Japanese Journal of Applied Physics
Vol. 43, No. 6A, 2004, pp. 3620–3623

Applicability of Insensitive Properties of Measured Equation of Invariance Coefficients for Modified Scattering Objects

N. M. Alam CHOWDHURY, Jun-ichi TAKADA[1] and Masanobu HIROSE[2]

Department of Electrical & Electronic Engineering, Dhaka University of Engineering and Technology (DUET), Gazipur 1700, Bangladesh
[1]*Department of International Development Engineering, Tokyo Institute of Technology, Tokyo 152-8552, Japan*
[2]*EM waves division, NMIJ, AIST, Ibaraki 305-8568, Japan*

(Received September 7, 2003; accepted January 9, 2004; published June 9, 2004)

The applicability of insensitive properties of measured equation of invariance (MEI) coefficients for the computation of scattering from modified structured bodies is presented in this letter. During scattering computation by using MEI technique (in IE-MEI or SIE-MEI method), the MEI coefficients of the whole scatterer is calculated in the conventional method. In contrast, using the insensitive properties of the MEI coefficients, the new method calculates the MEI coefficients only around the modified area if some portion of the scatterer is modified and reuse those for the other portion of the scatterer. Thus CPU time for solving the modified problem can be saved by using the new method. The numerical results verified the validity of the new technique which is compared with the available numerical solutions. [DOI: 10.1143/JJAP.43.3620]

KEYWORDS: Insensitive properties of MEI coefficients, IE-MEI, SIE-MEI, scattering from modified structure, cube

1. Introduction

The measured equation of invariance (MEI) technique[1)] and hence MEI coefficients were used by the integral equation formulation of MEI (IE-MEI) and scalar-field approach of IE-MEI (SIE-MEI) methods[2–4)] to solve the wave scattering problem effectively. In this letter, we focus on the insensitive properties of MEI coefficients with respect to the small modification of the scatterer shape which can broader the applicability of MEI technique. In the derivation of MEI coefficients, we use the property of the local geometrical dependence. Using these properties in the scattering computation of modified scatterer, the computational time can be saved.

The newly founded insensitive properties of MEI coefficients can be applied to any method that uses MEI technique for the scattering computation. In this letter we describe only with the SIE-MEI method for the 3D problem. For 2D case this technique can easily be implemented without any modification.

2. SIE-MEI Method

In the SIE-MEI method, the discretized version of local linear equation of the problem can be expressed as[4)]

$$\sum_m \left[a_{nm}\phi_1(\boldsymbol{r}_m) - b_{nm}\frac{\partial\phi_1(\boldsymbol{r}_m)}{\partial n}\right] = 0, \tag{1}$$

where a_{nm} and b_{nm} are the unknown MEI coefficients for the $n(=1,2,\cdots N)$-th node associated with the $m(=1,2,\cdots M)$ number of neighboring nodes, and ϕ_1 and $\partial\phi_1/\partial n$ are the scattered wave and its normal derivative generated by the suitable Q-sets of secondary sources called *metrons*.[4)]

In matrix form, eq. (1) becomes

$$[\mathbf{C}\ \mathbf{D}]\begin{bmatrix}\mathbf{a}\\ \mathbf{b}\end{bmatrix} = 0, \tag{2}$$

where $[\mathbf{C}\ \mathbf{D}]$ is the $[Q \times 2M]$ known matrix composed of metron fields and their normal derivatives and $\left[\begin{smallmatrix}\mathbf{a}\\ \mathbf{b}\end{smallmatrix}\right]$ is the column vector of unknown MEI coefficients composed of invariant local sources.

This local matrix [eq. (2)] is solved repeatedly for the whole scatterer surface by using least square solution. Therefore, two sparse matrices **A** and **B** of MEI coefficients which are invariant to excitation are obtained. Finally, by using these sparse matrices, the desired solution of the scattering problem can be derived.

3. Insensitive Properties of MEI Coefficients

The insensitive properties of MEI coefficients is effective when the scatterer structure is modified in order that the scattered field should be recalculated.

