Also it resembles Gaussian function for light nuclei while for heavier ones it goes over to the Fermi distribution [5]. Based on this line, Lukyanov *et al* have derived analytic expressions for calculating the reaction cross section [6] and the elastic differential cross sections [7] for intermediate-energy heavy-ion reactions taking into account the effect of the Coulomb field. On the same basis, elastic and inelastic scattering amplitudes for heavy ion collisions are obtained taking into consideration the multi step effect of the 2^+ rotational state excitation including all orders of the deformation parameters [8,9]. Also in the field of proton-nucleus scattering some attempt has been executed to calculate the elastic and inelastic differential cross sections using the Symmetrized Woods-Saxon optical potential [10]. The eikonal phase shift function has been calculated analytically and the results give good agreement to the numerical calculations and the experimental data.

The aim of this work is to introduce a microscopic approach that starts from the optical limit of the Glauber scattering theory. This allows us to study the proton-nucleus reaction cross section. We use Lukyanov expression for deriving the optical thickness function which is embedded in the eikonal phase shift function in case of zero or finite ranges. Modified Glauber models I and II have been used also to execute the calculations. In addition, we derive a new form of the strong absorbing radius R_{sab} which is used to calculate the geometrical reaction cross section and to introduce generalized form for the reaction cross section in Glauber models I and II. The reaction cross section is calculated for P- ^{6}Li, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ^{27}Al, ^{40}Ca, ^{58}Ni and ^{208}Pb at energy range 30 - 800 MeV.

2. Formalism:

For a finite range interaction, the proton-nucleus reaction cross section in the framework of the standard Glauber model can be written as [3, 11-16]:

$$\sigma_R(mb) = 20\pi \int_0^{\infty} \{1 - \exp[-\chi(b)]\} b\,db \tag{1}$$

where the imaginary part of the eikonal phase shift $\chi(b)$ is given by:

$$\chi(b) = 0.1\bar{\sigma}_{NN} \int d^2 s \int_{-\infty}^{\infty} dz \rho(\underline{s}, z) f(\underline{s} - \underline{b}) \tag{2}$$

Here $\rho(\underline{s}, z)$ is the density distribution function of the nucleus. It's considered in the Symmetrized Fermi form i.e.

$$\rho(s,z) = \rho(0) \frac{\sinh(c/a)}{\cosh(c/a) + \cosh(s/a)} \tag{3}$$

where $r = \sqrt{s^2 + z^2}$. Integrating the density function over z produces the optical thickness function:

$$t(s) = \int_{-\infty}^{\infty} \rho(s,z) dz \tag{4}$$

Following the same procedure given by Lukyanov *et al* [4], we get:

$$t(s) = 2a\rho(0) u_{SF}(s) P(s) \tag{5}$$

where:

$$\rho(0) = \frac{3A_T}{4\pi c^3} \left[1 + \left(\frac{\pi a}{c} \right)^2 \right]^{-1}, \tag{6}$$

$$P(s) = \frac{1}{\sqrt{1-x}} \ln \frac{1 + \sqrt{1-x}}{1 - \sqrt{1-x}}, \tag{7}$$

$$x = \frac{2}{\kappa} \frac{1}{1 + \frac{\cosh(c/a)}{\cosh(s/a)}} \left[1 + \frac{\kappa - 1}{\cosh(s/a)} \right], \tag{8}$$

$$\kappa = e^{\delta} \quad \text{and} \quad \delta = 1.10315 + 0.34597(c/a) - 0.00446(c/a)^2 \tag{9}$$

Then equation (2) becomes:

$$\chi(b) = 0.1\bar{\sigma}_{NN} \int d^2 s [t(s) f(\underline{s} - \underline{b})] \tag{10}$$

The nucleon-nucleon range function $f(b)$ will be represented in Gaussian form:

$$f(b) = \frac{1}{\pi r_0^2} \exp\left(-b^2 / r_0^2\right) \tag{11}$$

where r_0 is the range parameter. Carrying out the integral over the angle in equation (10) we get:

$$\chi(b) = 0.1\bar{\sigma}_{NN} \int_0^\infty t(s) F(s,b) s ds \tag{12}$$

where:

$$F(s,b) = \frac{2}{r_0^2} \exp\left[-\left(s^2 + b^2\right)/r_0^2\right] I_0\left(2sb/r_0^2\right) \tag{13}$$

