

## 3D Scalar-formulation of IE-MEI Method for Acoustic Scattering

N.M.Alam CHOWDHURY<sup>1</sup>, \*Jun-ichi TAKADA<sup>2</sup> and Masanobu HIROSE<sup>3</sup>

<sup>1</sup>Department of Electrical & Electronic Engineering, Dhaka University of Engineering & Technology, Gazipur 1700, Bangladesh, nmalamc@yahoo.com

<sup>2</sup>Tokyo Institute of Technology, Graduate School of Science & Engineering, O-okayama, Meguro-ku, Tokyo 152-8552, Japan, takada@ide.titech.ac.jp

<sup>3</sup>National Institute of Advance Industrial Science & Technology, Optoelectronics Division EM Field Measurement Lab., Ibaraki 305-8568, Japan, mahirose@etl.go.jp

### Abstract

The Scalar-formulation of IE-MEI method for 3D acoustic scattering problem is presented in this paper. The surface integral equation is derived from scalar reciprocity relation by using 3D scalar Helmholtz equation which satisfies the MEI postulates. As a result it preserve the matrix sparsity which can save computational time and storage memory. This paper also focuses on the insensitive property of MEI coefficients with respect to the small modification of the scatterer shape which can broader the applicability of MEI technique. The numerical results of arbitrary shape 3D problem are verified with the available numerical solution.

### 1 Introduction

The Measured Equation of Invariance (MEI) technique [1] was successfully applied [2] directly on the object surface with the surface integral equation by preserving the matrix sparsity which can save computational time and memory requirements. This Integral Equation formulation of MEI (IE-MEI) method is excellent for 2D EM problems [3] but not efficient for 3D arbitrary boundaries [4].

In this paper, we describes the Scalar-formulation of IE-MEI (SIE-MEI) method for 3D acoustic scattering problem, where the scalar-field integral equation is derived from scalar reciprocity relation according to Hirose's approach [3]. MEI postulates [1] are then applied to solve integral equation with suitable set of equivalent sources.

This paper also describe the insensitive property of MEI coefficients for the computation of scattering from modified structured bodies. According to this property, if some portion of the scatterer is modified, then the MEI coefficients shall be recalculated only around the modified area and the previous stored coefficients are reused for the other portion of the scatterer. This gives the same results as those obtained by considering the whole modified scatterer, while the computational (CPU) time is drastically reduced.

### 2 Scalar-formulation of IE-MEI method

Consider an acoustic problem as shown in Fig.1(a), where  $\partial V^+$  is a closed surface of region  $V^+$  placed very near to the scatterer that includes only a single source which is represented by the equivalent monopole source  $\rho_2$  and the dipole moment  $\mu_2$ . This leads the *Scalar-field Integral Equation* [5]

$$\oint_{\partial V} (\phi_1(\mathbf{r})\tilde{\rho}_2(\mathbf{r}) - \phi_1'(\mathbf{r})\hat{\mathbf{n}} \cdot \tilde{\boldsymbol{\mu}}_2(\mathbf{r})) dS = 0, \quad (1)$$

where  $\phi_1$  and  $\phi_1'$  are the scattered field and its normal derivative, respectively, and  $(\tilde{\cdot})$  terms represent the equivalent sources near the scatterer.

MEI postulates are then applied to get the localized and discretized version of Eq. (1) [5].

$$\sum_m [a_{nm}\phi_1(\mathbf{r}_{nm}) - b_{nm}\phi_1'(\mathbf{r}_{nm})] = 0, \quad (2)$$

where  $a_{nm}$  and  $b_{nm}$  are the unknown MEI coefficients for the  $n$  ( $= 1, 2, \dots, N$ )-th node associated with the  $m$  ( $= 1, 2, \dots, M$ ) number of neighboring nodes, and  $\phi_1$  and  $\phi_1'$  are the scattered wave generated by the suitable  $Q$ -sets of secondary sources called *metrons* [5].

In matrix form, Eq. (2) becomes

$$[\mathbf{C} \ \mathbf{D}] \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} = 0, \quad (3)$$

where  $[\mathbf{C} \ \mathbf{D}]$  is the  $[Q \times 2M]$  known matrix composed of metron fields and their normal derivatives and  $\begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix}$  is the column vector of unknown MEI coefficients.

