IE-MEI Method Implemented to the 3D Arbitrary Shape Scattering Problem : Scalar-field Approach

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Abstract

In this paper we introduce the Scalar-field approach of Integral Equation formulation of MEI (SIE-MEI) method to solve the 3D scattering problem. The scalar-field integral equation is derived from scalar reciprocity relation and applied on the scatterer surface which satisfies the MEI postulates. Spherical wave function is chosen as metron set for 3D implementation. Uniform and arbitrary shape 3D problems are considered to verify our method and compare the results with the available solutions.

1 Introduction

The IE-MEI method [1] was proposed for two-dimensional (2D) electromagnetic (EM) problem by preserving the advantages of sparse linear system, resulting in minimum storage memory and CPU time savings as of MEI [2] method. This method also was applied to 3D scattering problem [3] but the formulation becomes problem-dependent. This might be due to the unsuitable metron set, mesh generation, etc., which concluded that the IE-MEI method is inefficient to be extend for 3D arbitrary boundaries.

Alternatively, we propose the Scalar-field approach of IE-MEI method with new set of metrons for 3D scattering problem. Present implementation itself is not only useful and effective to the scalar-field problems, but also a preparatory study for further extension to 3D vector EM problems.

2 Scalar-field approach of IE-MEI method

In Scalar-field approach of IE-MEI method, at first let us derive the *Scalar Reciprocity Relation* from Green's theorem by using 3D scalar Helmholtz equation as

$$\int_V \left(\phi_2(\boldsymbol{r})g_1(\boldsymbol{r}) - \phi_1(\boldsymbol{r})g_2(\boldsymbol{r})\right) dV = \oint_{\partial V} \left(\phi_1(\boldsymbol{r})\frac{\partial \phi_2(\boldsymbol{r})}{\partial n} - \phi_2(\boldsymbol{r})\frac{\partial \phi_1(\boldsymbol{r})}{\partial n}\right) dS, \quad (1)$$

where ϕ_1 and ϕ_2 are the scalar fields due to the source distribution $g_1(\boldsymbol{r}')$ and $g_2(\boldsymbol{r}')$, respectively.

Let us consider an imaginary region V^+ near the scatterer which is bounded by the surfaces ∂V^+ and ∂V^+_∞ and assume that the volume V^+ includes only the source g_2 and represented by the equivalent monopole source $\rho_2(\mathbf{r})$ and the dipole moment $\boldsymbol{\mu}_2(\mathbf{r}) \cdot \hat{\boldsymbol{n}}$ (Fig.1(a)). By

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using Hirose's approach [4] with appropriate limiting conditions [5] this leads to the *Scalar-field Integral Equation*

$$\oint_{\partial V} \left(\phi_1(\boldsymbol{r})\tilde{\rho}_2(\boldsymbol{r}) - \frac{\partial \phi_1(\boldsymbol{r})}{\partial n} \tilde{\boldsymbol{\mu}}_2(\boldsymbol{r}) \cdot \hat{\boldsymbol{n}} \right) dS = 0, \tag{2}$$

where ($\tilde{\cdot}$) terms represent the equivalent sources near the scatterer.

MEI postulates [2] are then applied to Eq. (2). Discretizing and solving this integral equation repeatedly for the whole scatterer surface with suitable set of equivalent sources called *metrons*, two sparse matrices $\mathbf{A}$, $\mathbf{B}$ of local sources which are invariant to excitation are obtained (Eq. (3a)). Therefore, by using Dirichlet boundary condition, as an example of soft-body problem, the Eq. (3a) can be represented as the equation of equivalent surface sources $\frac{\partial \phi}{\partial n}$ on the scatterer (Eq. (3b)).

$$\mathbf{A}\,[\boldsymbol{\phi}_1] - \mathbf{B}\left[\frac{\partial \boldsymbol{\phi}_1}{\partial n}\right] = 0, \quad \text{(a)} \qquad \left[\frac{\partial \boldsymbol{\phi}}{\partial n}\right] = \left[\frac{\partial \boldsymbol{\phi}^{\text{inc}}}{\partial n}\right] - \mathbf{B}^{-1}\mathbf{A}\left[\boldsymbol{\phi}^{\text{inc}}\right], \quad \text{(b)} \tag{3}$$

where ϕ_1 and $\frac{\partial \phi_1}{\partial n}$ are the scattered field and its normal derivative, respectively, due to the metrons for some certain incident fields and ϕ^{inc} and $\frac{\partial \phi^{\text{inc}}}{\partial n}$ are the incident field and its normal derivative, respectively.

