Switching Gap Analysis for generalized butterfly networks

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***Abstract*— Ahlswede *et al.* introduced the concept of network coding in multicast networks, wherein they showed that maximum achievable information rate by network coding in a single-source multi-sink network exceeds, in general, that for the case of network switching. It is, hence, of interest to analyze the switching gap for a network, defined as the ratio of maximum achievable information rate using network coding (NC) to that of network switching (NS). In this paper, we find the switching gap for a class of symmetric networks that contains the well-known butterfly network, which is often used to illustrate the advantage of network coding.**

***Keywords*— information theory, network coding, network switching, switching gap, butterfly networks**

I. Introduction

In their fundamental work on network coding [1], Ahslwede *et al.* determined the capacity for multicasting information in a single-source multi-sink network. Specifically, the network is represented by a directed graph denoted by $G = (V, E)$, where V is the set of nodes of a point-to-point communication network (that also form the vertices of the graph), E is the set of edges such that information can be sent noiselessly from node p to node q for all $(p, q) \in E$. We associate a non-negative number called capacity, c to each edge, which denotes the maximum amount of information that can be transmitted using that edge or channel.

For single-source network coding [2], it is shown that the maximum achievable information rate for network coding is given by the max-flow min-cut bound. This bound cannot be achieved, in general, by conventional network switching and network coding has to be applied at selected nodes of the network. It is shown in [3] that linear network coding suffices to achieve the optimum.

Network switching and network coding are two techniques for data transmission across a network. Strictly speaking, network switching is a special case of network coding but we treat them separately to distinguish between the prevalent practice and the achievable.

In network switching, a node receives information from its input bit streams (or channels) and forwards one of the bit streams (switches) to all other output links. For example, in Fig. 1, assume that the link capacity of all the edges is unity and A and B are bits to the multicast, and A, B denotes the information sent across in two separate time instances (comma indicating the separation in time). Note that node W is switching the incoming information. It is clear that link (W, X) require two time cycles to send bit A to sink t_2 and B to sink t_1. So, at each odd cycle, one sink receives two bits and other sink receives only one bit. At even cycle, this is reversed. Hence the information rate of 3 bits in 2 cycles could be achieved.

Network coding is defined as the process of linear recombination of input bit streams coming at a node via its input links, into one or several output bit streams. Fig. 2 shows network coding. Here the node W, XORs the bits received from nodes U and V (A and B) and forwards the result. Such an operation meets the unit link capacity condition while providing an extra information to both the sinks enabling them to receive two bits in two time instances. Hence a rate of 2 bits/sec is achieved.

Switching gap is defined as the ratio of maximum achievable information rate using network coding (NC) to that of network switching (NS). Switching gap is an indicator of how much network coding will help over network switching. Larger than unity implies a good advantage while closer to unity implies little advantage. The Ahlswede-Cai-Li-Yeung's butterfly network [1] was analyzed in [4] to determine switching gap of the network and certain conditions on link capacities were given where that switching gap of the network comes out to be unity. We extend that work by taking modified versions of butterfly network, which are larger networks, and finding their switching gap under a given set of conditions on the link capacities of the network. In this process, we analyze singular-symmetric, dual-symmetric and triple-symmetric butterfly networks that are defined below. The switching gap increases as the size of the network grows.

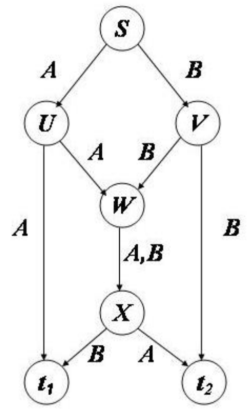

Fig. 1. Network Switching

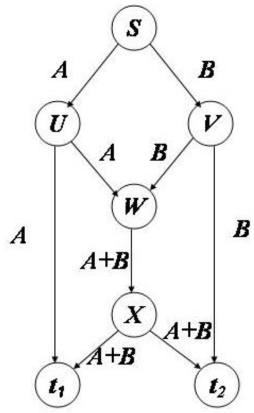

Fig. 2. Network Coding

II. Analysis for Singular Butterfly Network

Fig. 3 shows singular butterfly network. Here $w_i > 0$ denotes the link capacities of the edges in the graph.

