УДК 621.396

BASE LAWS OF TRANSLATION MOTION OF A BODY

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This article deals with main patterns of relationship between inertia and translation motion of various moving parts of instruments and mechanisms. The interrelation between momentum and inertia being capable to affect parts movement dynamics is established in the article. Here is justified the methodology of definition of the conveyed momentum between active and passive bodies considering their geometry. Equations and dependencies analysis is reported in the article to prove a relation between impulse of inertia and performed work. The vector equation establishing a relation between kinetic energy of a moving part on the one hand and work and impulse of force on the other hand.

Instruments and mechanisms being actually in use contain parts performing rotational, oscillatory or translation motion. Such parts have mass; therefore, their motion is accompanied with effects of inertia forces [1] sufficiently influencing the motion dynamic. Body's inertia is allowed for in the theory of oscillations being applied in various industries [2], in the shock theory and rotary machine dynamics. The approach considering inertia in the rotary machine dynamics [3] resulted in deduction of new equations describing motion of a rotor being installed on a flexible shaft.

The existing concept of inertia forces is described in the most detailed way in the paper [4]. According to the concept, inertia forces are considered mainly in non-inertial reference frames. At the same time, it is considered that inertia forces could also originate in Newtonian reference frames [5] in some particular cases. However, such cases are not systematized and they are considered as the ones corresponding to a specific problem. It results in difficulties relevant to application of inertial approach to the analysis of the body motion dynamics.

Such tasks as analysis of inertia forces origination conditions, consideration of cases of real effect upon the dynamics, deduction of main patterns of relationship and dependencies for definition and application of inertia forces to the solution of specific problems of the dynamics of moving parts of instruments and mechanisms performing a translation motion are posed as the main problems.

As is well known, motion of a body of mass m with a velocity of $\overline{v}$ is characterized by the momentum $\overline{K}$ of the body performing an inertial movement.

$$\overline{K} = m\overline{v}\,. \tag{1}$$

The body's momentum vector has the same direction as the velocity vector. The velocity vector direction coincides with a direction of the displacement vector, which indicates a body motion direction and tends to zero in modulus.

Body's momentum is a measure of inertia and characterizes a body's ability to keep on its inertial motion while affecting another body trying to prevent from such a motion. In a quiescent state a body possesses rest energy or momentum equal to zero.

The total vector sum of the momentum $\overline{K_C}$ of the system consisting of several bodies i having a mass m_i, and mobbing with a velocity of $\overline{v_i}$ shall be equal to zero at any time instant.

$$\overline{K_C} = \sum_1^i m_i \overline{v_i} = 0\,. \tag{2}$$

Momentum of the bodies system is equal to scalar sum of bodies momentums.

$$K_C = \sum_1^i m_i v_i = Const\,. \tag{3}$$

Two moving bodies of masses m_1 and m_2 and having velocities of $\overline{v_1}$ and $\overline{v_2}$ respectively are interacting and exchanging with some momentum, i.e. with some inertia $m_1\overline{\Delta v_1} = m_1\overline{\Delta v_2}$. Both bodies interact and oppose each other while exchanging with such momentum.

The difference between final and initial momentums demonstrates what happens with one body under effect of another one as well as evaluates a response of one body to the effect of another one.

An interaction between the bodies shall occur through immediate contact without origination of rotary motion providing that bodies' rectilinear motion is conserved both prior and after such interaction. It could occur in case of central interaction between bodies (the common normal to the surfaces of both bodies in contact points crosses their centers of mass [2].

Interacting bodies may be distinguished as active and passive ones.

An active body is always decreasing the velocity of motion and gives some momentum while interacting with passive body. A passive body could initially lose but finally increases its velocity as a result of interaction with another body and finally acquire some momentum.

Body's motion prior to interaction and after it remains rectilinear. Orientation of the velocity vector with respect to the body is changed.

