Global and Local Control of Spatiotemporal Chaos in Coupled Map Lattices

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We present a simple method, using constant pinnings, to suppress spatiotemporal chaos and achieve global control in coupled map lattice models under different situations, e.g., for uniform pinning, nonuniform pinning with regular or random distributions, and lattices with spatial heterogeneity in local dynamics and coupling strength. The method is easy to implement and does not require any *a priori* information of the system dynamics or explicit changes in its parameters. This method can also be used for local control of spatiotemporal dynamics, an aspect that has crucial importance in many natural systems. [S0031-9007(98)06896-3]

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Spatially extended systems are commonly described using coupled map lattice (CML) models which exhibit a wide variety of novel and complex spatiotemporal behaviors including spatiotemporal chaos (STC) for different levels of spatial coupling and nonlinearity in the local dynamics [1]. Along with physicochemical systems such as plasma devices, laser systems, and chemical reactions, the CMLs are also being increasingly used in modeling spatially extensive excitable media in biology, such as the cardiac, neural, or retinal tissue, and metapopulations in ecology, where the coupled discrete nature of the media and the resulting spatiotemporal dynamics has significance in both physical and biological functions [2]. Alterations in the normal functions in these systems lead to pathological conditions, and thus control of spatiotemporal dynamics has major implications in biological functions. However, only a few methods have been proposed for controlling such spatially extended systems [3–5].

The dynamical control of spatially extended systems can have two different motivations: (a) control of the full system by manipulating all or parts of the system, and (b) controlling only a localized spatial region, leaving the rest of the system unperturbed. The first is needed when one desires to exert global control over the system in the event of its exhibiting undesirable dynamics, e.g., instabilities in coupled chemical reactors, or in arrays of Josephson junctions, etc. The second is extremely useful in situations where local control is required without interfering with other parts of the system, viz., suppressing activities of an ectopic node in the heart, or introducing localized alterations in neural tissues.

In this Letter we propose a novel and simple method to control the spatiotemporal dynamics in CMLs by applying constant pinning in the spatial domain. The STC in the lattice can be suppressed by pinning all sites uniformly, and the high dimensional system can thus be stabilized in any desired periodic state on appropriately varying the pinning strengths in few time steps (∼50). Global suppression of STC (as measured by negative maximum Lyapunov exponent, $\lambda_{\max}$) can also be achieved by this method by the application of regularly spaced or randomly distributed nonuniform pinnings. The major advantage of the method is that, unlike other feedback or adaptive control algorithms [3,6], it does not require any *a priori* knowledge of the system dynamics, such as stable or unstable fixed points and periodic orbits, nor does it require modifying or tracking any of the system parameters or variables explicitly. Therefore this method can also be easily used to suppress chaos in lattices with heterogeneity in the local nonlinear parameter or in the coupling strength—a situation common in reality. In addition to global control, we show that this method can also effectively control spatiotemporal dynamics in spatially localized regions in CMLs.

The diffusively coupled map lattice model in one dimension is given by the following general form [1]:

$$x_{n+1}(i) = (1 - \epsilon)f(x_n(i)) + \frac{\epsilon}{2}[f(x_n(i-1)) + f(x_n(i+1))], \quad (1)$$

where $n = 1, 2, \ldots, N$ are the discrete time steps, $i = 1, 2, \ldots, L$ is the discrete lattice sites with periodic boundary conditions, $x_n(i)$ represents a continuous state, ϵ is the diffusive coupling strength to the nearest neighbor sites, and $f(x)$ governs the local dynamics. Here $f(x) = rx(1 - x)$ for $1 \leq r \leq 4$, $0 \leq x \leq 1$, which shows the universal bifurcation structure of period doubling route to chaos with increasing nonlinear parameter r [7].

The method proposed to control the spatiotemporal dynamics in CMLs involves applying constant pinnings on the lattice sites as follows:

$$x_{n+1}(i) = (1 - \epsilon)f(x_n(i)) + \frac{\epsilon}{2}[f(x_n(i-1)) + f(x_n(i+1))] + p_n(i), \quad (2)$$

where $p_n(i)$ represents the pinning strength at the ith site at nth time step. It can assume negative or positive values depending on the nature of the local dynamics [8] and can be either uniform and the same on all the lattice sites or nonuniform and site dependent; i.e., it assumes different constant values at different sites. For uniform global pinning, $p_n(i) = p$, for $i = 1, 2, \ldots, L$. For nonuniform pinning, $p_n(i) = \delta(i - i_p)p$, where i_p is the inverse of the pinning density, p_d: if $\delta(i - i_p) = 1$, the ith site is said to be pinned and takes a finite value p, or else $p_n(i) = 0$. For all numerical simulations the lattice length is $L = 60$, and the initial conditions are chosen randomly from the range $x_c \pm 0.01$, where x_c is the critical point of the local map function ($x_c = 0.5$ here) [9].

