Journal of Electrical Engineering
The Institution of Engineers, Bangladesh
Vol. EE 32, No. I & II, December 2005

Removal of Resonant Solution of Electromagnetic Scattering Problem

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ABSTRACT

A nonunique solution of integral equation of a region bounded by perfectly conducting walls a resonant cavity causes an internal resonant solution in addition with the true solution. Due to this resonant problem, the solution of electromagnetic scattering problem will degrade the accuracy at resonance frequencies. A combined electric and magnetic field integral equation is developed to solve this problem. In this paper, we observe the contamination of solution due to the resonant solution of the integral equation for transverse magnetic (TM) scattering from a conducting cylinder. The electric field and magnetic field integral equations fail to give unique solutions at resonance frequencies. The solution of combined-field integral equation can overcome this problem. The numerical results of electric field and magnetic field solution for two-dimensional (2D) perfectly conducting cylinder are verified with the combined-field solution that shows the good agreement.

1. INTRODUCTION

In the integral representation, the electric field integral equation (EFIE) [1-4] and the magnetic field integral equation (MFIE) [2,5-7] are widely used to solve the electromagnetic (EM) scattering problem. For a region bounded by perfectly conducting walls, a nonunique solution consists of the true solution plus an arbitrary number of resonant solutions arising during this solution. This resonant solution is nonphysical that gives a nontrival field inside the bounded region. The presence of resonance at certain values of wave number causes this internal resonant effects which adversely affects both the accuracy and computational time of the solution method. Therefore, the uniqueness of solution of EFIE and MFIE is not guaranteed at these interior resonant frequencies of closed body [8-10].

Many methods have been proposed to overcome this problem, such as the combined-source method [11], the extended boundary condition method [12,13], the combining interior and exterior field expression method [14], the combined field integral equation method [15,16], etc. Among of them the Combined Field Integral Equation (CFIE) method is suitable for a region bounded by perfectly conducting walls in either the electric or magnetic sense. In the combined electric and magnetic field solution, the resonant solutions that enter into the electric field solution do not satisfy the boundary condition on the magnetic field; the resonant solutions that enter into the magnetic field solution do not satisfy the boundary condition on the electric field. Therefore, the lack of the other boundary condition solves the resonance problem in the combined field solution.

In this paper we mainly concentrate on alleviating the resonance problem by using combined field integral equation of two-dimensional perfectly conducting body. The resonant solution that arises only at discrete values of the wavenumber is described in section 2. The formulation of electric, magnetic and combined field integral equations are given in section 3. Some numerical examples are given in section 4. Finally in section 5, concluding remarks of this paper are presented.

2. EIGEN WAVENUMBERS

Fredholm integral equations of first and second kinds are used to solve the electromagnetic scattering problem. Mathematically, these equations can be written as

$$f(x) = \int_a^b \mathbf{K}(x,t)\Phi(t)\,dt, \qquad (1)$$

and

$$f(x) = \Phi(x) - \lambda\int_a^b \mathbf{K}(x,t)\Phi(t)\,dt, \qquad (2)$$

where λ is a scalar parameter, functions $\mathbf{K}(x, t)$, $f(x)$ and the limits a and b are known, and $\Phi(x)$ is unknown. The function $\mathbf{K}(x, t)$ is known as the kernal of the integral equation containing Green's functions or derivatives of it.

Table 1 shows the Green functions that are commonly used to solve the electromagnetic scattering problem. Where G is the Green function, k is the wavenumber, δ is the Dirac delta function, $H_0^{(2)} = J_0 - jN_0$ is the Hankel function of the second kind of order 0.

Table 1: Green's Function.

Operator Equation	**Scalar Helmholtz Equation**
	$\nabla^2 G + k^2 G = -\delta(\boldsymbol{r} - \boldsymbol{r}')$
Solution Region	
1 dimensional	$\frac{1}{2jk}e^{-jk\lvert x-x'\rvert}$
2 dimensional	$\frac{1}{4j}H_0^{(2)}(k\lvert\rho-\rho'\rvert)$
3 dimensional	$\frac{e^{-jk\lvert\boldsymbol{r}-\boldsymbol{r}'\rvert}}{4\pi\lvert\boldsymbol{r}-\boldsymbol{r}'\rvert}$

A resonant solution of Eq.(1) or (2) arise only at discrete values of the wavenumber k called eigen wavenumbers, which degrade the accuracy of the numerical solution of an electromagnetic scattering problem not only at the eigen wavenumbers but also over the range of the wavenumber spectrum.

For a cylindrical body with circular geometry, the eigen wavenumbers are determined from the roots of Bessel functions

$$J_n(ka) = 0, \qquad n = 0, 1, 2, \cdots\cdots \qquad (3)$$

where J_n is the Bessel function of the first kind of order n and a is the radius of the circular cylinder.

Table 2 shows some roots of the Bessel function of first kind of order 0 and 1, for a circular cylinder of unit radius. More roots of J_n can be found in Ref.[17].

Table 2: Roots of J_n for a circular cylinder of unit radius.