A velocity increment of the body of mass m_1 from the value of $\overline{v_{1^0}}$ to the value of $\overline{v_{1^1}}$ results in increment of its momentum $\Delta\overline{K_1}$. The momentum increment is directed from the end of the $m_1\overline{v_{1^0}}$ vector to the end of the vector $m_1\overline{v_{1^1}}$ and forms an acute angle with the velocity vector. Its value could be found from the following expression:

$$\Delta\overline{K_1} = m_1\overline{v_{1^1}} - m_1\overline{v_{1^0}} = m_1\Delta\overline{v_1} \,. \qquad (4)$$

We will consider such a direction of the body's momentum with respect to the velocity vector direction as a positive one.

Reduction of velocity of the moving body of mass m_2 from the value of $\overline{v_{2^0}}$ to the value of $\overline{v_{2^1}}$ results in reduction of the body's momentum to the value of $-m_2\Delta\overline{v_2}$. Decrement of the momentum $\Delta\overline{K_2}$ is directed from the end of the vector $m_2\overline{v_{2^1}}$ to the end of the vector $m_2\overline{v_{1^0}}$ and forms an obtuse angle with the velocity vector. Its value could be found from the following expression:

$$\Delta\overline{K_2} = m_2\overline{v_{2^1}} - m_2\overline{v_{2^0}} = -m_2\Delta\overline{v_2} \,. \qquad (5)$$

Such a direction of the momentum vector we will consider as negative. It is evident that the passive body momentum of mass m_1 is increased under effect of the body of mass m_2.

If the momentum of the passive body of mass m_1 is increased uniformly during an interval of Δt, we may write considering (5) that

$$\frac{m_1\overline{v_{1^1}} - m_1\overline{v_{1^0}}}{\Delta t} = m_1\frac{\overline{v_{1^1}} - \overline{v_{1^0}}}{\Delta t} = m_1\overline{a} = \overline{F_2} \,. \qquad (6)$$

It follows from the equation that the velocity increment of the body of mass m_1 occurs under the effect of the force $\overline{F_2}$ acting from the side of another body of mass m_2 that is losing its velocity. The force direction coincides with the directions of acceleration and of variation of the momentum of the body of mass m_1. Direction of the force acting from the body of mass m_2 forms an acute angle with a direction of the velocity vector of the body of the mass m_1.

It is evident that while the bodies of masses m_2 and m_1 are interacting; the momentum of the body of the mass m_2 is decreased.

If the momentum of the active body of the mass m_2 is decreased uniformly during Δt interval, then considering the equation (5) we can state the following:

$$\frac{m_2\overline{v_{2^1}} - m_2\overline{v_{2^0}}}{\Delta t} = m_2\frac{\overline{v_{2^1}} - \overline{v_{2^0}}}{\Delta t} = -m_2\frac{\Delta\overline{v_2}}{\Delta t} = -m_2\overline{a_2} = \overline{F_2^u} \,, \qquad (7)$$

where $\overline{a_2}$ is an acceleration of the decrement of the momentum;

$\overline{F_2^u}$ is the body's inertia decreasing the velocity of motion.

Direction of the inertia of the body of mass m_2 is opposite to the acceleration and to the body's velocity variation. Inertia direction forms an obtuse angle with the body's velocity vector originating this force.

The case when initially the body of mass m_1 is braked by the force $\overline{F_2}$ and then the body's velocity starts increasing is possible. In such case the following expression is valid.

$$\frac{m_1\overline{v_{1^1}} - m_1\overline{v_{1^0}}}{\Delta t} = m_1\frac{\overline{v_{1^1}} - \overline{v_{1^0}}}{\Delta t} = -m_1\frac{\Delta\overline{v_1}}{\Delta t} = -m_1\overline{a_1} = \overline{F_1^u} \,. \qquad (8)$$

The obtained result witnesses that the passive body also creates the inertia while braking. It affects the active body. Equation of the passive body motion while being exposed to the force $\overline{F_2}$ may be written in the following way:

$$\frac{m_1\overline{v_{1^0}} - m_1\overline{v_{1^1}}}{\Delta t} = m_1\frac{\overline{v_{1^0}} - \overline{v_{1^1}}}{\Delta t} = -m_1\overline{a} = -\overline{F_2} = \overline{F_1^0} \,, \qquad (9)$$

where $\overline{F_1^0}$ is a passive inertia of the passive body of the mass m_1, which is a response to the effect of the force $\overline{F_2}$ not opposing the body's motion.