3.1 MEI postulates

As we know, MEI postulates[1)] are
(1) local sources exist near the scatterer, and these sources are:
(2) dependent on the scatterer geometry,
(3) dependent on the position,
(4) invariant to the excitation field.

The postulates 1 and 4 state that, MEI coefficients can be derived from the solutions of integral equations with different metrons. Postulates 2 and 3 state that these coefficients are depend on position and scatterer geometry.

According to the postulates, it has been considered that the derived MEI coefficients are also dependent on the whole scatterer geometry. But in fact, they do not depend so much on the whole scatterer geometry. As in eq. (2), the MEI coefficients of each node are derived from the metron fields at the node for the possible sets of metrons, combined with the interaction of metron fields at the neighboring nodes associated in the local region. Again, the metron field that are derived from the local sources, mostly depends on the local geometry of the scatterer and do not give significant effect on the other portion of the scatterer. This local geometrical dependency i.e., insensitive properties of MEI coefficients can be applied for the scattering computation of modified structure to save the computational time.

3.2 Flow-chart of conventional and proposed solution techniques

Figure 1 shows the flow-chart of conventional solution technique to derive the equivalent surface source by using SIE-MEI method. If some small portion of the scatterer is modified, we must follow the same procedure again, which

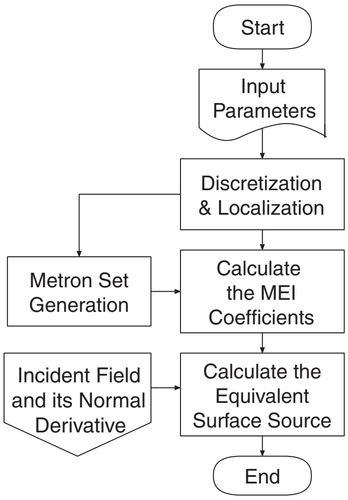

Fig. 1. Flow-chart of conventional solution technique.

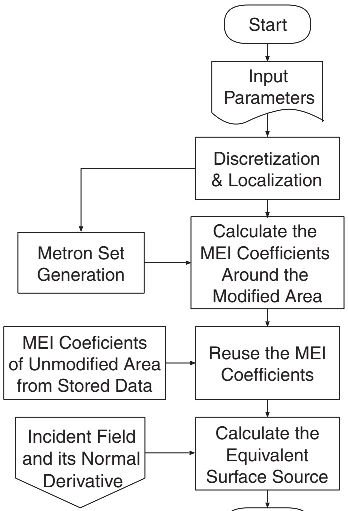

Fig. 2. Flow-chart of proposed solution technique.

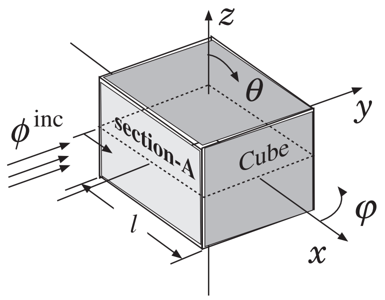

Fig. 3. Plane wave incident on a cube.

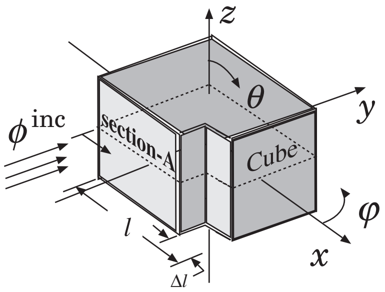

Fig. 4. Plane wave incident on a cube with concave modification.

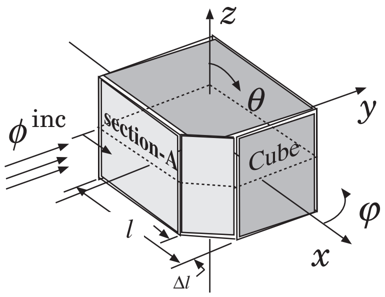

Fig. 5. Plane wave incident on cube with convex modification.

takes large time for the computation. On the other hand, we propose a new solution technique which is summarized in the Fig. 2, on the basis of the insensitive properties of MEI coefficients.

