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Formation Of Slow Wave Structure (SWS) Using A Partially Filled Smooth Walled Cylindrical Waveguide (SCW) With A Dielectric Liner And Prediction of Cerenkov Resonance

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ABSTRACT

A simple method of obtaining a slow wave structure has been proposed in this paper. When a cylindrical waveguide is partially filled then with a dielectric liner the waveguide mode will be found to propagate frequencies for which the phase velocity of the mode is less than the velocity of light. Hence prediction of cerenkov radiation may be possible. Different values of permitivity changes the value of cut-off and resonant frequencies of the structure.

1. INTRODUCTION

High power microwave sources are important for a number of advanced applications ranging from current drive and RF heating of magnetically confined plasmas in fusion devices to high resolution nano second radar [1]-[3].One of the ways of generating high power microwave is to couple slow space charge waves of an intense relativistic electron beam (IREB) with a slow-guided electromagnetic wave [4]. The modes that have electric field components in the direction of the beam propagation and which have velocities less than the electron beam velocity, can decelerate the beam and convert the beam kinetic energy into electromagnetic wave energy as is observed in TWT and Klystrons. Stimulated emission results, because the spontaneously emitted cerenkov radiation due to the presence of axial electron beam bunches the cerenkov emission. As the wave grows the beam electrons are slowed and become trapped between wave crests [5].

To establish the above facts we studied a Slow Wave Structure (SWS) [6]. Because without an SWS no interaction with an IREB is possible. To form an SWS we propose to incorporate a dielectric liner on the wall of a smooth cylindrical waveguide (SCW). An SCW always guides electromagnetic modes having phase velocities higher than the velocity of light. Practically an IREB can not possess such a speed. In this paper it is shown by theoretical analysis that a dielectric liner of suitable permittivity and thickness may form an SWS for Cerenkov interaction with an IREB in the wave guide.

In an empty SCW the well-known dispersion relation is

$$\omega^2=\omega_{c0}^2+k^2c^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

and

$$\omega_{c0}=\frac{\rho_{L\gamma}}{R_0}C \quad \ldots (2)$$

$\rho_{L\gamma}$ is the γth. root of the lth order Bessel's function of the first kind.

where

ω is the angular frequency,
k is the axial wave-number
ω_{c0} is the cut-off angular frequency
c is the velocity of light
Ro is the radius of the cylindrical wave guide.

Equation (1) reveals that the phase velocity of the electromagnetic mode is always greater than the velocity of light. Fig. 1 represents the fact that a beam line can not interact with this wave.

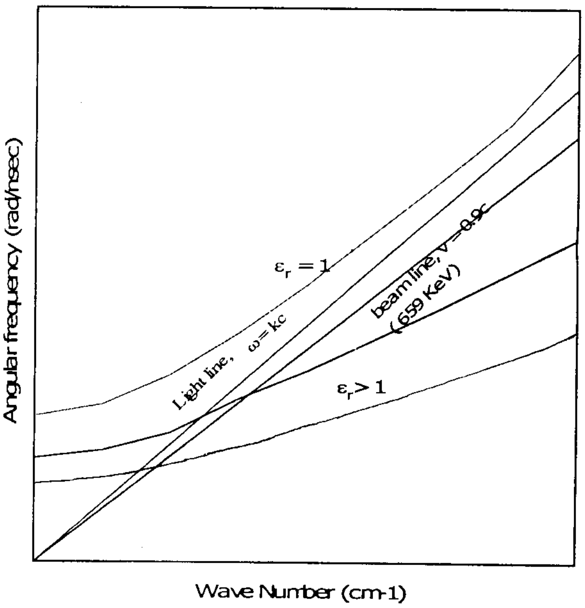

Fig. 1: Dispersion curves of an SWS for different permittivity. Here the beam line or light line intersects the dispersion curve predicting Cerenkov resonance

Section II of this paper contains the derivation of a dispersion relation for waves in an infinitely long dielectric loaded cylindrical waveguide. Section III presents the numerical analyses of the slow-wave phenomena. A discussion of this work along with conclusions is presented in section IV.

2. DISPERSION CHARACTERISTICS OF CYLINDRICAL WAVEGUIDE HAVING A DIELECTRIC LINER ON ITS WALL

As shown in Fig. 2 let us consider an axially symmetric cylindrical waveguide of radius 'b'. The region 0= r = a empty but the region a = r = b is filled with a dielectric liner.