Also, we have calculated the reaction cross section taking into consideration the zero range approximation *i.e.*

$$\chi(b) = 0.1\bar{\sigma}_{NN} t(b) \tag{14}$$

where the thickness function $t(b)$ has been defined previously in equation (5).The average NN cross section, averaged over neutron and proton numbers, is given by [17]:

$$\bar{\sigma}_{NN} = \frac{Z_T}{A_T}\sigma_{pp} + \frac{N_T}{A_T}\sigma_{np} \tag{15}$$

where the nucleon-nucleon cross sections are[15]:

$$\sigma_{pp} = 13.73 - 15.04/\beta + 8.76/\beta^2 + 68.76\beta^4 \tag{16}$$

and

$$\sigma_{np} = -70.67 - 18.18/\beta + 25.26/\beta^2 + 113.85\beta \tag{17}$$

where β is the ratio between projectile velocity to light velocity:

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} \quad , \gamma = \frac{E_L}{931.5} + 1.0 \tag{18}$$

and E_L represents the incident energy in the laboratory frame. But expressions (12) and (14) don't account for the deviation in the eikonal trajectory due to the Coulomb field. We incorporate this deviation schematically by evaluating the imaginary part of the eikonal phase shift at the distance of the closest approach given by [15, 18-23]:

$$b \rightarrow b' = \frac{\eta}{k} + \sqrt{b^2 + \frac{\eta^2}{k^2}} \tag{19}$$

where k is the wave number and η is the Sommerfeld parameter, given by:

$$\eta = \frac{Z_T e^2}{\hbar \upsilon}, \tag{20}$$

$$k = \frac{m_T \sqrt{E_L (E_L + 2m)}}{\sqrt{(m + m_T)^2 + 2m_T E_L}} \quad ; \; m_T = A_T m \tag{21}$$

and

$$\upsilon = \frac{\sqrt{E_L (E_L + 2m)}}{E_L + m} \tag{22}$$

The above treatment for the impact parameter results what is called modified Glauber model I. Cha has suggested modified Glauber model II in which the impact parameter can be modified as [24]:

$$b \to b'' = b' - \frac{V_N(b')}{V'_{eff.}(b')} \tag{23}$$

where b'' is the distance of the closest approach in the presence of the nuclear and Coulomb fields. The quantity b'' is a solution of the equation:

$$1 - \frac{V_{eff}(r)}{E_{C.M.}} = 1 - \frac{2\eta}{kr} - \frac{L^2}{k^2 r^2} - \frac{V_N(r)}{E_{C.M.}} = 0 \tag{24}$$

where $V_N(r)$ is the real part of the Woods-Saxon optical potential, $E_{C.M.}$ is the kinetic energy in the center of mass system and L is the angular momentum.

The strong absorption radius is defined as, the radial distance where the transparency function $T(b) = \exp[-\chi(b)] = 0.5$. It has been introduced previously by Charagi *et al* [15] depending on that the density function has the Gaussian form. Thus, the applied integrations to evaluate the eikonal phase shift can be easily carried out analytically. Now, we try to evaluate the strong absorption radius in case of Symmetrized-Fermi density distribution function. In [25], Lukyanov *et al* have referred to that, in the center of a nucleus, expression (7) takes the value $P(0) = 1$, and in the region of the main contribution $s : c \to \infty$ it changes a little by $\cong 0.4(a/c)$. This enables one to take $P(s)$ at one point only. Then, if $\cosh(c/a) >> \kappa$ one obtains:

$$P_a(c) \cong \ln(4\kappa) = 2.48945 + 0.34597(c/a) - 0.00446(c/a)^2 \tag{25}$$

and consequently expression (5) can be simplified to:

$$t(s) = 2a\rho(0)\mu_{SF}(s) P_a(c) \tag{26}$$

Substituting in the eikonal phase shift for zero range approximation (eq. (14)), then:

$$\chi(b) = 0.2a\bar{\sigma}_{NN}\rho(0)P_a(c)\frac{\sinh(c/a)}{\cosh(c/a)+\cosh(b/a)} \tag{27}$$

