NEWTON'S SECOND LAW AND CURVILINEAR MOTION OF A MATERIAL PARTICLE

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Summary The Newton's second law is considered with reference to motion of a material particle along a curvilinear trajectory. It is shown that a change in momentum of a material particle creates a tangential and centripetal force. The equations of motion of a particle were obtained. It is shown that motion of a particle requires application of a momentum force at presence of a reaction force. It is shown that a work of a tangential and centripetal force changes the magnitude and direction of a kinetic energy vector.

Newton gave the following definition of the second law of motion with reference to the rectilinear motion of a material particle with a mass m and acceleration $\bar{a}$: "Change of momentum $d\left(m\bar{v}\right)$ is in proportion to the applied momentum force $\bar{F}$ and takes place in the line of the straight line along which this force acts".

$$\bar{F} = m\bar{a} = d\left(m\bar{v}\right)/dt \tag{1}$$

This law provides for a change in momentum of the material particle without a change in the direction of its motion. However, while a free material particle moves along a curvilinear trajectory, the momentum magnitude and direction may vary [1]. Let us assume that the material particle moves along a convex curvilinear trajectory from left to right, the direction of the velocity changing clockwise. Let us assume that within a short period of time $t_1 - t_0$ the momentum of the material particle $m\bar{v}_0$ has increased and is equal to $m\bar{v}_1$, a small angle φ having been formed between the direction of the momentum vector $m\bar{v}_1$ and the direction of the momentum vector $m\bar{v}_0$ (Figure 1).

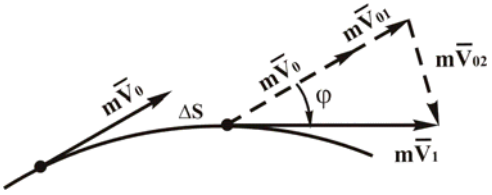

Figure 1

Let us introduce an additional momentum vector $m\bar{v}_{01}$ so that a magnitude of the momentum vector $m\bar{v}_0 + m\bar{v}_{01}$ is equal to that of the momentum vector $m\bar{v}_1$. Let us join the terminus of the additional vector $m\bar{v}_{01}$ and the terminus of the vector $m\bar{v}_1$ and we shall obtain a vector $m\bar{v}_{02}$. Therefore, we can write down the following vector equation in view of the small angle $\varphi = \sin\varphi$ and $\sin 0 = 0$

$$m\bar{v}_1 - m\bar{v}_0 = m\bar{v}_{01} + m\bar{v}_{02} \tag{2}$$

Proceeding to the limit, if $t_1 - t_0 \to 0$, we shall obtain

$$d\left(m\bar{v}\right)/dt = m\bar{a}_\tau + m\bar{a}_n = \bar{F}_\tau + \bar{F}_n \tag{3}$$

where $ma_n = mv_1\left(\sin\varphi - \sin 0\right)/dt = mv\varphi/dt = mvw$, ω is the angular velocity, $\bar{F}_\tau$ is the tangential force tangent to the curvilinear trajectory, $\bar{F}_n$ is the centripetal force with the direction perpendicular to the force $\bar{F}_\tau$.

In this definition of the forces bringing about the change in momentum of the material particle, the direction of the force $\bar{F}_\tau$ coincides with that of the vector $\bar{a}_\tau$, and the direction of the force $\bar{F}_n$, with that of the vector $\bar{a}_n$.
The equation of an accelerated motion of the free material particle along the curvilinear trajectory is as follows:

$$\bar{F}_\tau + \bar{F}_n = m\bar{a}_\tau + m\bar{a}_n \tag{4}$$

For the purpose of consideration, we selected a convex trajectory of the material particle motion. If we assume that the convex trajectory gradually turns into a concave trajectory, then the change of the velocity direction will happen anticlockwise. The position of the instantaneous radius of rotation will change consentaneously. The equations

(2), (3) and (4) will not change. If mechanical constraints impose requirements of kinematical or geometric nature, then the change of momentum of the material particle should be written as follows

$$d\left(m\bar{v}\right)/dt = \bar{F}_\tau + \bar{R}_n \tag{5}$$

where $\bar{R}_n$ is the constraint reaction force that requires an additional tangential force for own creation.

In this case, the motion equation for a constrained particle will be as follows

$$\bar{F}_\tau = m\bar{a}_\tau + m\bar{a}_n \tag{6}$$

Let us consider some features of the equation (4). If $\bar{F}_\tau = 0$, then $m\bar{a}_\tau = 0$. If $\bar{F}_n$ is the gravitation force, then from the equation (4) we shall obtain a certain equation of motion of the satellite in circular orbit.
The force $\bar{F}_n$ ensures the material point motion round a circle. This force is not a momentum force. In this case we obtain that $\bar{F}_\tau$ should be equal to $m\bar{a}_\tau + m\bar{a}_n$. If we additionally assume than the acceleration $\bar{a}_\tau$ can be equal to zero, we shall obtain that $F_\tau = mv\omega$. It results from this that at presence of constraints a uniform motion of the material particle round a circle requires application of a tangential force.