This local matrix is solved repeatedly for the whole scatterer surface, two sparse matrices  $\mathbf{A}$  and  $\mathbf{B}$  of MEI coefficients which are invariant to excitation are obtained (Eq. (4a)). For a soft-body problem, Eq. (4a) can be represented as the equation of equivalent surface sources  $\frac{\partial\phi}{\partial n}$  on the scatterer (Eq. (4b)).

$$\mathbf{A} [\phi^{\text{sc}}] - \mathbf{B} \left[ \frac{\partial\phi^{\text{sc}}}{\partial n} \right] = 0, \quad (\text{a}) \quad \left[ \frac{\partial\phi(\mathbf{r})}{\partial n} \right] = \left[ \frac{\partial\phi^{\text{inc}}(\mathbf{r})}{\partial n} \right] - \mathbf{B}^{-1} \mathbf{A} [\phi^{\text{inc}}(\mathbf{r})], \quad (\text{b}) \quad (4)$$

where  $\phi^{\text{inc}}$  and  $\frac{\partial\phi^{\text{inc}}}{\partial n}$  are the incident field and its normal derivative, respectively.

### 3 Insensitive Property of MEI Coefficients

The insensitive property of MEI coefficients is effective when the scatterer structure is modified so that the scattered field should be recalculated [6].

According to Eq. (3), the MEI coefficients of each node are derived from the metron fields at the node for the possible sets of metrons, combined with the interaction of metron fields at the neighboring nodes associated in the local region. Although these fields are generated by the metrons defined over the entire domain, they mainly depend on the local geometry of the scatterer and do not get significant effects from the other portion of the scatterer. This local geometrical dependency, we define here, the insensitive property of MEI coefficients.

In conventional technique, the MEI coefficients of whole structure of unmodified body are derived according to Eq. (3) and then by using Eq. (4b) the equivalent surface sources of the scatterer are derived. If some small portion of the scatterer is modified, we follow the same procedure again. But in our proposed technique, we store the MEI coefficients of the whole structure of original body during scattering computation. Now if some portion of the scatterer is modified, then we shall calculate the MEI coefficients only around the modified area and we can reuse the stored data for the other portion. Because of the insensitive property of MEI coefficients, new set of data gives almost the same result as the result obtained by the conventional solution process. Thus, by following the proposed solution technique we can avoid the extra computational burden and hence save the computational time.

## 4 Numerical Implementation

### 4.1 SIE-MEI method for 3D Acoustic Scattering

Let us consider a cube on which a plane wave is incident to  $+y$  direction as in Fig. 1(b). Figure 2(a) shows the 2D plot of equivalent surface source on the cube along the perimeter of section-A for the side of  $l = 2\lambda$  and compared with the numerical solution using Combined-field Method of Moments (CfMoM). The results show an excellent agreement between them.

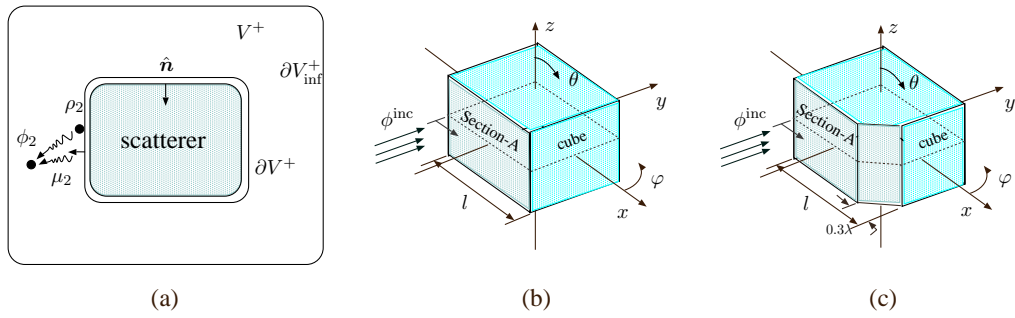


Figure 1: (a) Region  $V^+$  very near to the scatterer, (b) Cube, and (c)  $0.3\lambda$  modified Cube.