According to the flow-chart of Fig. 2, at first we store the MEI coefficients of the whole structure of original body during scattering computation. Then we modify the structure and calculate the MEI coefficients only around the modified area and reuse the stored data for the other portion. Because of the insensitive properties of MEI coefficients, new set of data gives almost the same result as the result obtained by the conventional solution process. Thus, by following the proposed solution technique we can avoid the extra computational burden and hence save the computational time.

4. Numerical Implementation

Let us consider a cube as shown in Fig. 3, on which plane wave is incident to the $+y$ direction. Figures 4 and 5 show the same cube with Δl concave and Δl convex modification, respectively, at one of the edges parallel to z axis. In all of these cases, the same type of plane wave is incident from the same direction.

4.1 Inner product between two vectors

As a measure of the insensitive properties of the MEI coefficients, we use the three kind of the inner products between the vectors **a** and **b** by eq. (2) at each node of the same (or unmodified) part between the original cube and the modified cube. The first inner product is between the vector **a** for the original cube and the vector $\tilde{\mathbf{a}}$ for the modified cube at each node of the unmodified part, then normalized by the magnitude of the two vectors, indicated by "$r_a = |\mathbf{a}^H\tilde{\mathbf{a}}|$" in Fig. 6. Similarly, the second one is between the vector **b** and $\tilde{\mathbf{b}}$, indicated by "r_b" and the third one is between the vector consisting of the vector **ab** and $\tilde{\mathbf{a}}\tilde{\mathbf{b}}$, indicated by "$r_{ab}$" in Fig. 6.

Figure 6 shows the three kind of the inner products between the vectors for both cubes at each node of the unmodified part along the section-A in Fig. 5. From Fig. 6, all inner products are seen to be nearly the unity. It ensures the insensitive properties of the MEI coefficients. Therefore, if some small portion of the scatterer is modified, we can get

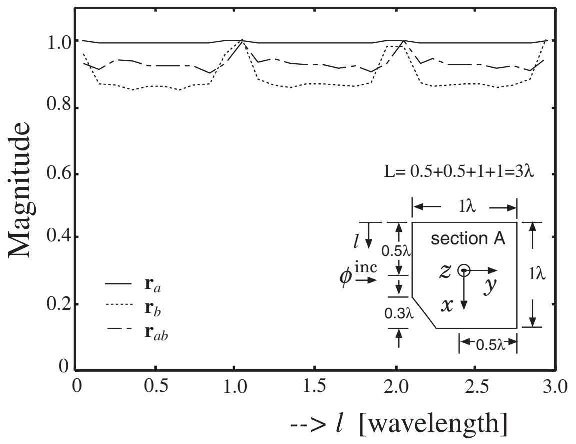

Fig. 6. Normalized inner product between the vectors.

the desired solutions and save the computational time by following the proposed procedure of Fig. 2.

4.2 *Equivalent surface source*

Figures 7, 8, and 9 show the 2D plot of equivalent surface source along the perimeter of section-A (Figs. 3, 4, 5) for the side of $l = 1\lambda$. Each of the graphs contains the results of *full cube* (or the original cube) by using conventional solution technique and of the modified cube by using *conventional* and *proposed* solution technique, in the SIE-MEI method.