Then the transparency function:

$$\exp\left[-0.2a\bar{\sigma}_{NN}\rho(0)P_a(c)\frac{\sinh(c/a)}{\cosh(c/a)+\cosh(b/a)}\right] = 0.5 \tag{28}$$

and finally the strong absorption radius takes the form:

$$R_{sab} = a\cosh^{-1}\left\{\frac{a\bar{\sigma}_{NN}\rho(0)\ln(4\kappa)\sinh(c/a)}{10\ln\left(\sqrt{2}\right)} - \cosh(c/a)\right\} \tag{29}$$

Firstly, to test the applicability of this new form, we have used it in calculating the geometrical reaction cross section:

$$\sigma_R^{Geom.}(mb) = 10\pi R_{sab}^2 \tag{30}$$

Also, we took into consideration the effect of Coulomb distortion in the straight line path of the impact parameter:

$$\sigma_R^{Geom.\&C.}(mb) = \left(1 - \frac{V_C}{E_{C.M.}}\right)\sigma_R^{Geom.} \tag{31}$$

We have compared between the obtained results and that obtained by Charagi et al [15] using Gaussian density distribution function. Finally, σ_R in the framework of modified Glauber models I, II making use of the new expression of R_{sab} are:

$$\sigma_R^{\mathrm{C}}(mb) = \left(1 - \frac{V_C}{E_{C.M.}}\right)\sigma_R \tag{32}$$

and

$$\sigma_R^{\mathrm{CN}}(mb) = \left(1 - \frac{V_C + V_N}{E_{C.M.}}\right)\sigma_R \tag{33}$$

respectively, where:

$$V_C = \frac{Z_T e^2}{R_{sab}} \tag{34}$$

and

$$V_N = \frac{-V_0}{1 + \exp\left[\left(R_{sab} - r_V A_T^{1/3}\right)/a_V\right]} \tag{35}$$

3. Results and Discussion:-

In the present analysis, the reaction cross section for the elastic scattering of protons on different nuclei (^{6}Li, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ^{27}Al, ^{40}Ca, ^{58}Ni and ^{208}Pb) at different bombarding energies has been calculated using the Glauber optical model and its modifications. We took into consideration the finite range and zero range interactions. The calculations have been carried out using Symmetrized-Fermi density distribution function. Analytical expression for the optical thickness function has been obtained, as shown in the previous section. The calculations have been performed using the parameters of nuclear Symmetrized-Fermi density distributions which are listed in table (1).

Table1. Parameters of the of nuclear Symmetrized-Fermi density distributions [25]

Nucleus	c, fm	a, fm	R_{rms}, fm
$^{6}_{3}Li$	1.364	0.620	2.535
$^{12}_{6}C$	2.214	0.488	2.496
$^{16}_{8}O$	2.562	0.497	2.711
$^{20}_{10}Ne$	2.74	0.572	3.004
$^{24}_{12}Mg$	2.934	0.569	3.105
$^{27}_{13}Al$	3.07	0.519	3.06
$^{40}_{20}Ca$	3.556	0.578	3.493
$^{58}_{28}Ni$	4.153	0.566	3.844
$^{208}_{82}Pb$	6.557	0.515	5.427

In addition, from the analysis of the Symmetrized-Fermi function, we have obtained a new form of the strong absorption radius R_{sab} which is used to introduce a generalized form of σ_R in modified Glauber models I, II. We have calculated the geometrical reaction cross section σ_R^{Geom} using the new form of R_{sab}. It's favorable to mention that we have compared the results with that obtained by relations in [15] using Gaussian density distribution function. Also, we have taken into consideration the effect of Coulomb distortion. We have compared the results with the available experimental data.

In table (2), we have presented the geometrical reaction cross section calculations. The 1st column represents the target nuclei and the 2nd column represents the energy of the incident beam of protons. In the 3rd column, σ_R^{Geom}, we have used the Symmetrized- Fermi density distribution function (*eq.* (30)). The 4th column is the same as the 3rd column except the Coulomb distortion has been included (*eq.* (31)). The 5th column is the geometrical reaction cross section using Gaussian function instead of Symmetrized - Fermi function.