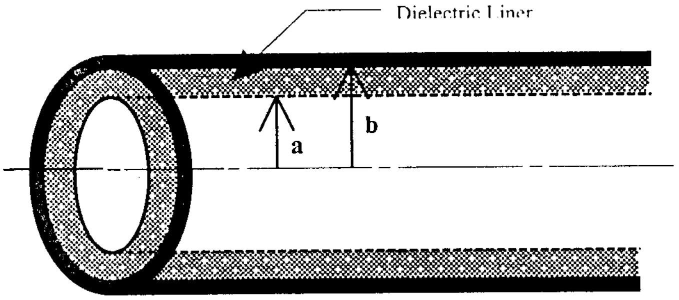

Fig. 2: A smooth cylindrical Waveguide(SCW) Loaded by a dielectric liner to form a Slow Wave Structure (SWS)

To find the solution for an axisymmetric TM mode, let us start from Maxwell's curl equations in a source free region with $e^{j\omega t}$ suppressed.

$$\nabla x \overline{E} = \frac{-\delta \overline{B}}{\delta t} = -j\omega\mu\overline{H} \qquad \ldots (3)$$

$$\nabla x \overline{H} = \frac{\delta D}{\delta t} = j\omega\mu\overline{E} \qquad \ldots (4)$$

Applying curl operation in equation (3) we have

$$\nabla x \nabla x \overline{E} = -\nabla x\ j\omega\mu\overline{H} . = \frac{\omega^2}{C^2}\varepsilon_r \overline{E} . \qquad \ldots (5)$$

Considering a uniform TM mode of propagation the radial component of the electric field is given by

$$\Rightarrow E_r = \frac{jk \qquad \delta E_z}{\frac{\omega^2}{C^2}\varepsilon_r - k^2 \quad \delta r} \qquad \ldots (6)$$

In the above equation E_z is the axial electric field component which is solution of

$$\frac{\delta^2 E_z}{\delta z^2} + \frac{1}{r}\frac{\delta E_z}{\delta r} + x^2 E_z = 0 \qquad \ldots (7)$$

where $x^2 = \frac{\omega^2}{C^2}\varepsilon_r - k^2$

For the proposed model the solution of equation is

$$E_z = AJ_0(xr) \qquad 0 \le r \le a \qquad \ldots (8)$$

where $x = \sqrt{\frac{\omega^2}{C^2} - k^2}$

$$E_z = BJ_0(yr) + DN_0(yr) \qquad a \le r \le b \qquad \ldots (9)$$

where $y = \sqrt{\frac{\omega^2}{C^2}\varepsilon_r - k^2}$

J_0 and N_0 are zeroth order Bessel's and Neuman's functions respectively. In the above A, B, and D are arbitrary constants. The RF electric fields must satisfy boundary condition that at the perfectly conducting wave-guide surface , the tangential electric field must be zero. E_z and $D_r = \varepsilon E_r$ must be continuous across the boundary between two regions. Applying boundary conditions, elimination of A, B and D yields the following dispersion relation.

$$\frac{\varepsilon_r\sqrt{\left(\frac{\omega^2}{C^2} - k^2\right)}\, J_0(xa)[J_1(ya)N_0(yb) - J_0(yb)N_1(ya)]}{\sqrt{\left(\frac{\omega^2}{C^2}\varepsilon_r - k^2\right)}}$$

$$= J_1(xa)[J_0(ya)N_0(yb) - J_0(yb)N_0(ya)] \qquad (10)$$

J_1 and N_1 are first order Bessel's and Neuman's functions.

3. NUMERICAL RESULTS

The effect of a dielectric liner on the dispersion characteristics of the wave guide is studied by numerical solutions of equation (10). For this purpose a FORTRAN programme has been developed for Bessel's and Neumans functions with complex arguments. The transcendental equation (10) has been solved for ω versus k. The oscillation frequency ω is real for a real-valued wave number k. The following parameters were used in the computations: relativistic factor =2.23 (v=0.9c). The thickness of a dielectric liner is taken as 0.5 cm.

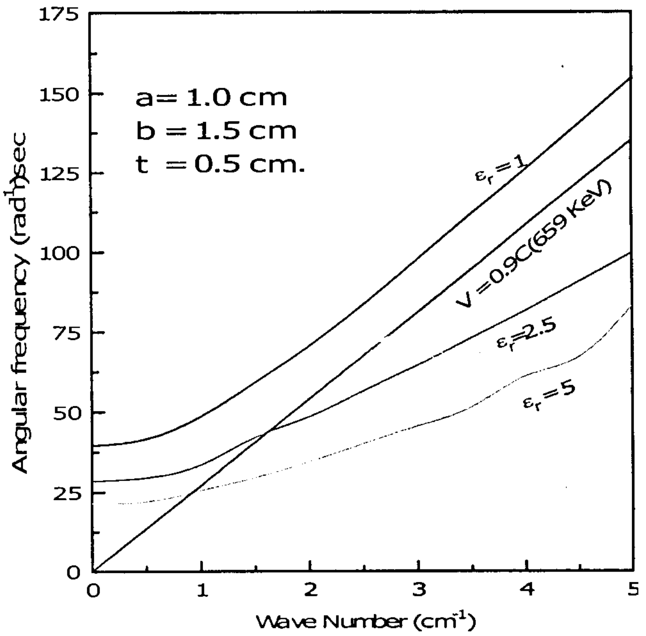

Fig. 3: Dispersion curves of an SWS with different values of permittivity of a dielectric liner.