According to Ahlswede-Cai-Li-Yeung's fundamental theorem for single-source network coding [1], the maximum achievable

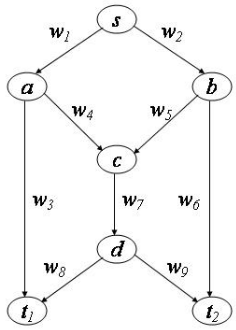

Fig. 3. Singular-Symmetric Butterfly network

information rate denoted by R^{**} is equal to the minimum of the s-t cuts for all source-sink pairs in the network. To enumerate the s-t cuts, consider the subgraph $G' = (V', E')$, where $V' \subset V$ and $E' \subset E$, which is formed from graph shown in Fig. 4 by removing all paths between nodes s and t_1.

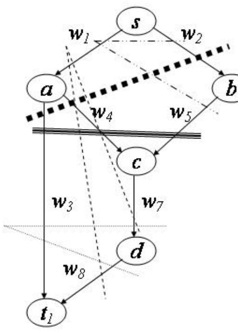

Fig. 4. Singular-Symmetric Butterfly sub network

All s-t cuts for this subgraph are shown as dashed lines in Fig. 4. The cut-set is enumerated as:

1) $\{(s, a), (s, b)\} = w_1 + w_2$.
2) $\{(s, a), (b, c)\} = w_1 + w_5$.
3) $\{(s, b), (a, c), (a, t_1)\} = w_2 + w_3 + w_4$.
4) $\{(s, a), (d, t_1)\} = w_1 + w_8$.
5) $\{(s, a), (c, d)\} = w_1 + w_7$.
6) $\{(a, t_1), (a, c), (b, c)\} = w_3 + w_4 + w_5$.
7) $\{(a, t_1), (c, d)\} = w_3 + w_7$.
8) $\{(a, t_1), (d, t_1)\} = w_3 + w_8$.

Since the network is symmetrical we can enumerate s-t cut sets for subgraph between s and t_2. These are as follows:

1) $\{(s, a), (s, b)\}$=$w_1 + w_2$.
2) $\{(s, b), (a, c)\}$=$w_2 + w_4$.
3) $\{(b, t_2), (a, c), (b, c)\}$=$w_4 + w_5 + w_6$.
4) $\{(s, b), (c, d)\}$=$w_2 + w_7$.
5) $\{(s, b), (d, t_2)\}$=$w_2 + w_9$.
6) $\{(s, a), (b, c), (b, t_2)\}$=$w_1 + w_5 + w_6$.
7) $\{(b, t_2), (c, d)\}$=$w_6 + w_7$.
8) $\{(b, t_2), (d, t_2)\}$=$w_6 + w_9$.

Combining min cuts of both the sinks, we get the maximum rate achievable by network coding on this network, R^{**} as minimum of following 5 terms.

1) $\alpha = (w_1 + \min(w_2, w_5))$,
2) $\beta = (w_2 + \min(w_1, w_4))$,
3) $\gamma = (\min(w_1, w_3) + \min(w_7, w_8))$,
4) $\delta = (\min(w_2, w_6) + \min(w_7, w_9))$,
5) $\zeta = (w_2 + w_4 + \min(w_3, w_6))$.

So,

$$R^{**} = \min(\alpha, \beta, \gamma, \delta, \zeta). \tag{1}$$

Maximum achievable information rate by network switching (NS), i.e., R^* of a communication network is computed by constructing its payoff matrix [5] and then solving it as per game theory principles [6]–[8] and finally taking the reciprocal of the final value to get R^*. Briefly we can say that network switching is a 2 person matrix game where in player 1 wants to choose an edge which is maximally present in the routes and player 2 wants to choose a tree which contains minimal edges of the graph G.