The forces being applied to the body from the side of another body usually are concentrated forces affecting the body in the point of the force application. Such forces are external and may be of artificial or inertial origin.

Inertia forces opposing the body's momentum variation occurs in each material particle of the body. Therefore, they are the body forces. That means the resultant force is applied to the body's center of mass. If there is no applied force, the inertia is not present either. The inertia forces may be generated by artificial means.

Action of an external force onto the body during certain period creates an impulse of force $\Delta\overline{S}$

$$\Delta\overline{S_2} = \overline{F_2}\Delta t \,. \qquad (10)$$

Impulse of force affecting the body is able to increase or to decrease a momentum

$$\Delta\overline{S_2} = m_1\overline{v_{1^1}} - m_1\overline{v_{1^0}} = \pm m_1\Delta\overline{v_1} \,. \qquad (11)$$

Body momentum decrement enables creation of an impulse inertia force of the moving body:

$$m_2\overline{v_{2^1}} - m_2\overline{v_{2^0}} = m_2\left(\overline{v_{2^1}} - \overline{v_{2^0}}\right) = -m_2\Delta\overline{v_2} = \Delta\overline{S_2^u} \ . \quad (12)$$

Impulse inertia force has an opposite direction with respect to the direction of the body's momentum variation. It forms an acute angle with the vector of body's velocity.

The momentum of the system involving, for example, two interacting bodies remains invariable because one of the bodies acquire some momentum, while another one is going to lose the same value of momentum. The vector sum of acquired and lost momentum of the interacting bodies is equal to zero, or

$$m_1\Delta\overline{v_1^*} = -m_2\Delta\overline{v_2^*} \ . \quad (13)$$

The asterisk at the parameter hereinafter indicates that in general case a variation of the parameter remains not defined.

After interaction, one body acquires an additional velocity, while another body loses its velocity in an inverse proportion to their masses. Vectors of the momentum variation of the bodies are oppositely directed.

The momentum is transferred from one body to another by means of immediate contact and as a result, it depends on a position of the contact points of the interacting bodies with respect to the direction of the rectilinear motion velocity of each body.

The maximum of the body's inertia value is achieved when the direction of inertia force coincides with the velocity direction. However, it could occur that the contact point of the moving body stays out of the line of the maximum inertia force application and the normal line to the body's surface in the contact point may form an angle with the moving body velocity vector direction. In such case, the inertia force being generated with the purpose of an obstacle overcoming will be less than the maximum value. Bodies may move toward each other or one of them may be on the point of overtaking of another one. In such cases, the momentum being transferred from on body to another will depend on the set of the velocities interacting with the bodies.

The scalar sum of the total momentum of the bodies being involved into the system prior to and after an interaction is a constant value.

$$m_1v_1 - m_1\Delta v_1^* + m_2v_2 + m_2\Delta v_2^* = m_1v_1 + m_2v_2 = Const \ . \quad (14)$$

Each body's momentum is changed in terms of both value and direction.

The active body momentum can be found from the following expression:

$$\overline{K_1^*} = m_1\overline{v_1} - m_1\Delta\overline{v_1^*} \ . \quad (15)$$

While the active body momentum can be found from the following expression:

$$\overline{K_2^*} = m_2\overline{v_1} + m_2\Delta\overline{v_2^*} \ . \quad (16)$$

Reasoning from the condition of equality and opposite directedness of impulse of forces (including the ones being created by inertia forces) during interaction,

$$\overline{F_1^*}\Delta t = -\overline{F_2^*}\Delta t \ , \quad (17)$$

it follows that the bodies interact between themselves with the forces being equal in value but opposite in direction (the Newton's law of action and reaction).

$$\overline{F_1^*} = -\overline{F_2^*} \ . \quad (18)$$

The Newton's law of action and reaction (the Newton's third law) distinguishes between acting and counteracting forces. An active body generates an acting inertia force, while a passive body originates a counteracting one.