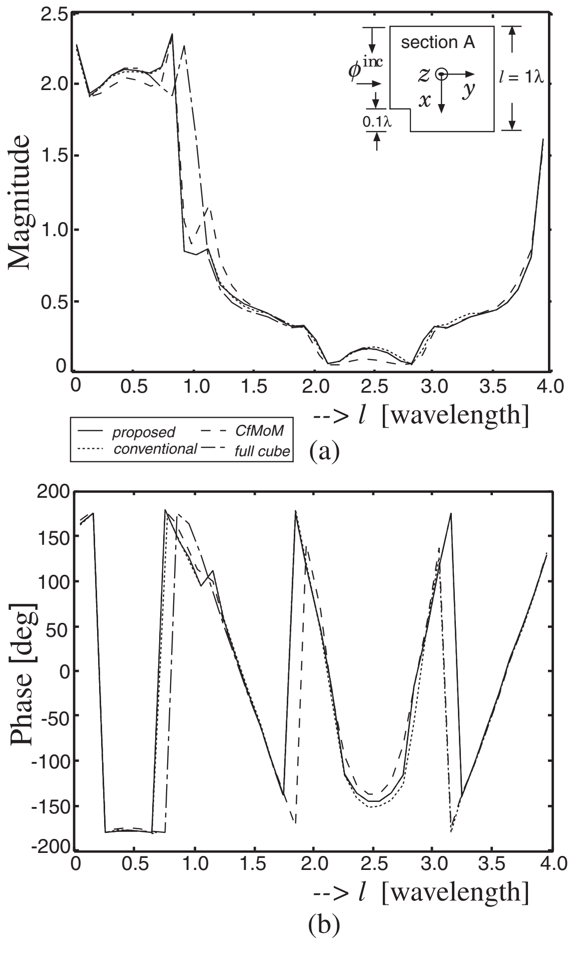

Fig. 7. Equivalent surface source on the cube with 0.1λ concave modification (a) magnitude, (b) phase variation.

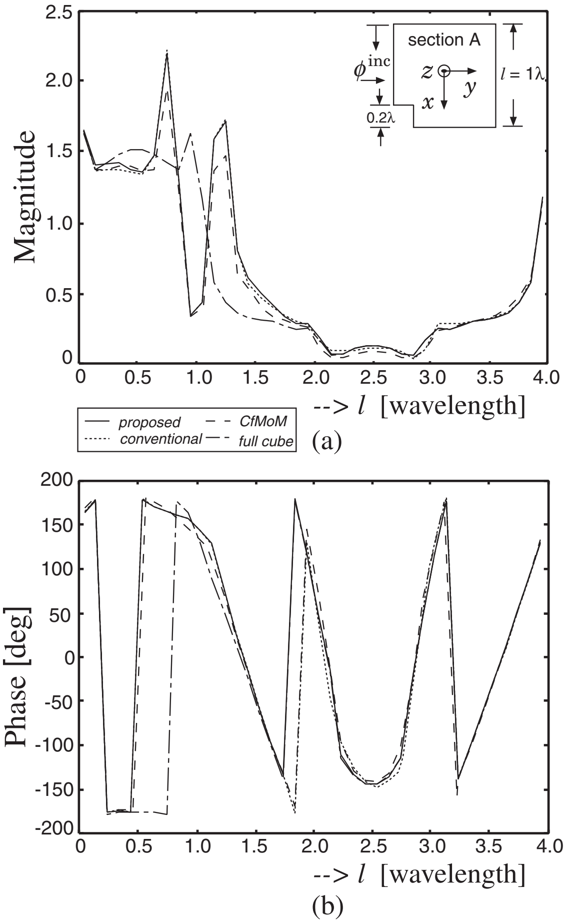

Fig. 8. Equivalent surface source on the cube with 0.2λ concave modification a) magnitude, b) phase variation.

The results of the modified cube are also compared with the numerical solution using Combined-field Method of Moments (*CfMoM*).[5)]

From the results shown in Fig. 7 and Fig. 8, it is seen that, due to the concave nature in the modified section, the results by the SIE-MEI method slightly deviate from those by the CfMoM. This deviation comes from multiple reflections in the concave parts of the modified section. But the results in the other parts are in very good agreement.

In contrast, the results in Fig. 9 corresponding to Fig. 5 are in excellent agreement with those by the CfMoM except the shadowed region, but it does not give any significant effect on the scattering characteristics.

4.3 *Comparison of CPU time*

In SIE-MEI method, most of the computational time is spent in the integration process to derive the MEI coefficients. Since the matrix is sparse, the time required for matrix inversion is very small compared to the time required in the integration process. In the proposed solution technique, we compute the MEI coefficients only around the modified region and reuse the stored MEI coefficients for the rest of the scatterer. Thus the new solution technique can save a large amount of computational time which increases rapidly as the size of the scatterer with the same modification increases.