To construct the payoff matrix for any single source communication network, first multicast routes from source to each of the sink nodes are determined. It is a path enumeration problem and can be solved by drawing open rooted trees for each of the sink and then concatenating one path each from all such trees. A rooted tree is constructed as an acyclic digraph with a unique node, called root node, which has the property that there exists a unique path from the root node to each other node and a set of links defined τ as a multicast route of the underlying digraph from the source node s to the sink nodes t_1, t_2, . . ., t_l, if the digraph (S, τ) induced by τ is a rooted tree of graph G with leaves as t_1, t_2, . . ., t_l, and root node as s. For each multicast route τ_j, one defines an indicator function as $\chi_{\tau_j}(e_i)$=1 if $e_i \in \tau_j$ and 0 otherwise, for $i = 1, 2, \ldots, I$ and $j = 1, 2, \ldots, J$. An $I \times J$ payoff matrix A is constructed with each element defined as

$$a_{ij} = \frac{1}{\Theta(e_i)} \chi_{\tau_j}(e_i) \tag{2}$$

for $i = 1, 2, \ldots, I$ and $j = 1, 2, \ldots, J$.

Rows of this matrix represent edges of graph G and columns represent multicast routes of G.

Next, principles of row and column dominance as well as successive elimination [6] are applied on the payoff matrix to get a simplified square matrix. A link is dominated by another link if every multicast routes including the former also includes the latter. A multicast route is dominated by another multicast route if each dominating link in the latter is also in the former.

Then, this matrix is solved to get a value whose reciprocal gives the desired rate R^*. Further for complicated networks, the payoff matrix could be directly constructed using only the dominated links and multicast routes instead of enumerating all routes.

For the network shown in Fig. 3, there are nine links and total of seven multicast routes which are shown in the Fig. 5.

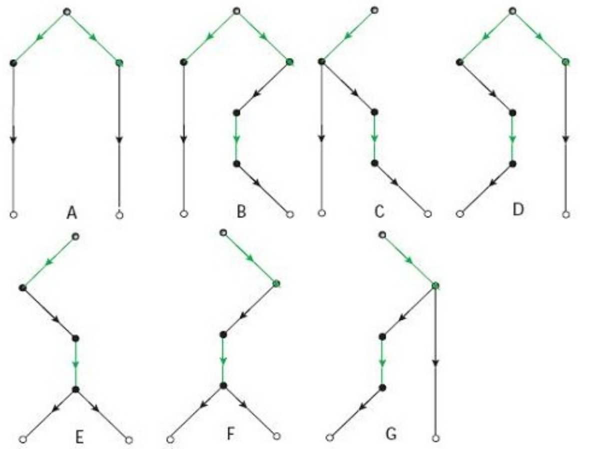

Fig. 5. Multicast routes in singular-Symmetric Butterfly network

Its payoff matrix A is given below.

$$\begin{vmatrix} \frac{1}{w_1} & \frac{1}{w_1} & \frac{1}{w_1} & \frac{1}{w_1} & \frac{1}{w_1} & \frac{1}{w_1} & 0 \\ \frac{1}{w_2} & 0 & \frac{1}{w_2} & \frac{1}{w_2} & 0 & \frac{1}{w_2} & \frac{1}{w_2} \\ \frac{1}{w_3} & \frac{1}{w_3} & \frac{1}{w_3} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{w_4} & 0 & \frac{1}{w_4} & \frac{1}{w_4} & 0 & 0 \\ 0 & 0 & \frac{1}{w_5} & 0 & 0 & \frac{1}{w_5} & \frac{1}{w_5} \\ \frac{1}{w_6} & 0 & 0 & \frac{1}{w_6} & 0 & \frac{1}{w_6} & 0 \\ 0 & \frac{1}{w_7} & \frac{1}{w_7} & \frac{1}{w_7} & \frac{1}{w_7} & \frac{1}{w_7} & \frac{1}{w_7} \\ 0 & 0 & 0 & \frac{1}{w_8} & \frac{1}{w_8} & \frac{1}{w_8} & \frac{1}{w_8} \\ 0 & \frac{1}{w_9} & \frac{1}{w_9} & 0 & \frac{1}{w_9} & 0 & \frac{1}{w_9} \end{vmatrix}$$

However, it is not necessary to enumerate of all the multicast routes to find R^*. Applying the game theory principles described above, we get the following simplified matrix:

$$A_1 = \begin{vmatrix} \frac{1}{w_1} & \frac{1}{w_1} & 0 \\ \frac{1}{w_2} & 0 & \frac{1}{w_2} \\ 0 & \frac{1}{w_7} & \frac{1}{w_7} \end{vmatrix} \tag{3}$$