The value of the relative permittivity of the dielectric liners were chosen as 1, 2.5 and 5. The ω–k plots are shown in Fig. 3. For every value of this permittvity the corresponding cut-off and resonant frequencies were noted. The cut-off frequency for =1 is 5.89 GHz. But the cut-off frequencies for ε_r =2.5 and 5 were 4.21 and 3.18 GHz respectively. The corresponding resonant frequencies were 6.37 GHz and 3.87 GHz respectively. No resonance was possible for ε_r = 1. The variations of cut-off and resonance frequencies with ε_r of the dielectric liner are shown in Fig. 4 and 5.

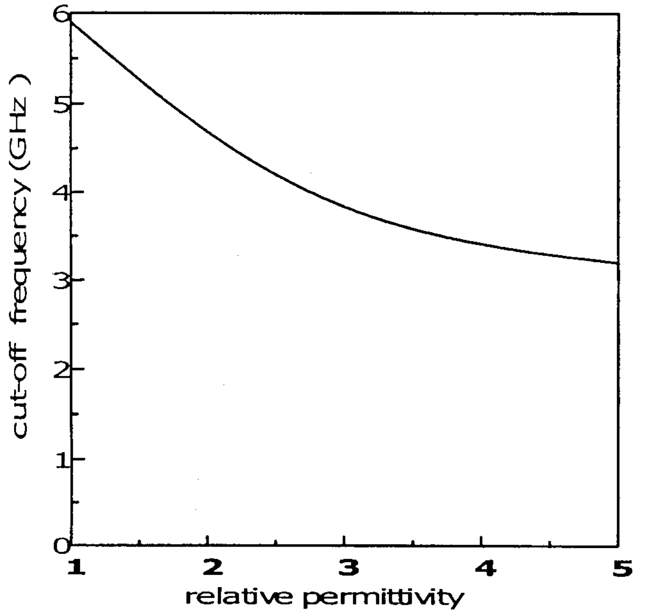

Fig. 4 : Relation between cut-off frequency and relative permittivity

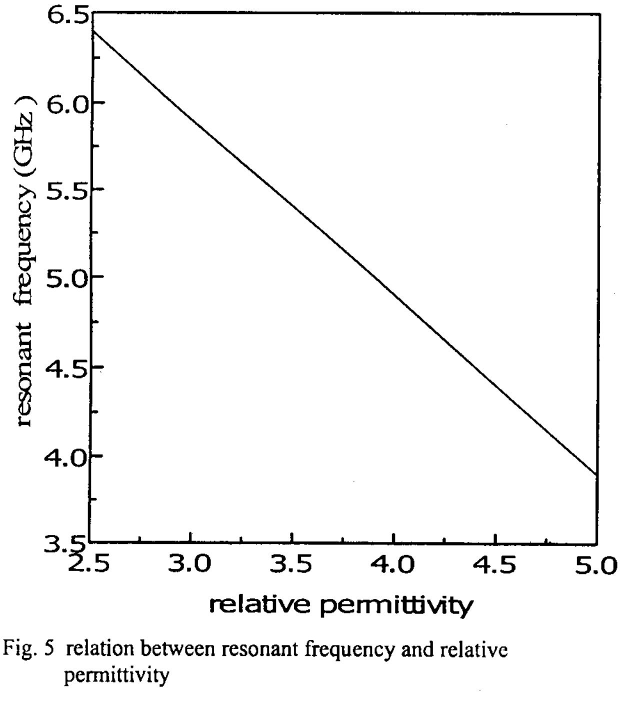

Fig. 5 relation between resonant frequency and relative permittivity

4. DISCUSSION AND CONCLUSION

The dispersion relation of an SWS formed in an SCW partially filled with a dielectric liner has been studied numerically. Due to incorporation of a dielectric liner on the surface of an SCW the phase velocity of the wave guide mode become less than that of light forming an SWS. Due to such slow wave phenomena interaction with an IREB is possible for cerenkov resonance

It is seen that with the increased value of the permittivity of the dielectric liner the cut-off and resonant frequencies decrease almost linearly.

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