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The FMM-MoM Method for Solving Electromagnetic Scattering Problem

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ABSTRACT

In this paper, a new method is proposed for the numerical analysis of electromagnetic scattering problem, which compromises between Fast Multipole Method (FMM) and Method of Moments (MoM). This method saves memory usage as well as computational (CPU) time requirement by matrix sparsity as done in FMM. It also improves the accuracy level by using the concept of MoM in the non-zero elements of the sparse matrix. Thus, the number of operations in this method is also optimized between the operations of FMM and MoM. Proposed method is implemented on infinitely long perfectly conducting body. Numerical results are verified with the solution of same problem by using FMM and MoM. The minimum storage memory requirements and CPU time savings of the proposed method are also verified with the existing methods.

1. INTRODUCTION

The advent of high-speed digital computers allowed more computations to be performed than ever before in the solution of electromagnetic field problem. It made practicable methods of solution too repetitious for hand calculation. Moreover, the accuracy level in the solution of electromagnetic field problems using digital computers lead the way to many people to devise many new and efficient numerical techniques. The most reliable and generally applicable numerical technique, Method of Moments (MoM) [1] is used to obtain such accuracy to solve a boundary value problem. The MoM generates the least unknowns but a dense matrix which requires a exceedingly large storage requirement of $O(N^2)$ (N = Number of unknowns) and direct solution time of $O(N^3)$, that become prohibitively large as the size of a scatterer grows. But the advent of very efficient acceleration methods for the numerical solution of electromagnetic problem and the fast growth of computer capacity has brought them within reach. Among the fast acceleration methods, Fast Multipole Method (FMM) [2~4] is an iterative approach, which generates a sparse matrix that can save both memory and CPU time. In the FMM, the solution time requires an operation cost of $O(N^{5/2})$, therefore, enabling to solve a more complex and large problem with ease. Most recently, Multilevel Fast Multipole Method (MLFMM) [5] further reduces more CPU time and memory usage in solving any problem related with matrix. Of course, these methods loss the accuracy levels in comparison with the MoM. Moreover, the FMM or MLFMM is not widely used as an algorithm as it should be [6]. It is considered by many to be hard to implement for those not familiar with data structures.

However, the accuracy level of MoM is better than any other methods so far have been developed. On the other hand, the FMM is considered as one of the fast solver for large-scale problems. In our research, these two methods are utilized to devise the computational technique to improve accuracy level in comparison with the FMM and to reduce the memory usage and CPU time requirement in comparison with the MoM.

A summary of the mathematical modelling and step by step solution of scattering problem by using

MoM, FMM and proposed method are given in Section 2. The numerical implementation and the results are given in Section 3. In Section 4, concluding remarks and future extension of the proposed method are described.

2. FORMULATION

In this paper, combined field integral equation [7] approach with dirichlet boundary condition is used to solve Helmholtz's equation for two-dimensional TM scattering problems, i.e.,

$$\frac{1}{2}J_z+\frac{1}{4}\int_C J_z\left[pkH_0^{(2)}(kR)+j\cos\psi H_1^{(2)}(kR)\right]dc'$$

$$=H_c^i+\frac{p}{Z_c}E_z^i, \tag{1}$$

where E_z^i is the axial component of incident electric field, H_c^i is the circumferential component of incident magnetic field, Z_c is the characteristic impedance of the medium, p is a positive real number that lies between zero and one for unique solution [8, 9], k is the free space wave number, R is the distance between observation point ρ and the source point ρ' (Fig.1), $H_0^{(2)}$ and $H_1^{(2)}$ are the Hankel's function of second kind of order zero and one, respectively, ψ the is angle between surface normal vector $\boldsymbol{n}'$ and distance vector $\boldsymbol{R}$ (Fig.1) and J_z is the induced surface current density.

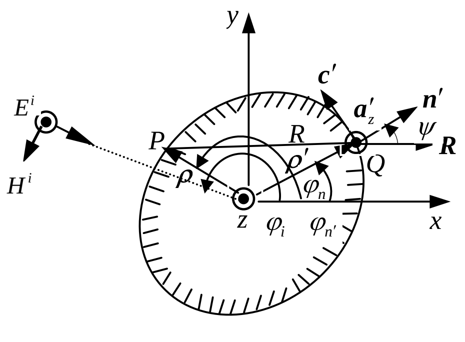


Fig. 1: Geometry for two-dimensional scatterer in TM^z polarization.