Now val(A)=val(A_1) [5]. Using Lemma 1 in [5], let (X^*, Y^*) be the mixed strategy pair for both players respectively at Equilibrium point. Applying the Lagrange Multiplier method [9] to A, we have following cases:

1) If $w_1 \le (w_2 + w_4)$, $w_2 \le (w_1 + w_7)$ and $w_7 \le (w_1 + w_2)$ then we have $X^* = ((w_1/(w_1 + w_2 + w_7)), (w_2/(w_1 + w_2 + w_7)), (w_7/(w_1 + w_2 + w_7)))$ and $Y^* = (((w_1 + w_2 - w_7)/(w_1 + w_2 + w_7)), ((w_1 - w_2 + w_7)/(w_1 + w_2 + w_7)), ((-w_1 + w_2 + w_7)/(w_1 + w_2 + w_7)))$ and val(A) is
$$\text{val}(A) = \frac{2}{(w_1 + w_2 + w_7)}. \tag{4}$$
2) If $w_7 > w_1 + w_2$, we have $X^* = (w_1/(w_1 + w_2), w_2/(w_1 + w_2), 0)$ and $Y^* = (0, w_1/(w_1 + w_2), w_2/(w_1 + w_2))$ and val(A) is
$$\text{val}(A) = \frac{1}{(w_1 + w_2)}. \tag{5}$$
3) If $w_1 > w_2 + w_7$, we have $X^* = (0, w_2/(w_2 + w_7), w_7/(w_2 + w_7), 0)$ and $Y^* = (w_2/(w_2 + w_7), w_7/(w_2 + w_7), 0)$ and val(A) is
$$\text{val}(A) = \frac{1}{(w_2 + w_7)}. \tag{6}$$
4) If $w_2 > w_1 + w_7$, we have $X^* = (w_1/(w_1 + w_7), 0, w_7/(w_1 + w_7), 0)$ and $Y^* = (w_1/(w_1 + w_7), 0, w_7/(w_1 + w_7))$ and val(A) is
$$\text{val}(A) = \frac{1}{(w_1 + w_7)}. \tag{7}$$

For the given singular butterfly network, let us assume the following conditions:

$$w_1 < \min(w_3, w_4).$$
$$w_2 < \min(w_5, w_6).$$
$$w_7 < \min(w_8, w_9).$$

And using above assumptions, maximum rate due to network coding, R^{**} (using 1) is

$$R^{**} = \min\left((w_1 + w_2), (w_2 + w_7), (w_1 + w_7)\right). \tag{8}$$

Thus using these conditions and above cases, if we organize w_1, w_2, w_3 in increasing order and denote one particular permutation of indices 1,2 and 7 as l, m, n then,

$$\text{val}(A) = \frac{2}{(w_l + w_m + \min((w_l + w_m), w_n))}. \tag{9}$$

Hence,

$$R^* = \frac{1}{\text{val}(A)} = \frac{(w_l + w_m + \min((w_l + w_m), w_n))}{2}. \tag{10}$$

Now using 8, we have

$$R^{**} = (w_l + w_m). \tag{11}$$

So, using 9 and 11, Switching gap of the network becomes

$$\frac{R^{**}}{R^*} = \frac{2(w_l + w_m)}{(w_l + w_m + \min((w_l + w_m), w_n))} \ge 1 \tag{12}$$

Using 12, if each edge has same capacity, say w then switching gain will be

$$\frac{R^{**}}{R^*} = \frac{4}{3}. \tag{13}$$

III. Analysis for Dual butterfly network

Fig. 6 shows dual butterfly network. It has two loops of singular butterfly network overlapped one after the other as shown in fig. 3.