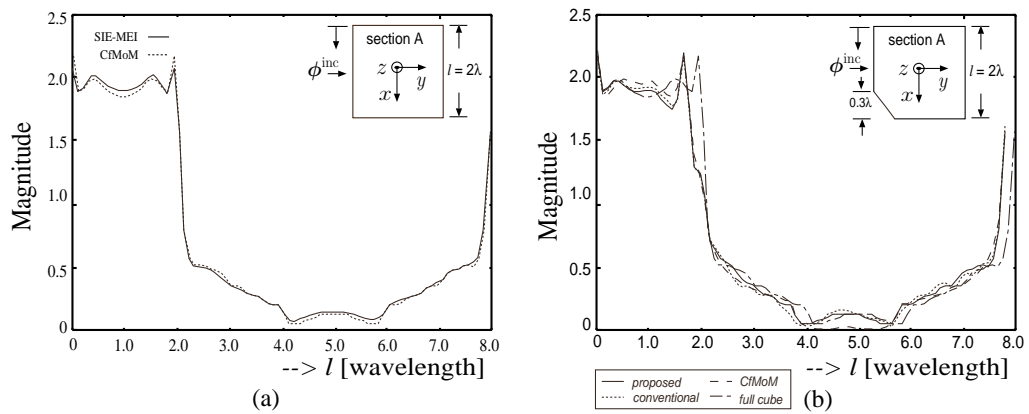


Figure 2: Equivalent surface source on the (a) Cube, and (b)  $0.3\lambda$  modified Cube.

## 4.2 Insensitiveness of MEI Coefficients

We then considered the same cube with  $0.3\lambda$  slice cut modification at one of the edges parallel to  $z$  axis, as shown in Fig. 1(c). In this case we also considered the same type of plane wave incident from the same direction. Figure 2(b) shows the 2D plot of equivalent surface source along the perimeter of section-A. The graph contains the result of *full cube* (or the original cube) by using conventional solution technique and of the modified cube by using *conventional* and *proposed* solution technique, in the SIE-MEI method. The results of the modified cube are also compared with the numerical solution using CfMoM, which gives a good agreement between them.

## 5 Conclusion

The 3D Scalar-formulation of SIE-MEI method for acoustic scattering problem is presented. The method uses the surface integral equation with MEI postulates, thus keeps the same advantages as of MEI and IE-MEI method. A new solution technique based on the insensitive property of MEI coefficients is also described in this paper. This technique is implemented on the scatterer when its geometry is modified and the scattering characteristics need to be recalculated. We can save the large amount of CPU time with the same accuracy in the result. We also verify our approach with the numerical solutions of the arbitrary shape 3D problem. Result shows an excellent agreement.

## References

- [1] K. K. Mei, R. Pous, Z. Chen, Y. W. Liu, and M. D. Prouty, "The measured equation of invariance: A new concept in field computations", IEEE Trans. Antennas & Propag., vol. 42, no. 3, pp. 320-327, March 1994.
- [2] J. M. Rius, R. Pous, and A. Cardama, "Integral formulation of the measured equation of invariance: A novel sparse matrix boundary element method", IEEE Trans. Magnetics, vol. 32, no. 3, pp. 962-967, May 1996.
- [3] M. Hirose, M. Miyake, J. Takada, and I. Arai, "New integral equation formulation of the measured equation of invariance and the extension to analyze two-dimensional cylinders with impedance boundary conditions", Radio Science, vol. 34, no. 1, pp. 65-82, Jan.-Feb. 1999.
- [4] J. M. Rius, J. Parrón, E. Úbeda, and J. R. Mosig, "Integral equation MEI applied to three dimensional arbitrary surfaces", Electronics Letters, vol. 33, no. 24, pp. 2029-2031, Nov. 1997.
- [5] N. M. Alam Chowdhury, J. Takada, M. Hirose, "Novel Formulation for the Scalar-Field Approach of IE-MEI Method to Solve the Three-Dimensional Scattering Problem", IEICE Trans. on Fundamentals, vol. E85-A, no. 8, pp. 1905-1912, August 2002.
- [6] N. M. Alam Chowdhury, J. Takada, M. Hirose, "Applicability of Insensitive Property of MEI Coefficients for Modified Scattering Objects", to be published in Jpn. J. Appl. Phys, 2004.