Global control of STC with uniform pinning.—(i) Homogeneous lattices: We show the effect of uniform pinning on the CML dynamics for a homogeneous lattice (same r and ϵ at all the sites). Figure 1(a) summarizes the long term spatiotemporal behavior of the CML when subjected to both negative and positive pinning strengths in the (r-p) parameter space. In a uniformly pinned lattice, the long term dynamics is similar at all the sites and depends only on the pinning strength p. Figure 1(b) shows the dynamical behavior of a lattice site in a CML with $r = 4$ for varying pinning strengths. A clear period-halving behavior is seen with a decrease in the magnitude of p, from spatiotemporally chaotic to periodic and equilibrium behavior. Thus it is possible to find the proper choice of pinning strength required to control spatiotemporally chaotic dynamics in homogeneous CMLs to desired stable states using this simple method. It is clear that one need not have any information about the system parameters or stable/unstable states of the CML to implement this method.

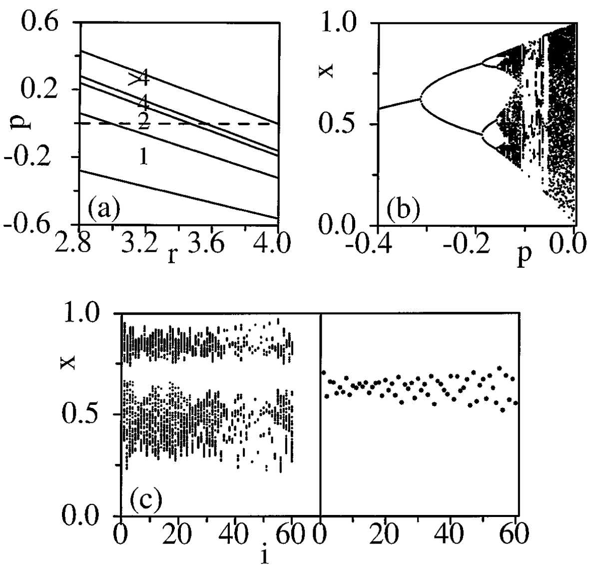

FIG. 1. (a) Spatiotemporal behavior of the CML in r-p parameter space. The regions marked 1, 2, and 4 exhibit fixed-point, 2 and 4 period dynamics; >4 includes higher periodic states and STC with small stable periodic windows; $x_n(i) \to -\infty$ in the rest of the region. (b) The period-halving behavior exhibited at $i = 30$ in the CML ($r = 4$, and $\epsilon = 0.8$) as a function of uniform p. (c) STC (shown in left) in a spatially heterogeneous CML ($r = 3.9 \pm 0.1$, $\epsilon = 0.7 \pm 0.01$) is suppressed to fixed point state (shown in right) by pinning uniformly with $p = -0.15$; here data for 100 consecutive time steps are superimposed after eliminating 1000 transients.

(ii) Heterogeneous lattices: Most natural systems are unlikely to have the same system parameters over the entire spatial domain [e.g., the growth rate (r) may not be identical in all the subpopulation patches, or the junctional coupling strengths (ϵ) may vary in arrays of coupled nonlinear oscillators or within cells in an excitable tissue [2]]. The left side of Fig. 1(c) shows the STC in a heterogeneous CML with small random variations in r and ϵ. In contrast to the homogeneous lattice where all sites exhibit identical dynamics, here, because of spatial parametric variations, suppression of STC is achieved with different lattice sites settling down to different fixed point dynamical states for a chosen value of pinning strength [see the right side of Fig. 1(c)]. It may be noted that all control algorithms which involve parametric corrections to control the system dynamics [3,6] will not work easily in such situations.

Global control of STC with nonuniform pinning.—Having uniform distribution of probes over the entire spatial domain to control the system dynamics is neither very efficient nor always practically feasible. This method can also be used to suppress STC (measured by $-\lambda_{\max}$) in the CMLs with nonuniform pinning at a lower density of pinned sites.