2.1 Method of Moments

In the MoM, the integral equation is discretized by writing the electric and magnetic sources as a sum of given basis functions. The moments of this equation are taken as inner products of basis functions with given testing functions. By using pulse function as basis function and characterizing weighting function by point matching method, Eq.(1) can be written as (assume p =1)

$$H_c^i+\frac{1}{Z_c}E_z^i=\frac{\delta_{mn}}{2}J_z(\rho_m)+\frac{k}{4}\sum_{m=1}^{N}J_z(\rho_m)$$
$$\times H_0^{(2)}(k|\rho_n-\rho_m|)\Delta C_m+\frac{jk}{4}\sum_{m=1}^{N}J_z(\rho_m)\frac{1}{|\rho_n-\rho_m|}$$
$$\times\left[(x_n-x_m)\cos\varphi_{n'}+(y_n-y_m)\sin\varphi_{n'}\right]$$
$$H_1^{(2)}(k|\rho_n-\rho_m|)\Delta C_m, \tag{2}$$

where $\varphi_{n'}$ is the angle shown in Fig.1 and δ_{mn} is the Kronecker delta function given by

$$\delta_{mn}=\begin{cases}1, & m=n\\ 0. & m\neq n\end{cases} \tag{3}$$

In matrix form

$$[\boldsymbol{Z}][\boldsymbol{J}]=\left[\boldsymbol{E}^i+\boldsymbol{H}^i\right],$$
$$\Rightarrow[\boldsymbol{J}]=[\boldsymbol{Z}]^{-1}\left[\boldsymbol{E}^i+\boldsymbol{H}^i\right], \tag{4}$$

where $[\boldsymbol{J}]$ is the unknown column vector representing the surface sources, $\left[\boldsymbol{E}^i+\boldsymbol{H}^i\right]$ are the known vectors describing the incident (or applied) electric and magnetic field, respectively. $[\boldsymbol{Z}]$ is the $[N\times N]$ known impedance matrix that can be defined as

$$Z_{mn}=\begin{cases}\dfrac{k}{4}\Delta C_m\left[H_0^{(2)}(k|\rho_n-\rho_m|)+\dfrac{j}{|\rho_n-\rho_m|}\right. \\ \times\left[(x_n-x_m)\cos\varphi_{n'}+(y_n-y_m)\sin\varphi_{n'}\right] \\ \left.\times H_1^{(2)}(k|\rho_n-\rho_m|)\right], & m\neq n \\ \dfrac{k}{4}\Delta C_m\left[1-j\left\{\ln\dfrac{\gamma_0 k\Delta C_m}{4}-1\right\}\right]+\dfrac{1}{2}. & m=n\end{cases} \tag{5}$$

2.2 Fast Multipole Method

The matrix formulation in FMM can be described by the following four steps :

Step 1:

The object surface is discretized into N number of unknowns.

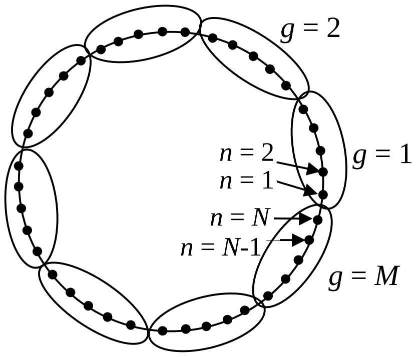


Fig. 2: Grouping of the discretized points.

Step 2:

Dividing the N number of unknowns into M number of localized groups as shown in Fig.2. Each coordinate point is labeled by their group number (g) and number of points (α) in that group as shown in Table 1.

Table 1: Grouping and indexing of discretizred points.

Grouping (g)	Indexing (α)	$n(m, \alpha)$
1	1	1
1	2	2
1	3	3
1	4	4
1	5	5
2	1	6
2	2	7
2	3	8
2	4	9
2	5	10

Step 3:

Consider the '*nearby*' basis functions within the same group and generate the sparse matrix of intra group. The elements of this near-field sparse matrix can be expressed as (by using same basis and weighting function as in MoM in section 2.1)

$$Z_{mn}^{n} = \begin{cases} \dfrac{k}{4}\Delta C_m \left[H_0^{(2)}\left(k\left|\rho_n-\rho_m\right|\right) + \dfrac{j}{\left|\rho_n-\rho_m\right|} \right. \\ \times\left[\left(x_n-x_m\right)\cos\varphi_{n'} + \left(y_n-y_m\right)\sin\varphi_{n'}\right] \\ \left. \times H_1^{(2)}\left(k\left|\rho_n-\rho_m\right|\right)\right], & m\neq n \text{ and } \left|m-n\right|\leq M \\ \dfrac{k}{4}\Delta C_m\left[1-j\left\{\ln\dfrac{\gamma_0 k\Delta C_m}{4}-1\right\}\right]+\dfrac{1}{2}, & m=n \\ 0. & \text{otherwise} \end{cases} \tag{6}$$

Step 4:

In this step another sparse matrix has to be created by considering the '*far-field*' interaction between inter group as shown in Fig.3.

