Newtonian Cosmology

Lecture notes from the First Tah Poe School on Cosmology: Introductory Cosmology (**TPCosmo I**), *1-8*

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Abstract

The aim of these lecture notes is to introduce elementary ideas of how to get cosmological dynamical equations by using Newtonian Dynamics. Newtonian dynamics approach can amazingly give main equations of cosmology e.g. Friedmann equation, fluid equation and acceleration equation. However to have the complete cosmology we need general relativity which is not in the interest of these notes. Discussion of dynamics of the universe filled with dust(non-relativistic particles) and radiation (relativistic particles) will be included here.

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1 Introduction

THE SKY IS DARK AND THE UNIVERSE IS BIG!

This could sound like an elegant beginning of this lecture. In fact human has been questioned this for thousand years. Cosmology could be one of the oldest subject of the world. It was strongly believed to be one part of astronomy until the past few decades. Around 1920s, Einstein 's general theory of relativity became the solid ground base for studying the universe. Later, the development of the big bang theory made particle physics (high energy physics) necessary for its development. To get the main dynamical equations of cosmology, fortunately, we don't really need to know general relativity but using Newton's theory of gravity is adequate for obtaining the major equations.

Newton tried to avoid the problem of finite universe. The problem of finite universe with Newtonian gravity states that if the universe has finite size, there must be the single centre of mass of the matter distribution and this will lead to falling of matter into the very huge mass sphere. Instead, he favored the idea of infinite universe that can allow his theory of gravity. If the universe is infinite, there would be no centre of mass to give the formation of the huge mass. He wrote in the second edition of his Principia that *The fixed stars, being equally spread out in all points of heavens, cancel out their mutual pulls by opposite attractions.* In this idea, the universe becomes static and only on smaller scale that the gravity can lead to the forming of stars and smaller objects.

1.1 Problems with Newton 's static universe

Newton's static infinite universe has some problems. One is the dark night sky riddle first asked by Thomas Digges in 1576 which is known today as Olbers' paradox (Olbers in 1826). Another one (which is similar) is the question asked by Edmund Halley in 1721. Olbers' paradox questions that *why the night sky is dark?* In figure 1, we are the observers standing at the centre of the imaginary sphere. r is the distance from us in the direction of radius of sphere. Each shell with thickness dr occupies the volume $4\pi r^2 dr$

Luminosity of stars L is proportional to number of stars N and number of stars is

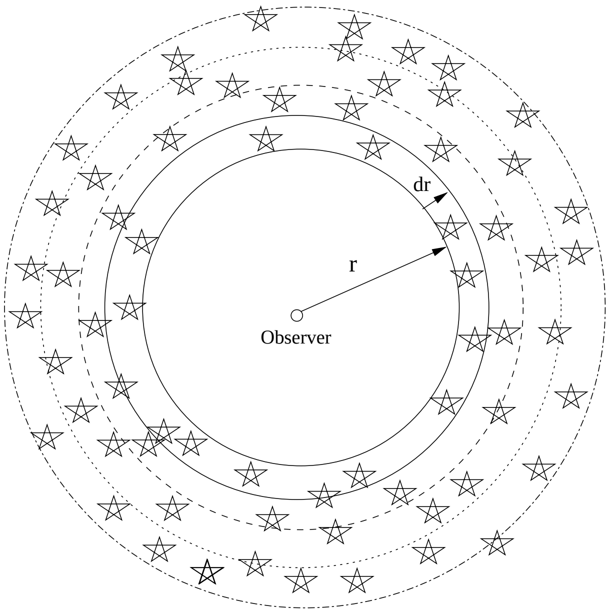


Figure 1: Observer's sphere with infinite sky

proportional to volume of shell. The flux light density is then

$$Flux = \frac{L}{4\pi r^2} \propto \frac{4\pi r^2 dr}{4\pi r^2} \propto dr \tag{1}$$

If $dr \to \infty$ then light we see in night sky should have infinite intensity. The other (similar) problem is from Halley. Any points in the uniformly star-distributed universe can have concentric sphere about them. As in figure 1, number of stars in each shell is proportional to r^2. The universe is isotropic about the point then the attraction force from stars in one side must be equal to the attraction force from stars in the opposite side. If we add up these shell more and more to infinite, the force from each side will get stronger and stronger to infinite but the total force is still zero. To keep the universe static and balance like this, very exact isotropy must be imposed, otherwise unbalance force could happen and pulls tremendously the matter toward one particular side! By this assumption, the net cosmic force is zero and the local motions are from only the local force.