The third law of Newton deals with interacting bodies being in motion with certain velocity. In a number of cases the Newton's law is used for the analysis of a body's motion being under exposure of the force $\overline{F}$. The feature of the case is as follows: the force $\overline{F}$, while affecting the body, keeps on moving together with the affected body. Inertias $\overline{F^0}$ became the reaction forces and do not counteract a motion defined in the Newton's second law of motion.

To eliminate uncertainties in equation terms indicated with asterisks, we have to proceed from the assumption that a moving body should spend minimum value of momentum (or impulse of force affecting the body) in order to overcome an occurred obstacle.

Under such an assumption, the body's inertia that could be generated by it decreases in proportion to the angle between the normal line to the body's surface at the point of a contact (occurred because of interaction) and the velocity vector. To enable origination of an inertia force, an angle between the normal line to the active body surface at the contact point and its velocity cannot exceed 90^o. In case, a body has a spherical shape, the normal line to its surface passes through the center of gravity.

Minimum value of the active body momentum being transferred during central interaction depends on the velocities of interacting bodies. It can be assumed that one of the interacting bodies is moving with a velocity equal to zero, while the velocity of another one can be considered as a sum velocity of both bodies.

The minimum value of momentum being transferred during central interaction of spherical bodies is defined considering the lower value of mass taking into account a vector difference between velocities of two bodies as well as the normal line slope angle β_a to the surface of acting active body at the contact point.

$$\Delta K_{\min} = m_{\min} \left| \left(\overline{v_1} - \overline{v_2} \right) \right| \cos \beta_a . \qquad (19)$$

If an angle β_a is less than 90^o, the body is active and lose some value momentum.

If an angle β_a is greater than 90^o, the body is passive and acquires an additional value of momentum.

If an angle β_a is equal to 90^o, then there is no interaction with another body resulting in a transfer of momentum.

If the angles for both bodies are equal, then the body with higher velocity is considered as an active one, which is catching up a passive body.

A coasting active body is able to create an impulse of inertia force $\overline{S^u}$ at the impact on obstacle prior to transfer of a body into the quiescent state. At the time point of an active body impact on the obstacle, the value of momentum is lowered by the value satisfactory to overcome an obstacle. The body maintains a residual value of momentum and remains able of overcoming another obstacle. That is why an active body of mass m performing coasting with velocity of $\overline{v_0}$ is able to produce certain number of inertia impulses prior to its transfer into quiescent state or achievement of the velocity value $\overline{v_1}$

$$m\left(\overline{v_1} - \overline{v_0}\right) = \sum_1^i \overline{S_i^u} . \qquad (20)$$

An external force being applied to the body's center of mass or a number of simultaneously acting forces also change a passive body momentum. Converging forces can be substituted with one resultant force. Therefore, a passive body motion depends on a sequence of forces application. A body will perform a broken line trajectory maintaining a rectilinear motion at each leg while being exposed to a number of sequential forces during certain period.

Similarly to the equation (20) and taking into account Impulse of force, we can derive the following expression

$$\sum_1^i \overline{S_i} = m\overline{v_1} - m\overline{v_0} \qquad (21)$$

The equation is valid for a free motion of a body being exposed to effect of forces, and only in case if such forces are applied to the center of mass. The first part of the equation may have a negative value in case a certain number of impulses of force are applied. The negative sign indicates that the direction of the impulses of force vector sum has formed an obtuse angle with the velocity vector of the passive body. This is another evidence of inertia impulse origination by the passive body.