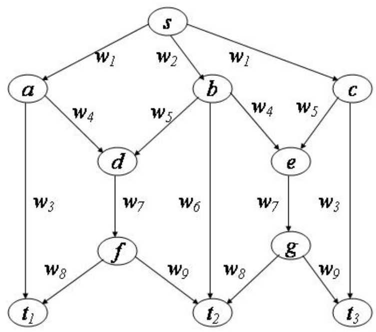

Fig. 6. Dual-Symmetric Butterfly network

Since it is symmetrical to singular symmetric case, it will have same min-cut values for sinks t_1, t_2 and t_3. Thus, R^{**} will be same as given in (1). Further using edge capacity assumptions, maximum rate due to network coding, R^{**} (using 1) is

$$R^{**} = \min\left((w_1 + w_2), (w_2 + w_7), (w_1 + w_7)\right). \tag{14}$$

Using assumptions about link capacities stated in previous section R^{**}, we have 5 dominating edges, namely $(s, a), (s, b), (s, c), (d, f), (e, g)$. We calculate network switching rate using multi cast routes formed taking subset of routes formed by these five edges. After elimination of dominated links and multicast routes, we get the following payoff matrix.

$$A = \begin{vmatrix} \frac{1}{w_1} & \frac{1}{w_1} & \frac{1}{w_1} & \frac{1}{w_1} & 0 \\ \frac{1}{w_2} & \frac{1}{w_2} & \frac{1}{w_2} & 0 & \frac{1}{w_2} \\ \frac{1}{w_1} & \frac{1}{w_1} & 0 & \frac{1}{w_1} & \frac{1}{w_1} \\ \frac{1}{w_7} & 0 & \frac{1}{w_7} & \frac{1}{w_7} & \frac{1}{w_7} \\ 0 & \frac{1}{w_7} & \frac{1}{w_7} & \frac{1}{w_7} & \frac{1}{w_7} \end{vmatrix} \tag{15}$$

Proceeding as in previous section, we solve A to get following cases.

1) If $(2w_1^2+w_2w_7) \geq 2w_1w_7$, $(w_2+w_7) > 2w_1$ and $w_1 > w_2$ then $X^* = ((w_1/(2w_1+w_2+2w_7)), (w_2/(2w_1+w_2+2w_7), (w_1/(2w_1+w_2+2w_7)), (w_7/(2w_1+w_2+2w_7)), (w_7/(2w_1+w_2+2w_7)))$ and $Y^* = (((2w_1^2+w_2w_7-2w_1w_7)/(2w_1(w_1+w_7))), ((2w_1^2+w_2w_7-2w_1w_7)/(2w_1(w_1+w_7))), ((w_7+w_2-2w_1)/(2(w_1+w_7))), ((w_1(w_1-w_2)+w_7(2w_1-w_2))/(w_1(w_1+w_7))), ((w_7+w_2-2w_1)/(2(w_1+w_7))))$ and val(A) is

$$\text{val}(A) = \frac{4}{2w_1+w_2+2w_7}. \tag{16}$$

2) If $(2w_1^2+w_2w_7) < 2w_1w_7$ then $X^* = (0,0,0,1/2,1/2)$ and $Y^* = (0,0,w_2/(w_2+2w_1), ((2w_1-w_2)/(w_2+2w_1)), w_2/(w_2+2w_1))$ and val(A) is

$$\text{val}(A) = \frac{1}{w_7}. \tag{17}$$

3) If $2w_1 < w_2$ and $(1+(w_1/w_7)) < (5-(3w_2/2w_1)+(w_7/2w_1))$ then $X^* = (1/3,1/3,1/3,0,0)$ and $Y^* = ((3w_1-w_2)/(w_1+w_7), (3w_1-w_2)/(w_1+w_7), (w_7+w_2-2w_1)/(2(w_1+w_7)), 0, (w_7+w_2-2w_1)/(2(w_1+w_7)))$ and val(A) is

$$\text{val}(A) = \frac{2w_1}{10w_1-3w_2+w_7}. \tag{18}$$

From these three cases, we get

$$\text{val}(A) = \min\left(\frac{4}{2w_1+w_2+2w_7}, \frac{1}{w_7}, \frac{2w_1}{10w_1-3w_2+w_7}\right). \tag{19}$$

Hence,

$$R^* = \min\left(\frac{2w_1+w_2+2w_7}{4}, w_7, \frac{10w_1-3w_2+w_7}{2w_1}\right). \tag{20}$$

Using (14) and (20), we have

$$\frac{R^{**}}{R^*} = \frac{\min((w_1+w_2),(w_2+w_7),(w_1+w_7))}{\min\left(\frac{2w_1+w_2+2w_7}{4}, w_7, \frac{10w_1-3w_2+w_7}{2w_1}\right)}. \tag{21}$$