1.2 The attempts to solve Newtonian static universe problem

- Johann Zllner (1872) suggested that space is spherical curve and universe is finite in extent. Johann was inspired by the work of Riemann on curved space.

- Carl Neumann and Hugo von Seeliger(late 19th century) tried to escape from the arising of the infinite unbalanced force. To do this, they proposed that gravity at large distance decrease faster than $1/r^2$ and they modified Newton's gravitational law to $1/r^{2+\epsilon}$. By this way, the force at cosmic distance decreases very fast and becomes negligible. Therefore the unbalanced force is also negligible.

-Charles Charlier (1908 and 1922) suggested the hierarchical structure of the infinite universe. In this scenario, the density of universe decreases when averaging over larger and larger region of the universe. So at infinite distance, the density could go to zero. At local scales gravity dominates the universe but at the large scale gravity become weaker and weaker. This idea can explain why the night sky is dark and also the problem of unbalanced infinite cosmic force.

At the time of these two riddles, gravity was thought to be simultaneously action. This means when one particle changes its position, other particles in the universe can feel this change instantaneously. In the early of the last century, Einstein proposed general theory of relativity. In his theory, light has finite speed and so does gravitational interaction. If we just add the idea of gravity propagating at the speed of light to the Newtonian theory in the finite age (let 's say 10 billion years), we don't feel the force from the distance beyond 10 billion light years and the force we can feel from that one side of the universe is very very tiny compare to the Earth's gravity at its surface. The hierarchical structure then can't solve anything for the unbalanced force problem because the force is so tiny and we don't need the concept of hierarchical structure. Also the Olbers's paradox is no longer a question since the light from beyond 10 billion light years has not reached us yet [2].

The real reason that we can't rely on Newtonian theory to answer these two questions is from the mathematical basis that is called the Dirichlet problem in the potential theory. To determine the force (size and direction), we need to know the boundary conditions. In the universe with uniform, finite and bounded distribution of matter, we can determine gravitational force at each point by boundary conditions. But in Newton's infinite, unbounded(edgeless) and centerless universe, the gravitational force becomes indeterminate. This will be discussed later on.

1.3 The non-static universe

As mentioned before, we can obtain cosmology similar to relativistic treatment from Newtonian gravity but however we must remember that Newtonian theory is not a complete theory for cosmology. Indeed Newton's theory is a special case of general relativity and general relativity predicts non-static universe. Here, to simplify context of the subject and to make it easier for the beginners, we will just add the idea of the expansion of the universe (discovered in the 1929 by Hubble [3] and [4]) to the Newtonian theory to get the dynamical characters of the universe.

Between 1920s to early 1930s, many theoreticians had been trying to apply general relativity to the universe. In the unbound (either infinite or finite) universe, according to general relativity, they found that the universe is non-static. Here is the order of events:

-1917 William de Sitter(Holland)
-1922 Alexander Friedmann (Russia)
-1927 Georges Lamaǐtre(Belgium)
-1927 to 1929 Arther Eddington(England), Howard Robertson(USA) and Richard Tolman(USA)
-1929 Edwin Hubble(USA).

In 1934 the subject was amazingly simplified by Edward Milne (England) and William McCrea (England) [5]. They discovered that the equation governing dynamics of the universe that has been derived from relativity can be also obtained from Newtonian gravity. Indeed, Newtonian theory is just approximation case. Why we can also get exactly the same equations from Newtonian gravity will be explained later.