Table2. Results of the geometrical reaction cross section

Target	E_L (MeV)	Calculations using S.F. density distribution function		Calculations using Gaussian density distribution function		$\sigma_R^{exp.}$
		$\sigma_R^{Geom.}$	$\sigma_R^{Geom.\&C}$	$\sigma_R^{Geom.}$	$\sigma_R^{Geom.\&C}$	
${}^{6}_{3}Li$	30	324.2406	307.2848	316.0525	299.3121	355±8
${}^{12}_{6}C$	30	415.7552	380.0979	381.2355	347.0906	447±20
	40	377.2234	351.7498	345.0025	320.6411	371±11
	50	347.9492	328.377	316.8551	298.1779	345±13
	65	314.4965	300.183	284.1658	270.5601	295±7
	800	240.2874	239.2709	211.1219	210.169	262±13
${}^{16}_{8}O$	30	504.3844	453.0254	483.4611	433.1787	499±20
	40	461.0547	424.2271	438.6857	402.7627	453±15
	65	390.1034	369.2569	363.5061	343.3828	365±15
${}^{24}_{12}Mg$	30	684.7048	596.7059	622.7316	538.8094	724±11
	40	599.5969	537.8355	570.0226	509.8036	645±11
${}^{40}_{20}Ca$	30	909.1896	742.8878	852.1874	691.1831	880±26
	40	842.237	722.1907	788.8578	672.678	807±24
	65	732.4616	663.5694	682.5247	616.0223	688±17
${}^{58}_{28}Ni$	30	1116.647	860.5789	1029.479	783.6085	1011±30
	40	1043.913	858.2216	960.0453	781.9694	955±34
	50	988.1593	843.6276	906.084	767.6848	856±29
	65	923.9282	816.4241	843.3807	740.6695	904.9±23.5
${}^{208}_{82}Pb$	30	2186.156	1149.693	2186.897	1150.259	2119±90
	40	2093.713	1332.979	2083.924	1324.971	2033±100
	50	2022.21	1424.105	2003.747	1408.379	1842±93
	65	1939.112	1488.583	1910.342	1463.168	2020±60

The 6th column is the same as the 5th column except including the Coulomb distortion effect. The 7th column represents the available experimental data. The experimental data are taken from [26-28]. We should note that, in the field of heavy ion collision, Charagi *et al* [3] have referred to that σ_R^{Geom} obtained using Gaussian function agrees with the reaction cross section calculated analytically in the framework of Glauber optical model within 3 percent. From the results tabulated in table (2), we can notice the following:-

(i) As expected, the Coulomb distortion effect decreases the value of σ_R^{Geom} in case of either Gaussian or Symmetrized-Fermi functions.

(ii) On the other side, the results, obtained using Symmetrized-Fermi function, are larger than that obtained using Gaussian function.

(iii) In general, the geometrical reaction cross section as a function of R_{sab} provides a good measure of the reaction cross section for Gaussian and Symmetrized-Fermi functions. This effective surface becomes smaller as the energy increases or in semi classical terms, the nuclear surface becomes more transparent.

Table3. *The reaction cross section for P- ^{6}Li, ^{12}C, ^{16}O and ^{20}Ne interactions at different energies*