3 Spherical Wave function as Metron Set

To get the reasonable solution, the choice of suitable metron set is an important factor. For 3D scattering problem, we propose spherical wave function as a metron set

$$\rho(r, \theta, \varphi) = \sum_{n=0}^{\infty} h_n^{(2)}(kr) \sum_{m=-n}^{n} P_n^{|m|}(\cos\theta) e^{jm\varphi}, \tag{4}$$

where $h_n^{(2)}(kr)$ is the n-th degree outward spherical Hankel function, $P_n^{|m|}(\cos\theta)$ is the n-th degree m-th order Associated Legendre function and $e^{jm\varphi}$ is the harmonic function.

4 Implementation Issues

We considered rectangular patch discretization with pulse-basis point-matching method and with ten unknowns per wavelength to keep the easier mesh generation and minimum number of integration points. The field incident on the scatterer ϕ^{inc} and the scattered field ϕ^{sc} generated by the metrons are

$$\phi^{\text{inc}}(\boldsymbol{r}) = e^{-j\boldsymbol{k}\cdot\boldsymbol{r}}, \quad \text{(a)} \quad \text{and} \quad \phi^{\text{sc}}(\boldsymbol{r}) = \oint_{\partial V} \rho(\boldsymbol{r}')\, G(\boldsymbol{r}, \boldsymbol{r}')\, dS', \quad \text{(b)} \tag{5}$$

where $\boldsymbol{k}$ is the wave propagation vector, $\rho(\boldsymbol{r}')$ is a metron given in Eq. (4), $G(\boldsymbol{r},\boldsymbol{r}')$ is the free space 3D Green's function, and $\boldsymbol{r}$ and $\boldsymbol{r}'$ are the position vectors of observation point and source point, respectively.

As an example of uniform shape body, let us consider an axially symmetrical body, e.g., sphere as in Fig.1(b), on which a plane wave (Eq. (5a)) is incident from $+z$ direction. Since the problem has axial symmetry with constant radial distance thus the spherical wave function (Eq. (4)) is reduces to *Zonal harmonics*, which is used as metron set for the sphere.

Let us consider a cube (Fig.1(c)), as an arbitrary shape body on which same type of plane wave (Eq.(5a)) is incident from $+z$ direction. Since the problem has radial, polar and equatorial variation, thus Eq. (4) is used as metron set.

Using Eq. (5b) and (3a), MEI coefficients $\mathbf{A}$ and $\mathbf{B}$ are obtained. Substituting these into Eq. (3b) with the Eq. (5a), the distribution of equivalent surface source are computed.

Figures 2(a)(b) and 3(a)(b) show the distribution of magnitude and phase of equivalent surface source on sphere of radius $a = 3\lambda$ along polar direction and cube of side 2λ along $y-z$ plane at $x = 0$, respectively. The results are compared with the series solution in case of sphere and with the numerical solution (e.g., CfMoM) in case of cube. In both cases they have excellent agreements except some errors in the shadowed regions, which are negligible because they do not give any significant effect in the scattering computation.