2 Newtonian Gravity

2.1 Newton's Law of Gravity

According to Newtonian gravity, the gravitational force is

$$\mathbf{F} = \frac{GM}{r^2} m\hat{r} \tag{2}$$

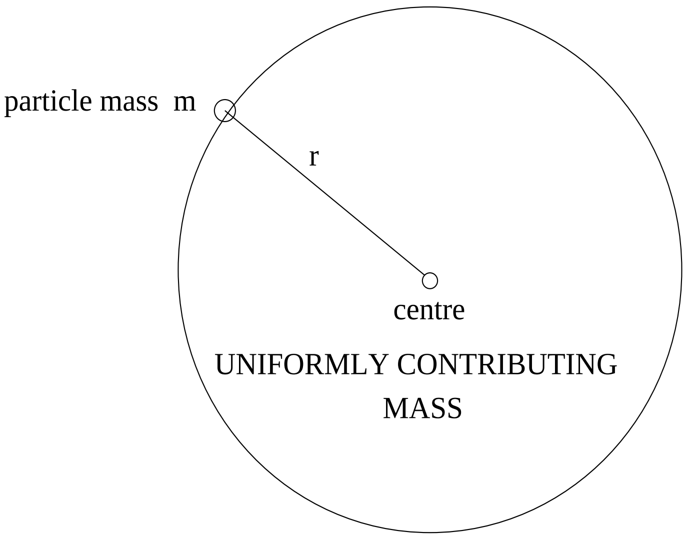


Figure 2: Sphere of uniformly distributing mass

where $\mathbf{F}$ is force from mass M acting on mass m, G is the gravitational constant, r is the distance between these two masses and $\hat{r}$ is a unit vector pointing from m to M. The force is always attractive and $\frac{GM}{r^2}\hat{r}$ is the gravitational field $\mathbf{g}$.

The potential energy is given by

$$P.E. = -\frac{GMm}{r} \tag{3}$$

Potential function is

$$\Phi = -\frac{GM}{r} \tag{4}$$

In the spherical symmetric and uniform distributed matter (see figure 2), similar to what happens in electrostatics, the potential is constant within the sphere due to the symmetrical property. Particles in this sphere feel no force according to

$$\mathbf{g} = -\nabla\Phi \tag{5}$$

The mass outside the sphere feels the force as if all mass of the sphere locates at the center and this force is independent of the radius of sphere R.

2.2 Homogeneity and Isotropy

Homogeneity of the universe means that the universe has the same property at any particular regions i.e. it looks the same at each point. **Isotropy** means that the universe looks

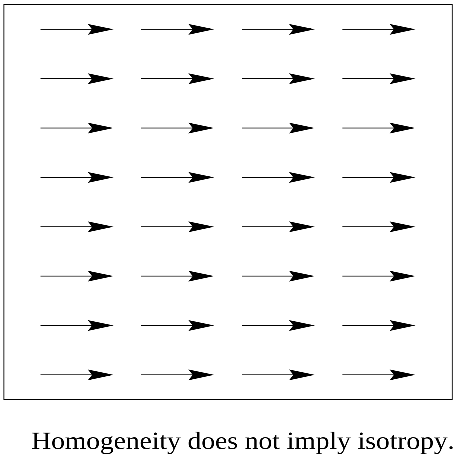


Figure 3: Homogenous vector field fails to hold isotropy

the same in all directions. These two properties are called the **cosmological principle**.

Figure 3 shows the homogenous of a vector field but obviously it can not imply isotropy since it look different from all directions. Figure 4 shows the isotropy about one point but it fails to include homogeneity. These two properties are therefore separated and we need both of them for the cosmological principle.

In the picture used by Milne and McCrea, the sphere of mass is expanding with a velocity $v = \dot{R(t)}$ where $\dot{R(t)} = \frac{dR(t)}{dt}$. The sphere keeps being homogenous and isotropic (as viewed from center of the sphere) during the expansion.

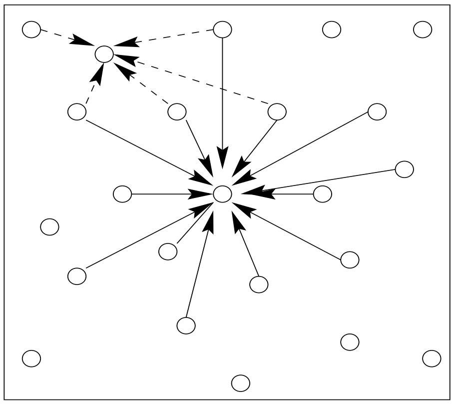
Let us look at the energy conservation law of a particle of mass m at the surface of this sphere. Here total mass of our sphere is ρV where ρ is mass density and volume $V = \frac{4}{3}\pi R^3$. Then the total energy of this particle is

$$\begin{aligned} U &= \frac{1}{2}m\dot{R}^2 + P.E. \\ &= \frac{1}{2}m\dot{R}^2 - \frac{4}{3}G\rho\pi R^2 m \end{aligned}$$

multiply both sides with $2/(mR^2)$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho + \frac{2U}{mR^2} \quad (6)$$

This equation is just another form of energy conservation law.



Isotropy does not imply homogeneity.