Target	E_L (MeV)	$\sigma_{R(0)}$ (mb)	$\sigma^{C}_{R(0)}$ (mb)	$\sigma^{C}_{R(0)}$ (mb) $\_R_{sab}$	$\sigma^{CN}_{R(0)}$ (mb)	$\sigma^{CN}_{R(0)}$ (mb) $\_R_{sab}$	$\sigma_{R(r_0)}$ (mb)	$\sigma^{C}_{R(r_0)}$ (mb)	$\sigma^{C}_{R(r_0)}$ (mb) $\_R_{sab}$	$\sigma^{CN}_{R(r_0)}$ (mb)	$\sigma^{CN}_{R(r_0)}$ (mb) $\_R_{sab}$	σ^{exp}_{R}
$^{6}_{3}Li$	30	353.169	334.5629	340.4739			447.957	423.4374	431.8606			355±8
	144	150.350	147.591	148.898	146.0229	146.9393	368.429	360.2289	364.882	359.744	360.081	
$^{12}_{6}C$	30	436.413	398.813	407.980	395.3393	382.8774	552.413	510.2365	516.202	507.813	484.432	447±20
	40	397.002	369.740	375.919	365.7503	355.9491	468.301	440.1224	444.611	436.829	419.875	371±11
	50	366.860	345.612	351.594	341.7419	334.0189	417.463	396.0918	400.092	392.629	380.092	345±13
	65	332.043	316.1968	321.0528			369.109	353.381	356.8925			295±7
	144	240.194	233.852	236.255	230.6633	232.7846	445.098	430.8351	437.798	429.599	431.367	
	398	219.030	217.137	217.701	216.406	217.5339	280.823	278.7346	279.118	278.151	278.905	
	800	251.816	250.831	251.082	249.6677	251.0578	270.612	269.2405	269.822	268.145	269.749	262 ± 13
$^{16}_{8}O$	30	526.605	472.8217	485.1225			649.919	591.3679	598.7225			499±20
	40	482.348	443.2907	452.9063			559.992	519.2849	525.9705			453±15
	65	409.003	386.242	392.731	380.7255	375.1652	449.472	426.8047	431.59	421.687	412.286	365±15
	200	277.923	271.668	273.917	268.0871	270.5775	481.979	476.9977	475.032	464.598	469.239	
	800	316.967	315.415	315.862	314.6606	315.8401	336.597	334.5831	335.418	333.876	335.395	
$^{20}_{10}Ne$	65	504.840	473.780	482.653	467.9578	466.5383	544.787	514.0125	520.845	508.564	503.454	
	800	390.403	388.235	388.896	387.9574	388.8792	409.833	407.0996	408.251	406.833	408.233	

***(Continue)Table3**. The same as table (1) but for P- ^{24}Mg, ^{27}Al, ^{40}Ca, ^{58}Ni and ^{208}Pb interactions.*

Target	E_L (MeV)	$\sigma_{R(0)}$ (mb)	$\sigma^{C}_{R(0)}$ (mb)	$\sigma^{C}_{R(0)}$ (mb) _R_{sab}	$\sigma^{CN}_{R(0)}$ (mb)	$\sigma^{CN}_{R(0)}$ (mb) _R_{sab}	$\sigma_{R(r_0)}$ (mb)	$\sigma^{C}_{R(r_0)}$ (mb)	$\sigma^{C}_{R(r_0)}$ (mb) _R_{sab}	$\sigma^{CN}_{R(r_0)}$ (mb)	$\sigma^{CN}_{R(r_0)}$ (mb) _R_{sab}	σ^{exp}_{R}
$^{24}_{12}Mg$	30	714.262	622.590	642.924			843.909	745.9269	759.632			724±11
	40	655.787	589.258	605.059			738.378	669.551	681.431			645±11
	65	559.444	520.7239	531.261	514.887	516.6042	602.996	654.356	577.619	558.9367	556.8212	
	800	440.0091	437.1486	438.0691	435.7918	438.0092	460.8174	457.3537	458.786	456.0581	458.723	
$^{27}_{13}Al$	30	710.2632	611.7167	632.2056			853.845	747.3188	760.0078			709±18
	40	656.9362	585.1951	601.1448			749.3842	674.4244	685.9286			645±35
	100	494.5932	468.4649	476.0518			553.3311	601.6419	607.9208			430±12
$^{40}_{20}Ca$	30	941.9236	769.9393	804.353			1094.385	910.83	934.5588			880±26
	40	873.8078	748.5885	775.2689			972.3301	842.082	862.8773			807±24
	65	761.2494	687.9355	705.6599	679.7684	683.4601	814.892	740.8932	755.3853	733.3762	731.6212	688±17
$^{58}_{28}Ni$	30	1150.587	886.7407	934.8205			1326.056	1044.795	1077.397			1011±30
	40	1076.602	883.841	921.1514			1191.0070	990.5255	1019.257			955±34
	50	1019.653	868.5287	899.3775			1103.731	948.5986	973.5376			856±29
	65	953.613	840.0392	864.8977	832.111	841.9404	1017.14	902.0065	922.5147	894.8315	898.028	905±24
	800	795.3549	785.5806	788.965	782.5825	788.8513	826.2553	819.9568	819.6171	812.9302	819.5	
$^{208}_{82}Pb$	30	2221.6330	1165.667	1301.254			2513.316	1385.935	1472.14			2119±90
	40	2127.705	1349.491	1454.444			2320.792	1507.879	1586.744			2033±100
	50	2057.086	1440.795	1528.775			2196.479	1562.128	1632.369			1842±93
	65	1974.295	1504.125	1577.517	1490.78	1483.687	2079.603	1597.346	1661.661	1585.952	1562.826	2020±60