It is also worthwhile noting that the definition of the forces (3) bringing about the change in momentum of the material particle along the curvilinear trajectory does not have an effect on the definition of the change in a moment of momentum relatively to the instantaneous center of rotation. The change of the moment of momentum of the material particle relatively to the instantaneous center of rotation has its usual view:

$$d[\bar{M}o\left(m\bar{v}\right)]/dt = \bar{M}o\left(\bar{F}_\tau\right) \tag{7}$$

This is explained by the fact that the direction of vector $\bar{F}_n$ passes through a point relatively to which the moment is defined, and the moment from the force $\bar{F}_n$ is therefore equal to zero.

The equation (3) allows a transition to the notion of a kinetic energy and a work of forces. Let us assume that in a time t the material particle moved from the point M_A to the point M_B of the trajectory. Let us break the translation s into sectors $\triangle s$. Let us multiply the right and left parts of the equation (3) by the mean velocity $\triangle s/\triangle t$ (a scalar). This operation does not change the magnitudes or directions of the vectors in the equation (3). As a result, we obtain the following vector equation:

$$d\left(m\bar{v}\right)\frac{\triangle s}{\triangle t} = m\bar{a}_\tau \frac{\triangle s}{\triangle t}dt + m\bar{a}_n \frac{\triangle s}{\triangle t}dt \tag{8}$$

or

$$m\bar{v}_1 v_1/2 - m\bar{v}_0 v_0/2 = \bar{F}_\tau \triangle s + \bar{F}_n \triangle s \tag{9}$$

where $F_n \triangle s = mv^2 \triangle\varphi$

In view of the translation s of the material particle from the point M_A to the point M_B within the time t, we obtain

$$m\bar{v}_{(M_B)}v_{(M_B)}/2 - m\bar{v}_{(M_A)}v_{(M_A)}/2 = A_{\tau(M_A,M_B)} + A_{n(M_A,M_B)} \tag{10}$$

The equation (10) shows that the change of the magnitude and direction of kinetic energy is accomplished by the work of force for translation and the work of force for changing the kinetic energy vector direction. Here, the work of force is a scalar.

CONCLUSIONS

The obtained research results allow obtaining differential equations of motion and solving problems of translatory motion of a material particle and a body along a curvilinear trajectory.

References

[1] Тарг С.М.: Краткий курс теоретической механики. Москва. Высшая школа. 1966. 416 стр. /Targ S.M. Concise Course in Classical Mechanics, Moscow, Vysshaya Shkola, 1966, 416 p./

Continuation on the following page!

PROFESSOR ERNIE TUCK
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Adelaide, April 22, 2008

Dear Dr Zhivotov,

I am writing on behalf of the Congress Committee of IUTAM with reference to your paper with ICTAM2008 10113 entitled

NEWTON'S SECOND LAW AND CURVILINEAR MOTION OF A MATERIAL PARTICLE

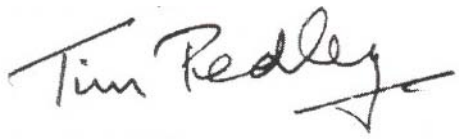
submitted to the 22nd International Congress of Theoretical and Applied Mechanics (ICTAM2008). Unfortunately the number of papers submitted greatly exceeded the number that could be accommodated in the program and I regret to inform you that we could not accept your paper for presentation.

I hope you will, nevertheless, be able to participate in the Congress, whose success depends on those who attend the lectures and participate in discussions as much as on those who make the formal presentations. Complete details about the Congress, including registration, may be found on the web site **http://ictam2008.adelaide.edu.au**. The organizers join me in expressing their hope that you will participate in this important mechanics event. A formal invitation to attend ICTAM2008 follows.

I look forward to seeing you in Adelaide.

With best wishes,

Yours sincerely



Timothy J. Pedley
Secretary, Congress Committee of IUTAM

Zhivotov's answer! (On 12/05/2008, at 9:20 PM)

ICTAM 2008, President — ictam2008@adelaide.edu.au

Professor Ernie Tuck, The University of Adelaide. — eotuck@internode.on.net

Dear Professor Ernie Tuck.