Figure 4: Isotropy does not imply homogeneity

3 Hubble 's law

In 1920s to 1930s, we began to realise that the distant nebulae are in fact other galaxies out of our galaxy. Our Milky way galaxy is not alone and the universe is much more bigger than people at that time thought. In 1929, Edwin Hubble found the expansion of the universe by studying galaxies' redshifts. The law that governs this expansion is called *Hubble 's law.* This law states that the further galaxy is, the faster it moves away from us. The law is found empirically from observation and it is

$$\mathbf{v} = H_0\mathbf{R} \tag{7}$$

where H_0 is **Hubble constant**. The law is not exact since at the smaller scale the universe is neither completely homogenous nor isotropic and there is also the peculiar velocities from local gravitational force.

The expansion of the universe leads us to the concept of expansion of space. Space *itself* is expanding and we need to introduce the new coordinate so-called **comoving coordinates** $\mathbf{x}$. This coordinate is moving along with the expansion of space. The **physical coordinate** $\mathbf{R}$ is given by

$$\mathbf{R} = a(t)\mathbf{x} \tag{8}$$

where $a(t)$ is **scale factor**. Here we assume homogeneity and isotropy of space to make $a(t)$ become function of time only. Figure 5 shows the expanding coordinate expand in 1 and 2 dimensions.

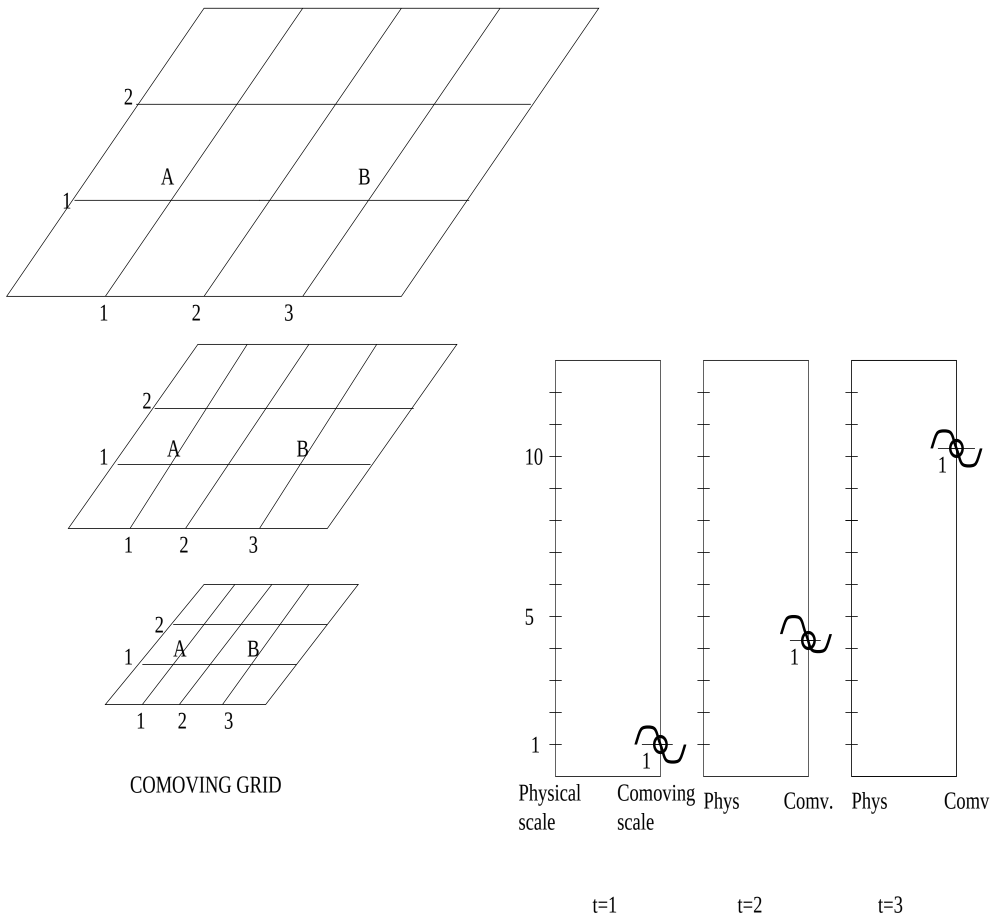


Figure 5: Expanding comoving grid and comving galaxy on a physical ruler

The recession velocity of galaxies $v(t)$ is on the same direction of the displacement from us $R(t)$ and is given by

$$\begin{aligned} \mathbf{v} &= \dot{\mathbf{R}} \\ &= |\dot{\mathbf{R}}|\frac{\mathbf{R}}{|\mathbf{R}|} \end{aligned} \tag{9}$$