In table (3), the reaction cross section has been calculated within the Glauber optical limit. The 1st column refers to the target nuclei and the 2nd column refers to the energies of the incident protons. The 3rd, 4th, 5th, 6th and 7th columns represent the reaction cross section taking into account the zero range approximation. $\sigma_{R(0)}$ in the 3rd column is the reaction cross section in the Glauber optical limit (*eq.'s* (1) and (14)). The 4th and 5th columns represent the reaction cross section results in modified Glauber model I but $\sigma_{R(0)}$ in the 4th column is calculated using *eq.'s* (1), (14) and (19) while $\sigma_{R(0)}\_R_{sab}$ is the generalized reaction cross section as a function of the new form of R_{sab} (*eq.'s* (33), (1), (14), (34) and (29)). Also, in the 6th and 7th columns, the results have been performed using zero range approximation in modified model II. $\sigma_{R(0)}^{CN}$ in the 6th column is calculated using eq.'s (1), (14) and (23) while $\sigma_{R(0)}^{CN}\_R_{sab}$ in the 7th column is calculated using eq. *'s* (33), (34), (35) and (29).Parameters of the optical model potentials are taken from [29-39]. In the 8th column $\sigma_{R(r_0)}$ is the reaction cross section in the Glauber optical limit but we have taken into account the finite range effect. The results in 9th and 10th columns are the same as that in 8th except the Coulomb distortion effect has been included. $\sigma_{R(r_0)}^{C}$ has been calculated using *eq.'s* (1), (12) and (19) while $\sigma_{R(r_0)}^{C}\_R_{sab}$ has been calculated using *eq.'s* (32), (1), (34) and (29). The 11th and 12th columns are the same as 9th and 10th columns respectively but in modified Glauber optical model II. $\sigma_{R(r_0)}^{CN}$ in 11th column is calculated using *eq.'s* (1), (13) and (23). $\sigma_{R(r_0)}^{CN}\_R_{sab}$ in the 12th columns is calculated using *eq.'s* (33), (1), (34), (35) and (29). It should be noted that the spaces in table (3) are due to the deficiency of the available parameters of the optical model potential. The input parameters of the real nuclear optical potential are listed in table (3). The last column represents the available experimental data for the corresponding listed interactions.

Generally, it is clear that the values of $\sigma_{R(r_0)}$ are higher than $\sigma_{R(0)}$ and the corresponding experimental data. We can also notice that the Coulomb distortion effect decreases the value of σ_R either in case of finite or zero ranges. Also, the nuclear distortion effect in modified Glauber model II decrease the values of σ_R slightly. The generalized form of the reaction cross section (*e.q.'s* (32) and (33)), in case of finite or zero ranges, increases the results than calculated normally (*i.e.* b' or b'' in $\chi(b)$). The agreement between finite and zero range results is improved as the energy increases and it's clear in P-^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg and ^{58}Ni reactions at 800 MeV. As shown in the table, the values of $\sigma_{R(0)}$ and $\sigma_{R(0)}^{C}$, either as a function of R_{sab} or not, are in reasonable agreement with the experimental data. This conclusion can be realized obviously for small energy ranges. It's contrary to the role that the finite range improves the results to the experimental data. This reason is due to the deficiency of the Glauber optical model at small energy ranges.

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