Zeros	J_0	J_1
1	2.4048	3.8317
2	5.5201	7.0156
3	8.6537	10.1735
4	11.7915	13.3237
5	14.9309	16.4706

3. INTEGRAL REPRESENTATION OF PERFECTLY CONDUCTING CYLINDER

Let us consider a 2D electromagnetic scattering problem in the open space, as shown in Fig.1. Integral equations for this perfectly conducting scatterer are obtained by applying Dirichlet boundary condition to integral representations. If a TM wave incident perpendicularly on this z-directed scatterer, then the incident electric field has only the z-component E_z^{inc} and the incident magnetic field has only the l-component H_l^{inc}.

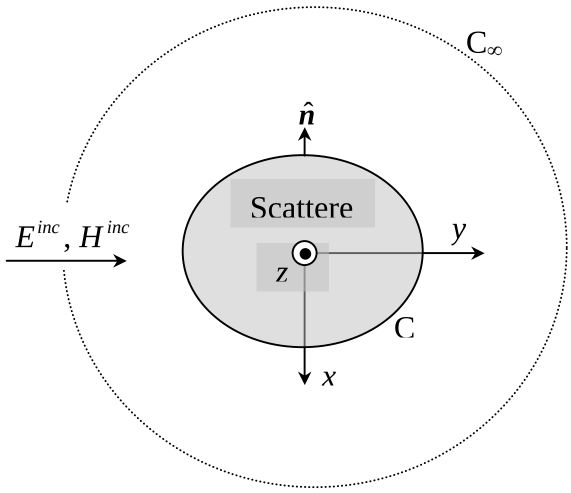

Fig. 1: 2D electromagnetic scattering problem.

Mathematically, the z-component of electric field integral equation for TM scattering from the 2D conducting scatterer is

$$E_z^{inc}(\rho) = \frac{kZ_c}{4}\int_C \mathbf{J}_z(\rho')H_0^{(2)}(k\lvert\rho-\rho'\rvert)\,dl', \qquad (4)$$

and the l-component of magnetic field integral equation is

$$H_l^{inc}(\rho) = \frac{1}{2}\mathbf{J}_z(\rho') - \frac{j}{4}\int_C \mathbf{J}_z(\rho')\frac{\partial H_0^{(2)}(k\lvert\rho-\rho'\rvert)}{\partial n'}\,dl', \qquad (5)$$

where Z_c is the characteristic impedance, $\mathbf{J}_z$ is the *z*-component of surface current density, ρ and ρ' are the position vector of the observation and source point, respectively.

Since the Eqs.(4) and (5) are the Fredholm integral equations of first and second kinds, these gives the unique solution only if their homogeneous equations has only the trivial solution. Due to the presence of eigen wavenumber in the range of the wavenumber spectrum a nontrivial solution is possible. A nontrivial solution consists with the true solution and the resonant solution which contaminated the solution of Eqs.(4) and (5).

To eliminate these resonant solutions, a combined electric and magnetic field integral equation can be used [2]. For the TM incident field on the z-directed perfectly conducting cylinder, the combined field integral equation can be written as

$$\frac{p}{Z_c}E_z^{inc} + H_l^{inc} = \frac{1}{2}\mathbf{J}_z + \frac{1}{4}\int_C \mathbf{J}_z\left(pk - j\frac{\partial}{\partial n}\right)\cdot \cdot H_0^{(2)}\left(k\left|\rho-\rho'\right|\right)dl', \quad (6)$$

where *p* is the positive real number lies between zero and one.

4. NUMERICAL EXAMPLES AND RESULTS

As for numerical example, consider a uniform shape perfectly conducting circular cylinder in the free space. Assume that the structure is uniform and infinitely long along the *z*-direction as shown in Fig.2.

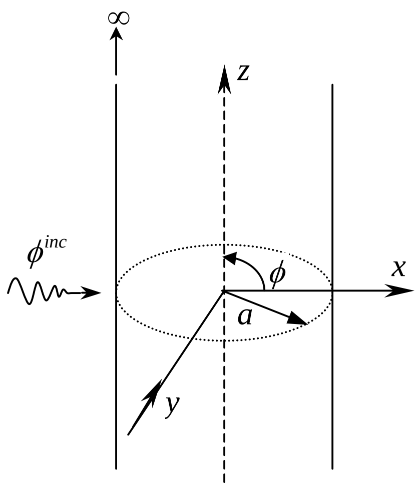

Fig. 2: Plane wave incident on a circular cylinder.

4.1 General Considerations

Let us consider the scatterer illuminated by the forwarded directed plane wave

$$\phi^{inc} = e^{-j\boldsymbol{k}\cdot\boldsymbol{\rho}}, \quad (7)$$

where $\boldsymbol{k}$ is the wave propagation vector, ρ is the position vector.

Piecewise linear discretization with pulse-basis point matching method and ten segments per wavelength are considered. Method of Moments (MoM) technique is used for numerical solution.