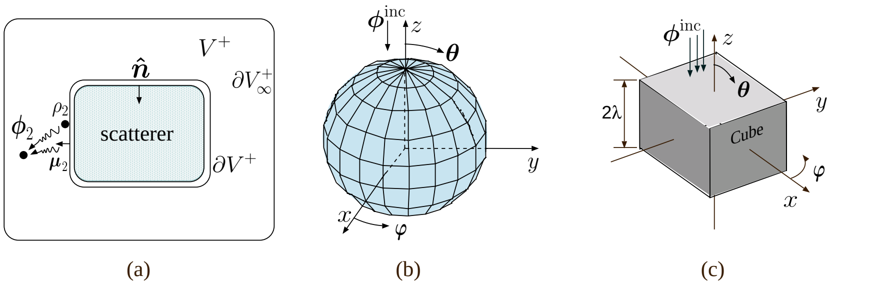

Figure 1: (a) Region V^+ very near to the scatterer, (b) Sphere, and (c) Cube.

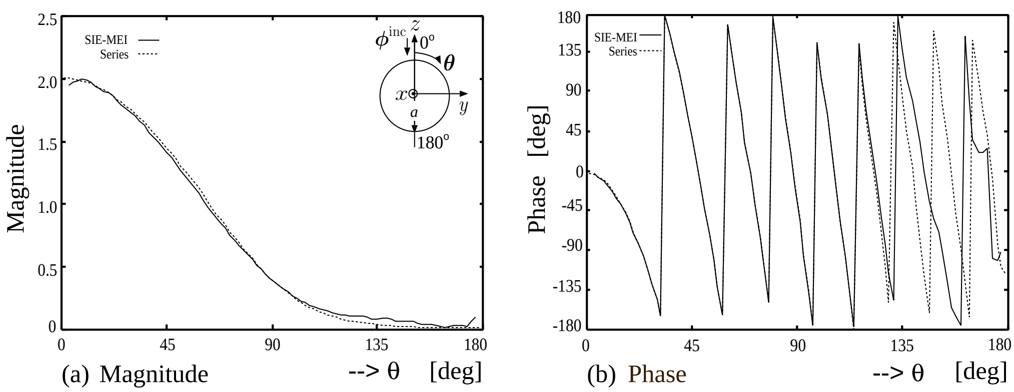

Figure 2: Equivalent surface source distribution on the Sphere.

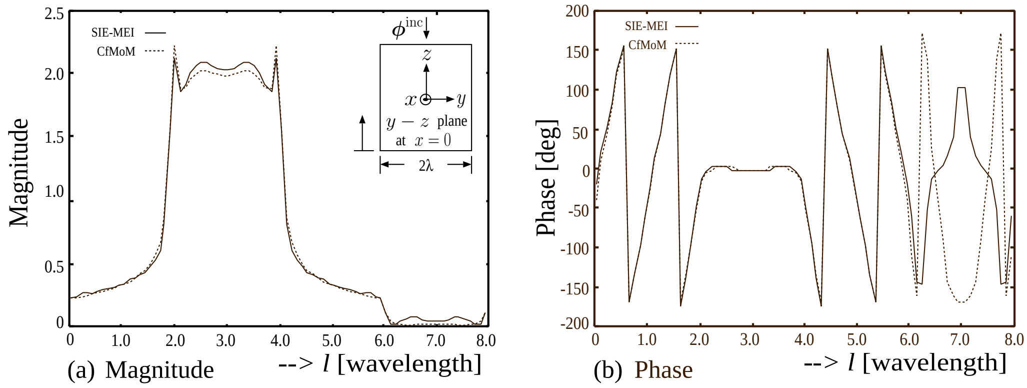

Figure 3: Equivalent surface source distribution on the Cube.

5 Conclusion

In this paper we propose Scalar-field approach of IE-MEI (SIE-MEI) method with new set of metrons which uses the surface integral equation with MEI postulates, thus keeps the same advantages as of MEI and IE-MEI method. We also verify our approach with the typical uniform shape and arbitrary shape 3D problems. Results shows an excellent agreement. The importance of scalar implementation is its simplicity compared with 3D vector EM implementation. In spite of its simplicity, it reveals many important features, i.e., choice of metrons, mesh generation, set of adjacent nodes, etc. To establish our method and metron set we are on research to implement it to other large and more complex 3D structure.

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