In the comparison we use cube of 1λ and 2λ side length

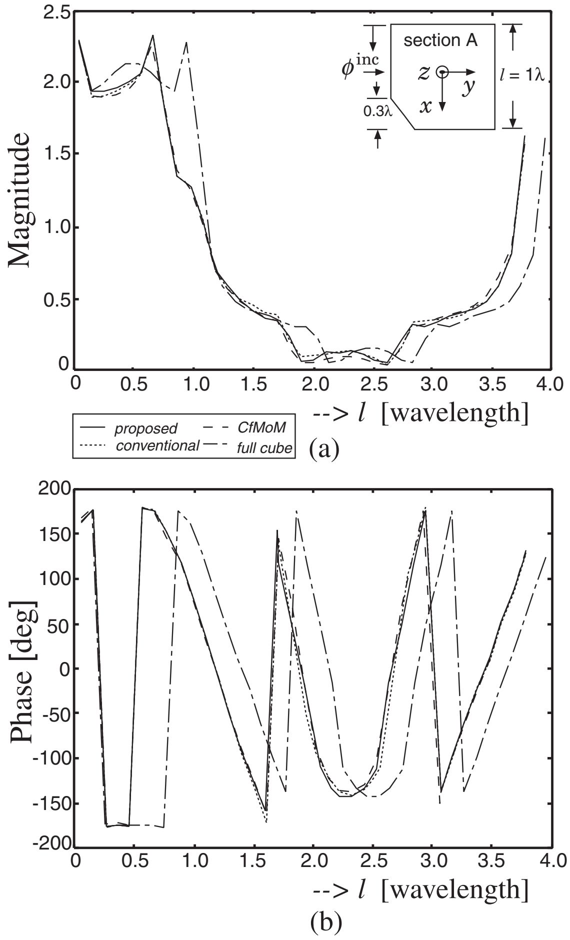

Fig. 9. Equivalent surface source on the cube with 0.3λ convex modification a) magnitude, b) phase variation.

with 10 segments per wavelength and 5 segments in the local region. As the cube is modified as Fig. 5, for 1λ cube the required number of MEI coefficients in the conventional and proposed solution techniques are 94% and 15.6%, respectively, with respect to the full cube. Similarly, for 2λ cube these are 97% and 6.8% with respect to the full cube. Thus by reducing the computation of MEI coefficients we can save the CPU time which is increases with the increase of the scatterer size.

From the comparison, it is clear that, by using proposed solution technique based on the insensitive properties of MEI coefficients for the scattering computation of modified scatterer we can save the computational time.

5. Conclusion

A new solution technique is proposed for the scattering from modified body based on the insensitive properties of MEI coefficients. This technique is implemented on the scatterer when its geometry is modified and its scattering characteristics need to be recalculated. In the SIE-MEI method, the dominant part of the computation is to derive the MEI coefficients. Thus, by avoiding the computation for some part of coefficients we can save large amount of CPU time with the same accuracy in the result. To verify our proposed technique the results are compared with the available numerical solutions and they are in good agreement.

If the whole scatterer size is much larger than its modified area, the propose technique gives more similar result to the original one as expected and saves much more CPU time.

Acknowledgment

The authors are indebted to Professor Makoto Ando of Tokyo Institute of Technology for his valuable suggestion on the locality of SIE-MEI method and its impact on the scattering computation of modified structured body.

1) K. K. Mei, R. Pous, Z. Chen, Y. W. Liu and M. D. Prouty: IEEE Trans. Antennas Propag. **42** (1994) 320.
2) J. M. Rius, R. Pous and A. Cardama: IEEE Trans. Magn. **32** (1996) 962.
3) M. Hirose, M. Miyake, J. Takada and I. Arai: Radio Sci. **34** (1999) 65.
4) N. M. Alam Chowdhury, J. Takada and M. Hirose: IEICE Trans. Fundament. **E85-A** (2002) 1905.
5) N. Morita, N. Kumagai and J. R. Mautz: *Integral Equation Methods for Electromagnetics* (Artech House, Boston, 1991) Chap. 3, p. 215.