Use equation (7), then

$$\begin{aligned} \mathbf{v} &= \frac{\dot{a}}{a}\mathbf{R} \\ &= H\mathbf{R} \end{aligned} \tag{10}$$

where

$$\boxed{H = \frac{\dot{a}}{a}} \tag{11}$$

is **Hubble parameter**. What different between H_0 and H is that H_0 is H at present time and it is historically called a constant because it looks like a constant of proportion in the Hubble law. It is indeed not a constant with time.

There is a subtle question about why are not ourselves expanding? Why does not the distance between the sun and the earth increase. The answer is that the expanding has a dominant effect on large scale (inter-galactic scale). At the scale smaller than this scale we hardly see any effect since it is dominated by gravity.

As we see the distant galaxy moving away from us very fast, can it go away faster than the speed of light? Yes, it can but how is about special relativity? This expansion does not violate special relativity since special relativity discusses about the speed of objects that just passing one another. That speed can not exceed the speed of light. Here it is the distant cosmic expansion of space, not the two objects passing each other.

4 Friedmann Equation

Now we will include the idea of expansion to the energy conservation by pluging $R(t) = a(t)x$ into equation (6), we obtain

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{2U/mx^2}{a^2} \tag{12}$$

$\frac{2U}{mx^2}$ is a constant. When space expands, x value remains the same since it *co moves* with the space also the total energy U still remains constant. Let us define kc^2 as $-2U/(mx^2)$. Then the equation becomes

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}} \tag{13}$$

where c is the speed of light. k is the **curvature of space** and it also denotes the **geometry of the universe**.This equation is known as **Friedmann equation**.

The universe is assumed to be homogenous, therefore the equations we have here can be applied to any regions of the universe. Friedmann equation is the core equation for cosmology. It describes how the scale factor a evolves in relation to the density ρ and curvature k of the universe.

5 Fluid Equation

The fluid equation bases on the idea that matter and radiation of the universe behave like perfect fluid e.g. no viscosity and heat flow. We shall start from looking at the first law of thermodynamics

$$\begin{aligned} dU &= TdS + dW \\ &= TdS - PdV \end{aligned} \tag{14}$$

where S, T, W, P are entropy, temperature, work done and pressure consequently. This law is also energy conservation law. Here we assume that the equation of state is

$$P = P(\rho) \tag{15}$$

That is to say P is the explicit function of ρ only and we assume that there are no other external forces. Matters here we consider in the universe are either dust or radiation. Dust is the term for non-relativistic matter e.g. particles with small speed compared to the speed of light. Other species are relativistic particles which move with the speed closed to that of light. Dust usually means baryons, electrons etc. and radiation here means photons, neutrinos. Here we are not interested in other forms of matter like scalar fields or other exotic particles. For dust $P = 0$ and for radiation $P = \rho c^2/3$. dS vanishes since reversible expansion is also assumed here. dV is $d(\frac{4}{3}\pi R^3) = 4\pi R^2 dR$. Equation (14) finally becomes just

$$dU = -P4\pi R^2 dR \tag{16}$$

The total relativistic energy of particles in sphere is

$$\begin{aligned} U &= Mc^2 \\ &= \rho V c^2 \\ dU &= 4\pi R^2 \rho c^2 dR + \frac{4}{3}\pi R^3 c^2 d\rho \end{aligned} \tag{17}$$

If we differentiate equation (16) with respect to t, equation (17) becomes

$$-P4\pi R^2 \dot{R} = 4\pi R^2 \rho c^2 \dot{R} + \frac{4}{3}\pi R^3 c^2 \dot{\rho} \tag{18}$$

Rearrange terms in this equation and write it in the terms of scale factor $a(t)$, we finally obtain **fluid equation**

$$\boxed{\dot{\rho} + 3\frac{\dot{a}}{a}\Big(\rho + \frac{P}{c^2}\Big) \;=\; 0} \tag{19}$$

Equation of state (15) can be rewritten as

$$P \;=\; \rho c^2 w \tag{20}$$

where $w = 0$ and $1/3$ for dust and radiation. Fluid equation is then simply

$$\boxed{\dot{\rho} + 3H\rho\Big(1 + w\Big) \;=\; 0} \tag{21}$$

Fluid equation is in fact energy conservation law. The first term $\dot{\rho}$ tell us how fast density changes (e.g. dilutes) and the second term is the lost of kinetic energy from fluid into gravitational potential energy.