4.2 2D Circular Cylinder

Figure 3 shows a cross-section of 2D uniform shape circular cylinder, on which a *z*-directed TM plane wave is incident to +*x* direction. Therefore, *z*-component of incident electric field is

$$E_z^{inc} = e^{-jka\cos\phi}, \quad (8)$$

and the *l*-component of incident magnetic field is

$$H_l^{inc} = -\cos\phi e^{-jka\cos\phi}, \quad (9)$$

where *k* is the wave propagation number, *a* is the radius of the cylinder and ϕ is the angular position on the surface boundary of the cylinder.

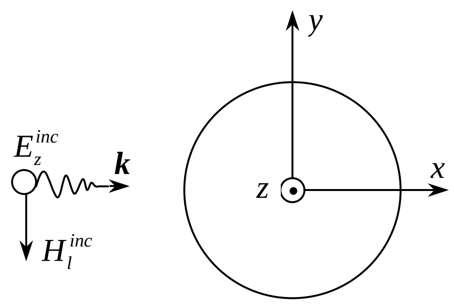

Fig. 3: Field component of incident wave.

Using the above numerical considerations in Eqs.(4), (5) and (6) the surface current density on the cylinder can readily be obtained.

Figures 4, 5, and 6 show the numerical results of surface current density on a circular cylinder of unit radius at the roots 8.6537, 11.7915 and 14.9309, respectively of Bessel function of first kind of zero order. From the figures it is seen that, the electric field and magnetic field solutions are contaminated by the resonant solutions, which can be overcome by using combined field integral equation.

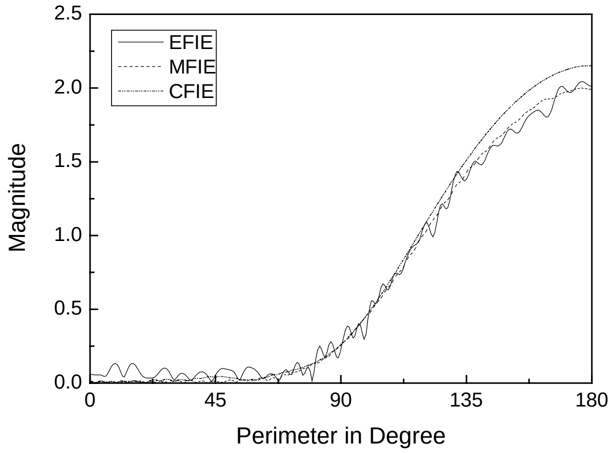

Fig. 4: Surface current density of unit radius circular cylinder at the eigen wavenumber 8.6537.

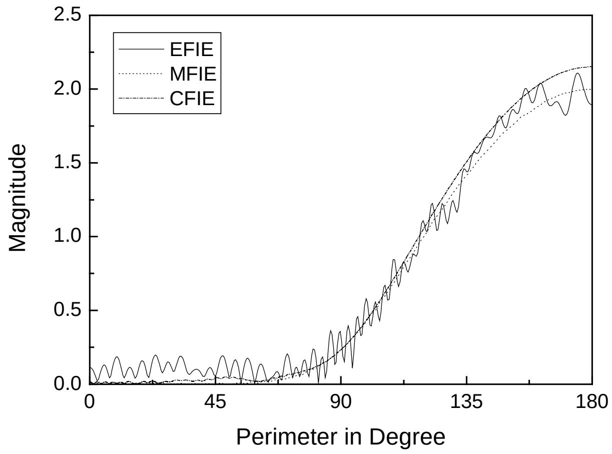

Fig. 5: Surface current density of unit radius circular cylinder at the eigen wavenumber 11.7915.

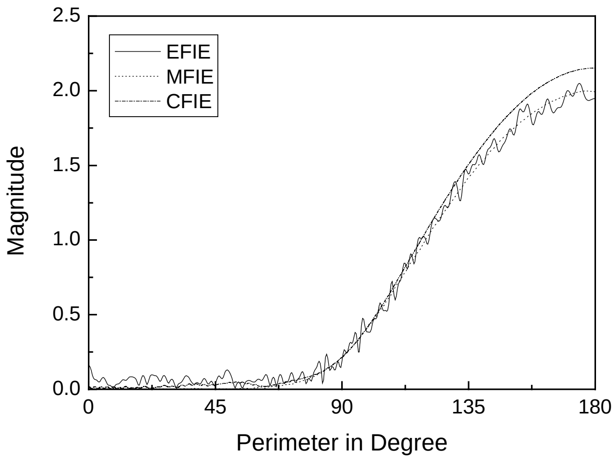

Fig. 6: Surface current density of unit radius circular cylinder at the eigen wavenumber 14.9309.

5. CONCLUSIONS

The combined field integral equation is used to solve the resonance problem of integral equation for a region bounded by perfectly conducting walls. The resonant solution that arises at eigen wavenumber degrades the solution accuracy and CPU time.

At certain values of eigen wavenumber, we observe the contamination of electric and magnetic field solution for TM scattering from a two-dimensional circular conducting cylinder. The removal of these resonant solutions of electromagnetic scattering problem by using combined field integral equation is also observed here. Necessary numerical results are derived to verify these solutions.

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