I have prepared a paper «**NEWTON'S SECOND LAW AND CURVILINEAR TRANSLATIONAL MOTION OF THE BODY**» (identification number 10113) for participation in the XXII International Congress of Theoretical and Applied Mechanics, to be held in Adelaide, Australia between 24–30 August 2008. In a paper it is shown, that:
In a paper it is shown, that:

- the modern formulation of the Newton's second law is erroneous: the formulation of the law substitutes idea of Lagrange to express any force through force of inertia.
The note: The modern formulation of the second law substitutes the formulation of the Newton's third law;

- change of quantity of movement of a body can occur both on magnitude, and on a direction. Thus forces should make work for change of quantity of movement of a body on magnitude and a direction;

- change of quantity of movement demands the appendix to a body of two mutually perpendicular forces: driving force and centripetal force.
Centripetal force very much frequently is absent in engineering practice. In this case driving force should compensate absence of centripetal force. I.e. in mechanisms centripetal forces are created due to work of driving forces;

- even uniform movement of a material point concerning the center of rotation demands the appendix of driving force which makes work on change of a direction of its movement.
Hence, uniform rotation of a material point on a circle demands expenses of energy;

- the existing mechanics is not suitable for the description of curvilinear movement of a body and the decision of this problem is resulted.

The given paper is the basic document, which advances new ideas in theoretical mechanics. These ideas will result in radical processing mechanics.

Organizers of the congress have sent the following message: “I am writing on behalf of the Congress Committee of lUTAM with reference to your paper with ICTAM2008 10113 entitled

NEWTON'S SECOND LAW AND CURVILINEAR MOTION OF A MATERIAL PARTICLE

submitted to the 22nd International Congress of Theoretical and Applied Mechanics (ICTAM2008). Unfortunately the number of papers submitted greatly exceeded the number that could be accommodated in the program and I regret to inform you that we could not accept your paper for presentation”.

However organizers of the congress have not sent comments of reviewers on a paper. Absence the comment does not allow to reveal a mistake of reviewers and to exclude artificial obstacles for promotion of new ideas in mechanics.

In this connection, I ask you to direct to my address of comments of reviewers.

Best regards,
Yu. Zhivotov

The answer to the Zhivotov's letter! (On 16.05.2008 AM 18:08:27)

Dear Professor Zhivotov,

Thank you for your e-mail. Unfortunately the number of papers submitted greatly exceeded the number that could be accommodated in the program and we regret to inform you that we cannot accept your paper for presentation.

We hope you will, nevertheless, be able to participate in the Congress, whose success depends on those who attend the lectures and participate in discussions as much as on those who make the formal presentations. To participate in this important mechanics event please go to our registration page.

Best regards
Jim Denier

Associate Professor Jim Denier
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Zhivotov's answer! (Date: 09.06.2008)

ICTAM 2008, President — ictam2008@adelaide.edu.au
Professor Ernie Tuck, The University of Adelaide. — eotuck@internode.on.net

Dear Professor Ernie Tuck.

I in the previous letter (On 16.05.2008 AM 18:08:27) asked to direct responses of reviewers which have served as the reason for a deviation of my paper: "NEWTON'S SECOND LAW AND CURVILINEAR MOTION OF A MATERIAL PARTICLE" to my address.

However from Secretary General ICTAM2008 I have received the inadequate answer – "Thank you for your e-mail. Unfortunately the number of papers submitted greatly exceeded the number that could be accommodated in the program and we regret to inform you that we cannot accept your paper for presentation".

I ask you once again will direct responses of reviewers to my address!

Best regards,
Yu. Zhivotov

Last answer to the Zhivotov's letter! (11.06.2008 04:02:01)

Dear Professor Zhivotov,

Thank you again for your e-mail. All papers for the congress were reviewed by the International Papers Committee appointed by IUTAM.
As local organisers, we are not involved in the review process nor are we privy to their deliberations. For these reasons I am unable to provide you with any feedback from the International Papers Committee.

Best regards
Jim Denier
Continuation on the following page!

New answer of Zhivotov! (Date: July 9, 2008)

President IUTAM
Professor L.B. Freund
freund@brown.edu

Copy

ICTAM 2008, President
Professor Ernie Tuck
ictam2008@adelaide.edu.au
eotuck@internode.on.net

Dear President IUTAM,
Professor L.B. Freund.

I have addressed to organizers of congress ICTAM 2008 with the request to give responses of reviewers on a paper «NEWTON'S SECOND LAW AND CURVILINEAR TRANSLATIONAL MOTION OF THE BODY» (identification number 10113).

Prof. Jim Denier has informed, that he has no responses of reviewers and organizers of the congress have no feedback with International Papers Committee.

I hope, that the management of the IUTAM has a feedback with International Papers Committee.

Therefore I ask you to assist in reception of responses of reviewers on my paper.

Best regards

Yu. Zhivotov

P. S.
I inform also, that the same history has happened to a paper “TORQUE EQUATION FOR RIGID ROTOR WITH ELASTIC SHAFT” (ID 10856), which has been sent on the congress ICTAM04. The given paper has been published in magazine and placed for the review
http://www.geocities.com/newrotor/Images/basmomE.pdf

Similar actions International Papers Committee do not promote promotion of mechanics.