6 Acceleration equation

Acceleration equation is important. It will tell us how the rate of expansion of the universe changes e.g. slower down or faster up. The equation is in fact a mixture of Friedmann and fluid equation and these two equations are in fact energy conservation law in mechanics and thermodynamics! After differentiate Friedmann equation with respect to time and pluging in fluid equation, we finally obtain

$$\boxed{\frac{\ddot{a}}{a} \;=\; -\frac{4\pi G}{3}\Big(\rho + \frac{3P}{c^2}\Big)} \tag{22}$$

The good feature of acceleration equation is that it does not contain k and we can use this equation regardless of the geometry of the universe. From the equation it seems that universe is decelerated. But if the pressure P has negative value, it could make $\ddot{a}$ positive and will accelerate the universe. [1] We will not consider this issue here since it is not in the scope of our lectures.

[1]Indeed the recent observation from Type Ia Supernovae strongly supports that universe now is accelerating! [6][7][8][9]. From view points of high energy physics we can have scalar field which can give negative pressure. This brings possible explaination to the observation.

7 Solutions of equations

We are now looking for the solution for the above equation we get. We can simply assume that $k = 0$ then Friedmann equation is [2]

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \tag{23}$$

For more discussion, please see [1].

7.1 The dust-filled universe

Dust (non-relativistic particles) is pressureless in the isotropic universe. Therefore the fluid equation becomes

$$\begin{aligned} \dot{\rho} + 3\frac{\dot{a}}{a}\rho &= 0 \\ \frac{1}{a^3}\frac{d(\rho a^3)}{dt} &= 0 \\ \rho a^3 &= constant \\ \rho &\propto \frac{1}{a^3} \end{aligned} \tag{24}$$

This says that the density falls down with the expanding volume of the universe. We are free to re-scale scale factor a since now we assume $k = 0$ in Friedmann equation then two sides of equation can be re-scaled on the equal basis. Here we let ρ_0 be density at present time then

$$\rho = \frac{\rho_0}{a^3} \tag{25}$$

Plug in equation (25) into equation (23), we obtain all solution below

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3} \tag{26}$$

$$\rho(t) = \frac{\rho_0}{a^3} = \frac{\rho_0 {t_0}^2}{t^2} \tag{27}$$

$$H(t) = \frac{2}{3t} \tag{28}$$

where t_0 is the present time (age of the universe). Equation (28) tells us that the universe will stop expanding ($H = 0$) at $t \to \infty$.

[2]Inflation theory of universe predicts that universe has *flat geometry* ($k = 0$). Moreover cosmic microwave background (CMB) data from BOOMERanG Mission(2000) and MAXIMA Mission(2000)[10] also confirms that universe has $k \sim 0$.

7.2 The radiation-filled universe

Carrying on the same procedure we use for the dust case, here $P = \rho c^2/3$ and finally we obtain

$$\rho \propto \frac{1}{a^4} \tag{29}$$

$$a(t) = \left(\frac{t}{t_0}\right)^{1/2} \tag{30}$$

$$\rho(t) = \frac{\rho_0}{a^4} = \frac{\rho_0 {t_0}^2}{t^2} \tag{31}$$

The radiation-filled universe expands slower than the dust-filled universe. Density in both cases decreases with t^2 but the radiation density decreases with a^4 not a^3 as in dust case. The extra a comes from the redshift effect in light wavelength. We can obtain the relation between redshift and scale factor from Hubble law. At galactic scale, for a small distance apart dr the recession velocity defers by dv. According to Hubble law, hence

$$dv = H dr = \frac{\dot{a}}{a} dr \tag{32}$$

H is constant within the variation of distance dr but is not constant with time. Doppler effect of light makes wavelength λ defer by $d\lambda = \lambda_{observe} - \lambda_{emit}$ and Doppler's law gives us that

$$\frac{d\lambda}{\lambda_{emit}} = \frac{dv}{c} \tag{33}$$

Time duration that light travels in distance dr is $dt = dr/c$. Insert equation (32) into (33) we get

$$\frac{d\lambda}{\lambda_{emit}} = \frac{(\dot{a}/a)dr}{c} = \frac{1}{a}\frac{da}{dt}dt = \frac{da}{a} \tag{34}$$

We can see that $\lambda \propto a$ and then frequency $\nu \propto a^{-1}$. Density of radiation (ρ_{rad}) is just energy density ϵ which equals to number density $\times$ Energy. Energy $= h\nu$ where h is Planck's constant. Therefore redshift can give factor a^{-1} to the decreasing of ρ_{rad} and finally makes $\rho \propto a^{-4}$ instead of a^{-3}.