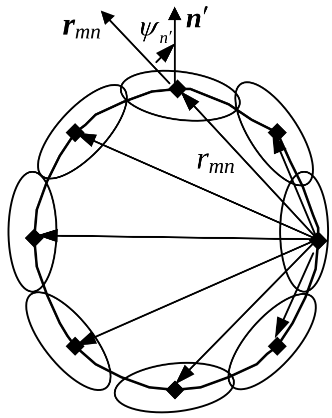


Fig. 3: Interaction among the inter group.

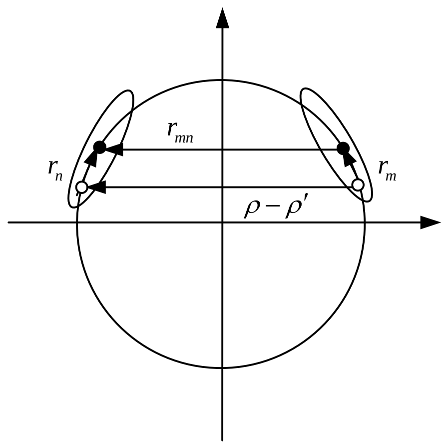


Fig. 4: Approximation while considering far-field.

The distance between observation point and source point can be approximated by using the concept of inter group of Fig.4 as

$$\boldsymbol{r}_m + \boldsymbol{r}_{mn} - \boldsymbol{r}_n = \rho - \rho'. \tag{7}$$

If $\boldsymbol{r}_m - \boldsymbol{r}_n$ is too small, then

$$\boldsymbol{r}_{mn} \cong \rho - \rho', \tag{8}$$

where $\boldsymbol{r}_{mn}$ is the distance between source points' group centre and the observation points' group centre.

Therefore, the elements of far-field sparse matrix can be expressed as

$$Z_{mn}^{f} = \begin{cases} \dfrac{k}{4}\Delta C_m \left[H_0^{(2)}\left(k|\boldsymbol{r}_n - \boldsymbol{r}_m|\right) + \dfrac{j}{|\boldsymbol{r}_n - \boldsymbol{r}_m|} \right. & (9) \\ \times\left[(X_n - X_m)\cos\psi_{n'} + (Y_n - Y_m)\sin\psi_{n'} \right] & \\ \left. \times H_1^{(2)}\left(k|\boldsymbol{r}_n - \boldsymbol{r}_m|\right) \right], & m \neq n \text{ and } |m-n| \le M \\ \dfrac{k}{4}\Delta C_m \left[1 - j\left\{ \ln\dfrac{\gamma_0 k \Delta C_m}{4} - 1 \right\} \right] + \dfrac{1}{2}, & m = n \\ 0. & \text{otherwise} \end{cases}$$

Finally, total current density $\boldsymbol{J}$ can be found as

$$\left[\boldsymbol{J}_n\right] = \left[\boldsymbol{Z}^n\right]^{-1}\left[\boldsymbol{E}^i + \boldsymbol{H}^i\right],$$

$$\left[\boldsymbol{J}_f\right] = \left[\boldsymbol{Z}^f\right]^{-1}\left[\boldsymbol{E}^i + \boldsymbol{H}^i\right],$$

$$\left[\boldsymbol{J}\right] = \left[\boldsymbol{J}_n\right] + \left[\boldsymbol{J}_f\right]. \qquad (10)$$

2.3 Proposed Method

To overcome the shortcomings of FMM and MoM [10], here we propose another method by using both the concepts of FMM and MoM.

In this method approximation of Eq.(7) is considered only when the groups are far apart. But when the groups are side by side, these are the fake approximations as shown in Fig.5.

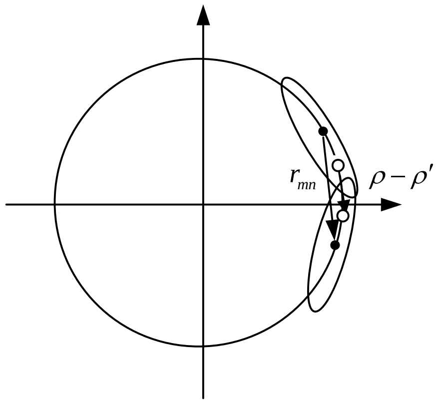


Fig. 5: Error in approximation when the groups are side by side.