If we take each edge has same capacity, say w then switching gain will be calculated on basis of case 1 and it is

$$\frac{R^{**}}{R^*} = \frac{8}{5} = 1.60. \tag{22}$$

IV. Analysis for Triple butterfly network

Fig. 7 shows triple butterfly network. Since it is symmetrical to singular symmetric case, it will have same min-cut values for sinks t_1, t_2 and t_3. Thus, R^{**} will be same as given in (1). Further using edge capacity assumptions, maximum rate due to network coding, R^{**} (using 1) is

$$R^{**} = \min((w_1+w_2),(w_2+w_7),(w_1+w_7)). \tag{23}$$

Using assumptions about link capacities stated in previous section R^{**}, we have 7 dominating edges, namely $(s,a),(s,b),(s,c),(s,d),(e,h),(f,i),(g,j)$. We calculate network switching rate using multi cast routes formed taking subset of routes formed by these seven edges. After elimination of dominated links and multicast routes, we get the following payoff matrix.

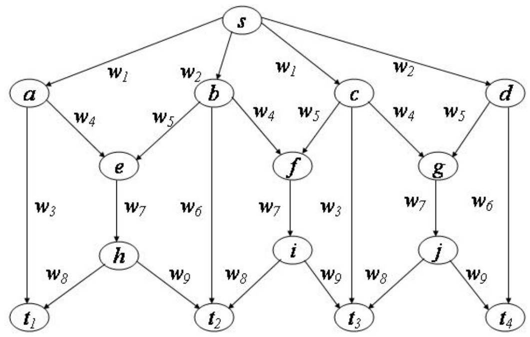

Fig. 7. Triple-symmetric Butterfly network

$$A = \begin{vmatrix} \frac{1}{w_1} & \frac{1}{w_1} & \frac{1}{w_1} & \frac{1}{w_1} & \frac{1}{w_1} & \frac{1}{w_1} & 0 \\ \frac{1}{w_2} & \frac{1}{w_2} & \frac{1}{w_2} & \frac{1}{w_2} & \frac{1}{w_2} & 0 & \frac{1}{w_2} \\ \frac{1}{w_1} & \frac{1}{w_1} & \frac{1}{w_1} & \frac{1}{w_1} & 0 & \frac{1}{w_1} & \frac{1}{w_1} \\ \frac{1}{w_2} & \frac{1}{w_2} & \frac{1}{w_2} & 0 & \frac{1}{w_2} & \frac{1}{w_2} & \frac{1}{w_2} \\ \frac{1}{w_7} & \frac{1}{w_7} & 0 & \frac{1}{w_7} & \frac{1}{w_7} & \frac{1}{w_7} & \frac{1}{w_7} \\ \frac{1}{w_7} & 0 & \frac{1}{w_7} & \frac{1}{w_7} & \frac{1}{w_7} & \frac{1}{w_7} & \frac{1}{w_7} \\ 0 & \frac{1}{w_7} & \frac{1}{w_7} & \frac{1}{w_7} & \frac{1}{w_7} & \frac{1}{w_7} & \frac{1}{w_7} \end{vmatrix} \tag{24}$$

Proceeding as in previous section, we apply lagrange multipliers [9] to A and get following cases.

1) If $2(w_1+w_2) \geq 3w_7$ and $(3w_7+2w_2) \geq w_7w_1$ then $X^* = ((w_1/(2w_1+2w_2+3w_7)), (w_2/(2w_1+2w_2+3w_7)), (w_1/(2w_1+2w_2+3w_7)), (w_2/(2w_1+2w_2+3w_7)), (w_7/(2w_1+2w_2+3w_7)), (w_7/(2w_1+2w_2+3w_7)), (w_7/(2w_1+2w_2+3w_7)))$ and $Y^* = (((2w_1+2w_2-3w_7)/(2w_1+2w_2+3w_7)), ((2w_1+2w_2-3w_7)/(2w_1+2w_2+3w_7)), ((2w_1+2w_2-3w_7)/(2w_1+2w_2+3w_7)), ((2w_1-4w_2+3w_7)/(2w_1+2w_2+3w_7)), ((-4w_1+2w_2+3w_7)/(2w_1+2w_2+3w_7)), ((2w_1-4w_2+3w_7)/(2w_1+2w_2+3w_7)), ((-4w_1+2w_2+3w_7)/(2w_1+2w_2+3w_7)))$ and val(A) is