Additionally it would be worth talking about definition of **Redshift** here. The redshift z is defined by

$$z \equiv \frac{\lambda_{observe} - \lambda_{emit}}{\lambda_{emit}} = \frac{\lambda_{observe}}{\lambda_{emit}} - 1 = \frac{a_{observe}}{a_{emit}} - 1 \tag{35}$$

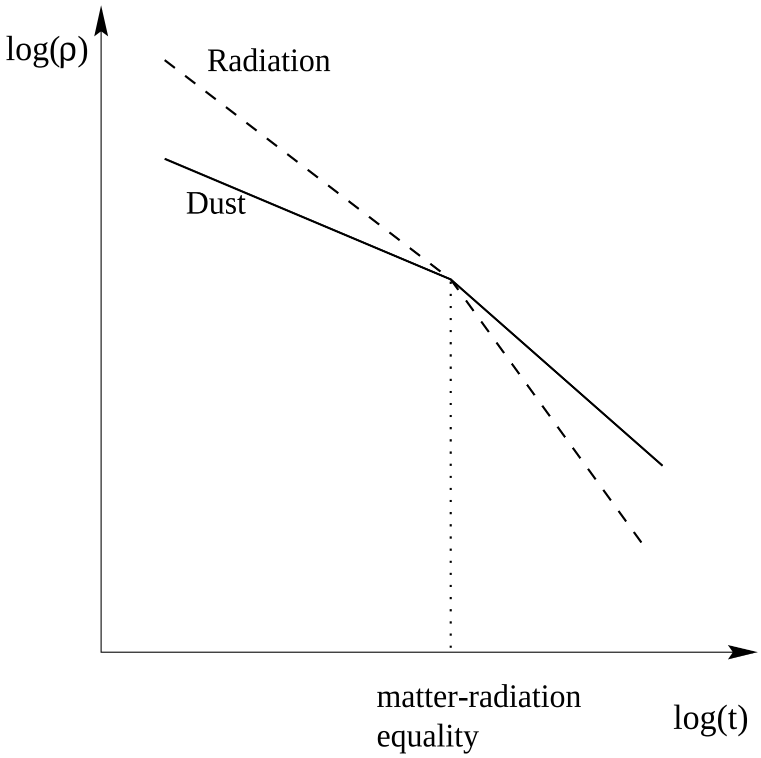


Figure 6: Schematic curve for the evolution of dust and radiation in the universe

7.3 The dust and radiation-filled universe

Now in this dust-radiation mixed universe we have two fluid equations which independently give equations (24) and (29). Our Friedmann equation now is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\Big[\rho_{dust} + \rho_{rad}\Big] \tag{36}$$

To get the solution for equation (36) is not so easy. In this universe containing two components of matters, dust falls off slower than radiation hence after sometimes from the beginning there must be the time that $\rho_{rad} = \rho_{dust}$ (**matter-radiation equality**) and after that dust will start to dominate the universe. In the early time, if we assume Big Bang, it should be radiation that dominates the universe and we can use all solutions of radiation case i.e. equations (29),(30) and (31). After the equality we shall use equations (24)-(28) which are of the dust-dominated case. After dust becomes dominant, expansion will be faster (please compare equations (26) and (30)). Figure 6 illustrates schematically the evolution of dust and radiation in the dust-radiation mixed universe.

We can see from figure 6 that after dust becomes dominant at equality both radiation and dust density fall off faster than before. This is because the scale factor in dust case increases faster than in the case of radiation.

8 Fate of the universe

The spatial curvature k can play important role in the evolution of the universe. We have already looked in detail when $k = 0$. In the situation that $k \neq 0$, it will dominate Friedmann equation very quick since the term k/a^2 falls off much slower than ρ both in dust and radiation case.