Therefore,

$$\boldsymbol{r}_{mn} \neq \rho - \rho'. \qquad (11)$$

So, it is reasonable to use MoM concept for the '*nearby*' group.

Thus, the first two steps of FMM will remain same for the proposed method. The third step is applied on a certain group and its adjacent two groups, i.e.; matrix elements of three '*nearby*' groups are calculated using Eq.(6). The fourth step is applied on the remaining '*far-field*' group, where the approximation of Eq. (8) is quite reasonable.

3. NUMERICAL IMPLEMENTATION AND RESULTS

For numerical implementation, we considered a uniform shape perfectly conducting cylindrical body.

3.1 General Considerations

Let us consider the scatterer illuminated by the forwarded directed plane wave

$$E_z^i = e^{-j\boldsymbol{k}\cdot\rho}, \qquad (12)$$

where $\boldsymbol{k}$ is the wave propagation vector, ρ is the position vector.

The scatterer is discretized such that, the segment length is of around one tenth of wavelength, which gives good result with minimum number of unknowns N.

Linear segmentation with pulse-basis point-matching method is used to keep the minimum number of integration points, which can avoid the extra computational burden.

For simplicity, choose the characteristic impedance Z_c and constant p to be unity. To avoid the computational error each term is made unitless. For accuracy, all the floating point and complex data are taken in double precision in the MATLAB code.

3.2 Perfectly Conducting Body

As for example of perfectly conducting body, let us consider a cylinder as shown in Fig.6, on which a plane wave is incident from $-x$ direction as given by

$$E_z^i = e^{-jka\cos\phi}, \qquad (12)$$

and

$$H_c^i = -\cos\phi e^{-jka\cos\phi}, \qquad (13)$$

where k is the wave propagation number, a is the radius of the cylinder and ϕ is the angle in equatorial direction.

The numerical results of surface current densities on a circular cylinder using our proposed method are given in Fig. 7. Figures show the results for a cylinder of radius 7(a) a = 2 and 7(b) a = 7 wavelengths, respectively as a function of observation angle ϕ and compares it with the solution of MoM and FMM. The comparison shows that the curve obtained from our proposed method has the same shape as MoM and some improvement compare to FMM. It is observed that, there is some deviation in magnitude. But this error is negligible since it does not give any significant deviation in the scattering computation.

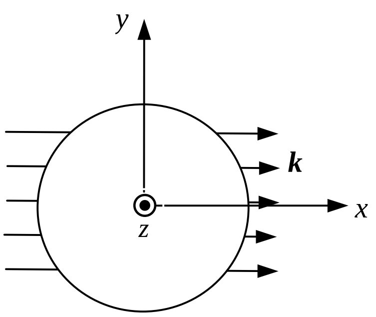
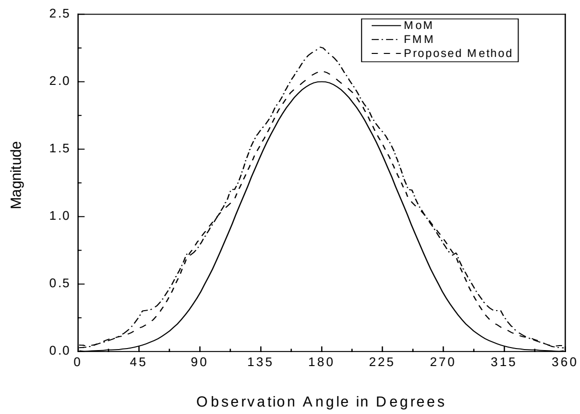
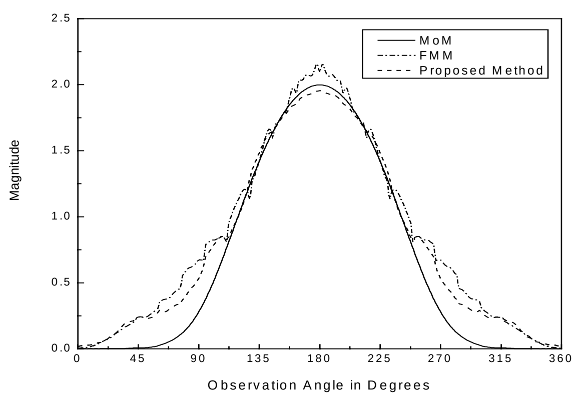


Fig. 6: Cross-section of cylindrical scatterer illuminated by *x*-directed plane wave.