$$\text{val}(A) = \frac{6}{2w_1+2w_2+3w_7}. \tag{25}$$

2) If $2w_1+2w_2 < 3w_7$ then $X^* = (0,0,0,((2w_1-w_7)/2w_7), (w_7-w_1)/w_7, ((2w_1-w_7)/2w_7), (w_7-w_1)/w_7)$ and $Y^* = (0,0,0,(w_4-w_1)/w_4, w_1/3w_4, w_1/3w_4, w_1/3w_4)$ and val(A) is

$$\text{val}(A) = \frac{6}{2w_1+2w_2+3w_7}. \tag{26}$$

3) If $2(w_1+w_2) < 3w_7$ and $(3w_1-2w_7)/w_1 < (4w_1-3w_7)/2w_7$, then $X^* = (0,1/2,0,1/2,0,0,0)$ and $Y^* = (0,0,0,((2w_1-w_7)/2(4w_1-3w_7)), ((w_1-w_2)/(4w1-3w_7)), ((2w_1-w_7)/2(4w_1-3w_7)), ((w_1-w_2)/(4w1-3w_7)))$ and val(A) is

$$\text{val}(A) = \frac{6w_1-5w_7}{2w_2(4w_1-3w_7)}. \tag{27}$$

4) If $2(w_1+w_2) < 3w_7$ and $(6w_1-5w_7)/w_2 < (4w_1-3w_7)/w_7$, then $X^* = (1/2, 0, 1/2, 0, 0, 0, 0)$ and $Y^* = (0, 0, 0, ((2w_1-w_7)/2(4w_1-3w_7)), ((w_1-w_2)/(4w1-3w_7)), ((2w_1-w_7)/2(4w_1-3w_7)), ((w_1-w_2)/(4w1-3w_7)))$ and val(A) is

$$\text{val}(A) = \frac{3w_1-2w_7}{w_1(4w_1-3w_7)}. \tag{28}$$

From these cases, we get

$$\text{val}(A) = \min\left(\frac{6}{2w_1+2w_2+3w_7}, \frac{6w_1-5w_7}{2w_2(4w_1-3w_7)}, \frac{3w_1-2w_7}{w_1(4w_1-3w_7)}\right). \tag{29}$$

Hence,

$$R^* = \min\left(\frac{2w_1+2w_2+3w_7}{6}, \frac{2w_2(4w_1-3w_7)}{6w_1-5w_7}, \frac{w_1(4w_1-3w_7)}{3w_1-2w_7}\right). \tag{30}$$

Using (23) and (30), we have

$$\frac{R^{**}}{R^*} = \frac{\min\left((w_1+w_2), (w_2+w_7), (w_1+w_7)\right)}{\min\left(\frac{2w_1+2w_2+3w_7}{6}, \frac{2w_2(4w_1-3w_7)}{6w_1-5w_7}, \frac{w_1(4w_1-3w_7)}{3w_1-2w_7}\right)}. \tag{31}$$

If we take each edge has same capacity, say w then switching gain will be calculated on basis of case 1 and it is

$$\frac{R^{**}}{R^*} = \frac{12}{7} = 1.92. \tag{32}$$

V. Conclusion and Future work

In this paper, we analyze a specific class of butterfly network under suitable assumptions about its link capacities. Based upon analysis of its three variants, we find that the information rate due to NC is coming out to be same in all versions of the network while information rate due to NS is decreasing as we are increasing the complexity of the network. Maximum information rate due to network coding is upper bounded by max-flow min-cut theorem. However as per our analysis analysis, we could conclude that maximum information rate due to network switching is hard to find for a generic class of networks. There is no formal way to apply game theory rules on a generic network. Matrix simplification rules lack a formal mathematical approach. In our future work, we will give an alternate graph theoretic approach to find maximum achievable information rate due to network switching for general single source multicast network.

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