(i) Regularly spaced pinned sites: The left side of Figs. 2(a) and 2(b) show weak and strong STC in CMLs. This chaotic dynamics can be suppressed (shown in the right sides of the figures) by applying nonuniform pinning at a very low density $p_d = 0.1$ [i.e., one in ten sites in Fig. 2(a)] and a denser distribution of $p_d \leq 0.25$ for a highly chaotic lattice [in Fig. 2(b)]. Fixed point control of STC can be achieved by pinning the alternate sites ($p_d = 0.5$) of the CML.

When $p_d < 1$, ϵ plays an important role in the suppression of STC at higher values of r. In Fig. 3 is shown the behavior of the $\lambda_{\max}$ for the full CML, for increasing values of ϵ at different pinning densities. There exists more than one window in ϵ at pinning densities $p_d \geq 0.25$ where STC is suppressed in a highly chaotic CML ($r = 4$). Control is easily achieved in a weakly chaotic lattice ($r = 3.6$) even for $p_d \leq 0.25$ over a large range of ϵ. Thus, control of STC is possible in CMLs over a wide range of ϵ and p_d at low r values, but at higher r, only a narrow range of ϵ and p_d is available for control.

(ii) Random distribution of pinned sites: Instead of pinning a few regularly spaced lattice sites, in some practical situations it may be advantageous to be able to suppress chaos by pinning sites randomly. For example, to suppress certain types of cardiac arrhythmia (e.g., tachycardia), electrical stimulations are generally applied to different parts of the whole tissue [3,5,10]. In Fig. 4 we show the results culled from 100 experiments each on randomly

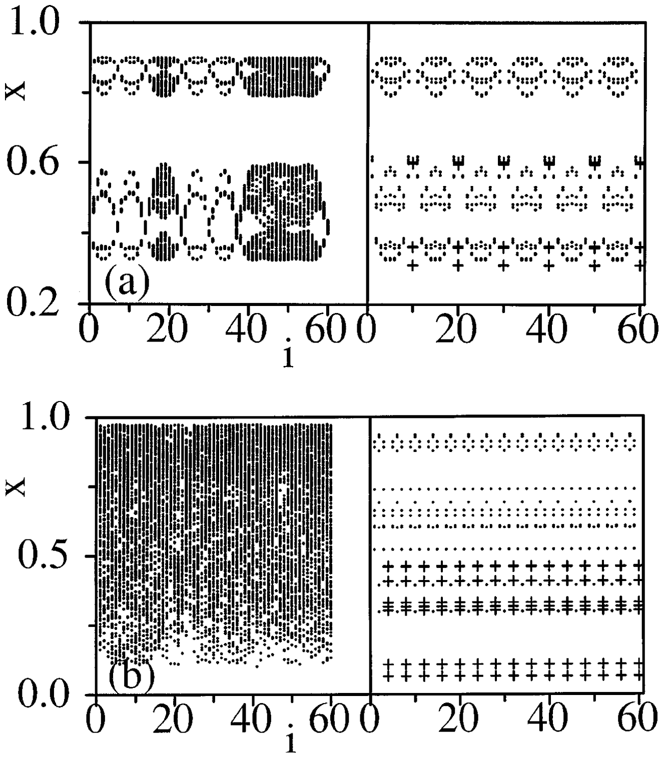

FIG. 2. Space-amplitude plots for nonuniform pinning ("+" denote the pinned sites). (a) CML ($r = 3.6$) exhibiting two-band STC (left side) and suppression of STC ($\lambda_{\max} = -0.13$) with $p = -0.25$ and $p_d = 0.1$ (right side); (b) CML ($r = 3.9$) exhibiting fully developed STC (left side) and suppression of chaos ($\lambda_{\max} = -0.07$) with $p = -0.45$ and $p_d = 0.25$ (right side). Here $\epsilon = 0.7$.

pinned CMLs for increasing pinning densities, where the suppression of STC is scored by the percentage of cases with negative $\lambda_{\max}$. It is clear that for a weakly chaotic CML (denoted by dashed lines), control of STC is possible even by randomly pinning only a few sites. However, as expected, control of STC in fully turbulent CML (denoted by the solid line) is more difficult—though it is still possible in about 15% of the cases by randomly

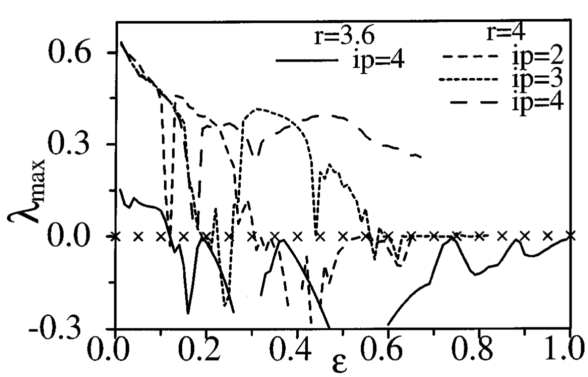

FIG. 3. Role of coupling strength ϵ in suppression of STC (regions of negative $\lambda_{\max}$) in CMLs at different pinning densities for $p = -0.4$.