(a)



(b)

Fig. 7: Surface current density on (a) $a = 2\lambda$, (b) $a = 7\lambda$ cylinder.

3.3 Storage Memory and CPU Time Requirements

In the propose method, the concept of FMM is used with matrix sparsity, which keeps the advantages of storage memory and CPU time savings. At the same time, by using the MoM concept for nearby groups, it improves the accuracy in results.

3.3.1 Storage Memory Requirements

Figure 8 shows the total memory usage (in Megabytes) in MoM, FMM and the proposed method. In each of the cases, same parameters are used for the same PC. In the sparse matrix, FMM require $N^{3/2}$ number of non-zero elements and proposed method require $3N^{3/2}$ number of nonzero elements, where N is the number of unknowns. But the dense matrix of MoM require N^2 number of non-zero elements, which causes requirements of higher memory, especially whenever the number of unknowns increases.

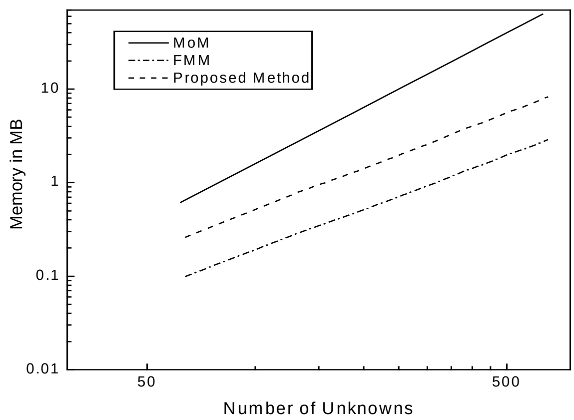


Fig. 8: Total storage memory required by MoM, FMM and Proposed method.

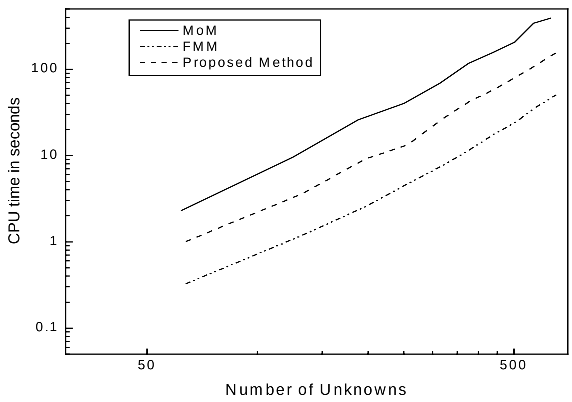


Fig. 9: Total CPU time required by MoM, FMM and Proposed method.

3.3.2 CPU Time Savings

Figure 9 shows the total CPU time (in sec) requirements in MoM, FMM and the proposed method. In FMM, it requires number of operations is only $O(N^{5/2})$. In contrary, the proposed method requires $O(3N^{5/2})$ number operation, which is only three times of that of FMM. But, due to the dense matrix, MoM require $O(N^3)$ operations.

3.4 Cost Comparison

Table 2 shows the computational cost savings of our proposed method by sacrificing the accuracy in minimum level with comparison to MoM. It sacrifices the accuracy level only in between 0.06% to 1.63% at the luminous zone with the 80% and 67% storage memory and computational time savings, respectively. Whereas, FMM saves 92% and 88% of storage memory and computational time requirements, respectively but losses the accuracy in between 5.6% to 7.0%, which is much more than the proposed method. Therefore, it is clear that, proposed method can improve the accuracy level with sufficient savings in storage memory and computational time requirements.

Table 2: Cost comparison for typical 4λ cylinder.

Methods	Storage Memory Savings (%)	CPU Time Savings (%)	Deviation of Current Densities at Luminous Point (%)
FMM	92	88	5.60~7.00
Proposed	80	67	0.06~1.63

4. CONCLUSIONS

A new computational technique based on FMM-MoM for the solution of electromagnetic scattering problem has been proposed in this paper. As we know, MoM forms dense matrix that requires large amount of storage memory and CPU time. On the other hand, FMM generates a sparse matrix, that can save storage memory and CPU time. However, in the accuracy level of MoM is better than FMM. In our proposed method we optimized in between two methods to get the maximum advantages.

Our proposed method is tested for analyzing scattering from a simple conducting cylindrical body and the results are compared with those obtained by MoM and FMM. Results show the appropriateness of our method. For a body with large dimension and complex structure, extra care must be taken for the discretization, choice of expansion function, etc. The proposed method may be applied to other arbitrary shaped 2D as well as 3D electromagnetic problems.

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