♣ For $k < 0$ after k/a^2 becomes dominant, Friedmann equation should look like

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{k}{a^2} \tag{37}$$

The solution of this equation is just

$$a \propto t \tag{38}$$

The universe will expand forever!

♣ For $k > 0$, the only possible way to solve this equation without being mess with imaginary number is to consider when density and spatial terms are in balance

$$\frac{8\pi G}{3}\rho = -\frac{k}{a^2} \tag{39}$$

and this will give

$$H = 0 \tag{40}$$

The universe stops expanding and then scale factor starts to decrease. After the time that $\frac{8\pi G}{3}\rho = -\frac{k}{a^2}$, the spatial curvature term will become dominant and brings about the cosmic re-collapsing. Notice that if we substitute $-t$ to t, Friedmann equation remains unchanged. This means that the equation is time-reversible and the universe will evolve reversely to when it expands.

We can see the schematic evolution curve in figure 7.

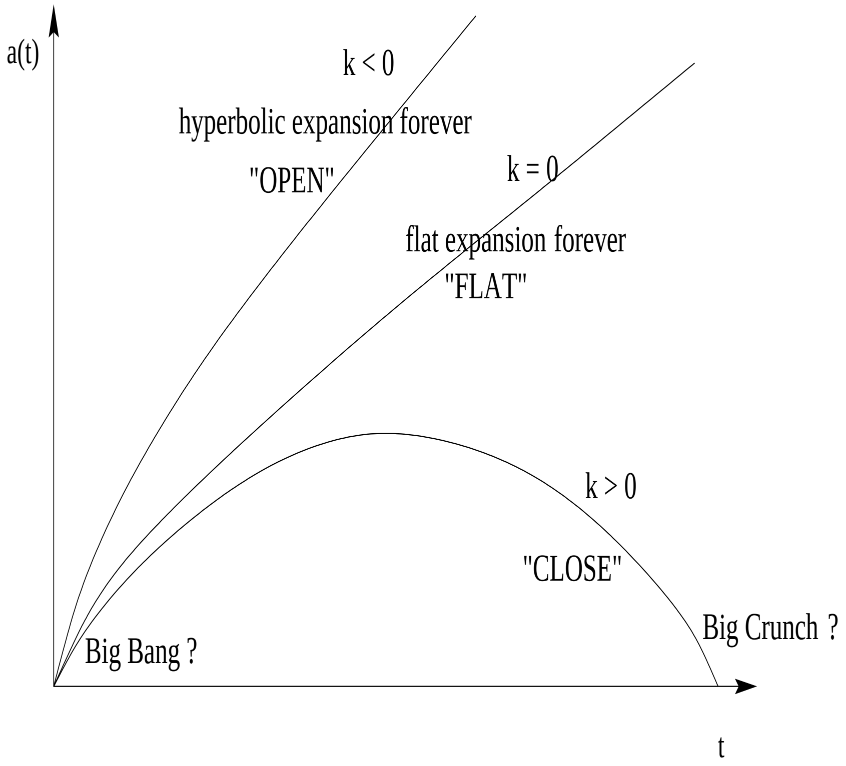


Figure 7: Figure 7: Fate of the universe, expand forever or re-collapse

9 Why Newtonian cosmology gives similar results as relativistic cosmology?

Newtonian cosmology bases on isotropic and homogenous Euclidean space which is flat but relativistic cosmology bases on curve space. The way out to explain why they give similar result is to take a look at the nature of these theories. Milne and McCrea had the picture of the sphere of unlimited size(unbounded) with edgeless and centerless when they first proposed Newtonian cosmology. In the uniform unbounded universe, gravity is proved to be indeterminate by David Layzer in 1954 [11]. Then matter should not have motion in particular direction then the universe should not expand! The way out is to consider our sphere in section 1 as a small bounded part of the whole infinite universe. In this small scale (small compare to the present Hubble length[3]), the curvature of space and expansion velocities are so small then Newtonian dynamics is well valid. By adding this small parts

[3]Hubble length is the distance that light travels within the time from Big Bang until today.

(spheres) of universe together we can have the picture of how the whole universe evolve.

We must be careful when considering large scale since curvature brings about general relativistic effect. If the model is not either isotropic or homogenous, Newtonian cosmology does not agree with relativistic cosmology. For more detail discussion, please see [2].

10 conclusion

We have derived basic dynamical equations of cosmology using Newtonian dynamics. The evolution of the universe defers up to the density and types of matters in it.

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