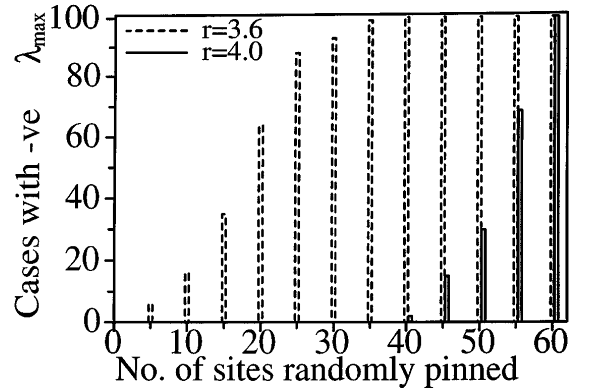

FIG. 4. Suppression of STC in CMLs ($\epsilon = 0.7$, $p = -0.4$) with randomly distributed pinned sites.

pinning only 75% of the sites. With the increase in pinning density, success rate increases and randomly pinning 90% of the sites yield control of STC in >50% of the cases. Thus, this method can also be useful in systems where regularly spaced distribution of pinning may not be feasible.

Local control of spatiotemporal dynamics.—Control of the dynamics in spatially localized regions, leaving the rest of the system undisturbed, is usually not addressed in spatially extended systems, though this has important applications in both physical and biological systems [11]. With many of the existing algorithms, spatially localized control is not possible since parametric changes are needed to achieve control. The present method can be effectively used for controlling the dynamics in spatially localized regions in a CML. Figure 5 shows that when a region of ten sites in a CML exhibiting spatiotemporal chaos is pinned locally leaving the rest of the sites undisturbed, it shows suppression of STC in that localized region, and the rest of the lattice continues to show STC. The desired

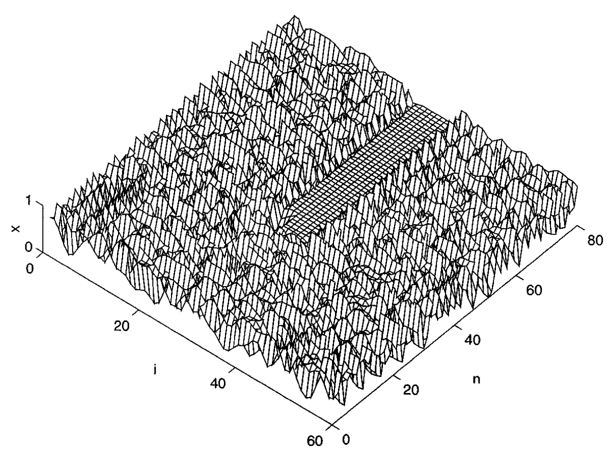

FIG. 5. Space-time-amplitude plot for local control. Sites $i = 26, \ldots, 35$ in the CML ($r = 4.0$, $\epsilon = 0.8$) exhibiting STC are pinned with $p = -0.3$ and controlled to equilibrium state while the rest of the lattice continues to exhibit STC.

dynamics in the pinned region can be obtained by appropriately choosing the pinning strength (from Fig. 1). In this method a pinned site can exert its influence to at most three neighboring sites depending on the coupling and pinning strengths. Thus suppression of STC is indeed quite strictly localized when coupling is weak, and the pinning region can be appropriately chosen at high coupling strengths.

To summarize, we have shown that it is possible to suppress spatiotemporal chaos in coupled map lattice models both globally and locally by applying constant pinning signals. Using this simple method we have shown the following: (a) Spatiotemporal dynamics of the CML can be controlled to desired dynamics by appropriately pinning all the sites. A few test experiments on the map function controlling the local dynamics can suffice for the learning phase of the method. (b) Control of dynamics is possible with heterogeneity in the parameters. (c) Global control of STC is possible by pinning fewer sites that are distributed in a regular or random fashion throughout the lattice. This provides considerable relaxation in the density and distribution of the pinned sites and allows targeting control signals to be spatially fairly nonspecific—a useful measure in experimental protocols. (d) Spatially localized suppression of STC without disturbing the rest of the lattice can be effectively achieved to the desired dynamics by choosing appropriate pinning strengths—a property that can have important applications.

The suppression of STC achieved in this method is due to the fact that spatially localized negative pinning effectively reduces the nonlinear parameter of the local dynamics (logistic map here), thereby inducing stability in the local dynamics and, in turn, in the whole CML. This method works for different forms of the one-dimensional, single humped functions [7] in the local dynamics with negative and positive pinnings by either pushing the system towards the region of reduced slope of the hump, or towards the region of inflection, if one exists [8,12]. Our preliminary investigations show that this method also works in CMLs where the local dynamics is governed by two-dimensional maps and continuous dynamics. The method is easily applicable for controlling STC in two-dimensional CMLs without any modification. Persistence of pinning required for control in this method is quite realistic, since many of the therapeutic measures of control involve regular application of drugs, or the presence of *in situ* pacemakers to control pathologies till physical alteration or removal are performed. In physicochemical systems, this allows the controlled state to be robust against perturbations. Thus this theoretical approach has important potential for implementation on a variety of physical and biological systems.

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[1] J. P. Crutchfield and K. Kaneko, in *Directions in Chaos*, edited by Hao Bai-Lin (World Scientific, Singapore, 1987); K. Kaneko, in *Chaos, Order, and Patterns*, edited by R. Artuso *et al.* (Plenum Press, New York, 1991), p. 237, and references therein; D. Stassinopoulos and P. Alstrom, Phys. Rev. A **45**, 675 (1992).

[2] D. Winston, M. Arora, J. Maselko, V. Gaspar, and K. Showalter, Nature (London) **351**, 132 (1991); A. V. Holden, J. V. Tucker, and B. C. Thomson, Physica (Amsterdam) **D49**, 240 (1991); M. S. Spach, Int. J. Bifurcation Chaos **6**, 1637 (1996); W. C. De Mello, Prog. Biophys. Mol. Biol. **61**, 1 (1994); S. J. Schiff, K. Jerger, D. H. Duong, T. Chang, M. L. Spano, and W. L. Ditto, Nature (London) **370**, 615 (1994); M. P. Hassell, H. N. Comins, and R. M. May, Nature (London) **353**, 255 (1991); P. Rohani, R. M. May, and M. P. Hassell, J. Theor. Biol. **181**, 97 (1996).

[3] A. Garfinkel, W. L. Ditto, and J. N. Weiss, Science **257**, 1230 (1992); H. Gang and Q. Zhilin, Phys. Rev. Lett. **72**, 68 (1994).

[4] M. Ding, W. Yang, V. In, W. L. Ditto, M. L. Spano, and B. Gluckman, Phys. Rev. E **53**, 4334 (1996); N. Parekh, V. Ravi Kumar, and B. D. Kulkarni, Pramana J. Phys. **48**, 303 (1997); Chaos **8**, 301 (1998).

[5] L. Glass and W. Zeng, Int. J. Bifurcation Chaos **4**, 1061 (1994).

[6] E. Ott, C. Grebogi, and J. A. York, Phys. Rev. Lett. **64**, 1196 (1990).

[7] R. M. May, Nature (London) **261**, 459 (1976); R. M. May and G. F. Oster, Am. Nat. **110**, 573 (1976).

[8] S. Parthasarathy and S. Sinha, Phys. Rev. E **51**, 6239 (1995); S. Sinha and S. Parthasarathy, Proc. Natl. Acad. Sci. U.S.A. **93**, 1540 (1996); S. Sinha and P. K. Das, Pramana **48**, 87 (1997); L. Stone, Nature (London) **365**, 617 (1993).

[9] The range of allowed initial conditions for chosen values of r and p is $[x^*, (1 - x^*)]$, where $x^* = [(r-1) - \sqrt{(r-1)^2 - 4rp}\,]/2r$.

[10] *The Computation Biology of the Heart*, edited by A. V. Paniflov and A. V. Holden (J. Wiley, Chichester, 1997).

[11] H. Zhang, Int. J. Bifurcation Chaos **6**, 1935 (1996); T. Elbert, W. J. Ray, Z. J. Kowalik, J. E. Skinner, K. E. Graf, and N. Birbaumer, Physiol. Rev. **74**, 1 (1994).

[12] R. L. Devaney, *An Introduction to Chaotic Dynamical Systems* (Addison-Wesley, Menlo Park, CA, 1985); R. C. Hilborn, *Chaos and Nonlinear Dynamics* (Oxford University Press, Inc., Oxford, 1994).