The active body coasting with a velocity of $\overline{v_0}$ creates an impulse of inertia during its impact on the obstacle lowering the velocity of rectilinear motion to the value of $\overline{v_1}$. The body's motion direction can change. In such case, the body producing an inertia impulse passes the distance exactly equal to Δs. Multiplying both left and right-hand sides of equation (12) by Δs we will obtain

$$m\left(\overline{v_1} - \overline{v_0}\right)\Delta s = \overline{F^u}\Delta t \Delta s , \qquad (22)$$

or

$$m\left(\overline{v_1} - \overline{v_0}\right)\frac{\Delta s}{\Delta t} = \overline{F^u}\Delta s . \qquad (23)$$

The expression $\frac{\Delta s}{\Delta t}$ represents an average velocity of rectilinear motion of the body during the period of Δt. Therefore, the equation (23) may be transferred into

$$m\left(\overline{v_1} - \overline{v_0}\right)\frac{v_1 - v_0}{2} = -\frac{m\Delta \overline{v}\Delta v}{2} = \overline{F^u}\Delta s = \overline{F^u}\frac{\Delta v}{2}\Delta t . \qquad (24)$$

An initial kinetic energy of a rectilinearly moving body is defined by the initial velocity $\overline{v_0}$

$$\overline{T_u} = \frac{m\overline{v_0}v_0}{2} . \qquad (25)$$

The work $\overline{A_u}$ of the inertia force of a body, which loses a velocity, can be derived from this expression:

$$\overline{A^u} = \overline{F^u}s = \overline{F^u}\frac{v_c}{2}t , \qquad (26)$$

where v_c is an average velocity of the body during the period t.

Direction of the kinetic energy vector coincides with the body's velocity vector direction.

The vector of the active body rectilinear motion kinetic energy variation is directed oppositely to the inertia force (to the contact point of interaction between active and passive bodies).

Upon a completion of interaction of the active and passive bodies, the first one keeps on moving with the constant velocity v_k and possesses a residual kinetic energy, which can be used for overcoming of other obstacles up to reaching a quiescent state.

The initial kinetic energy of the active body (considering its consuming) can be represented as:

$$\frac{m\overline{v_0}v_0}{2} = \sum_1^i \frac{m\Delta\overline{v_i}\Delta v_i}{2} + \frac{m\overline{v_k}v_k}{2} , \qquad (27)$$

or

$$\frac{m\overline{v_0}v_0}{2} - \frac{m\overline{v_k}v_k}{2} - = \sum_1^i \overline{F_i}\Delta s_i . \qquad (28)$$

For the moving active body the vector sum of the residual kinetic energy and the work has been performed by such body is equal to initial kinetic energy of the body.

In such a case, the kinetic energy of the body is transformed into the work intended for an obstacle overcoming.
The passive body increases its kinetic energy while interacting with the active body providing that the velocity is not lowered at the initial moment.

$$\frac{m\overline{v_k}v_k}{2}=\frac{m\overline{v_0}v_0}{2}+\sum_{1}^{i}\frac{m\overline{\Delta v_i}\Delta v_i}{2}. \qquad (29)$$

Therefore, the work of sequentially applied forces results in change the body's kinetic energy.

$$\sum_{1}^{i}\overline{F_i}\Delta s_i=\frac{m\overline{v_k}v_k}{2}-\frac{m\overline{v_0}v_0}{2}. \qquad (30)$$

The body's kinetic energy is defined by the final velocity of its motion.

If we consider the motion of the body as a complex motion, then the action of any force will be aimed at increasing of the kinetic energy of the body in the direction of the applied force.

$$\overline{F}\Delta s=\overline{F}\frac{\Delta v}{2}\Delta t=\frac{m\Delta\overline{v}\Delta v}{2}. \qquad (31)$$

In this case, a direction of the force coincides with a direction of the velocity vector variation, and it is directed towards the passive body.
It is necessary to point-out that during interaction one body loses while another one acquires some amount of a kinetic energy, and such amounts are not equal each other.
The work is performed by one body of the system, while another one acquires a kinetic energy of the rectilinear motion: that is a particular feature of the interaction.
The work of the force is a vector. The work of the force as a vector is utilized for the purpose of recovery of the location or velocity of the moving active or passive body upon a completion of the work.
If the task is to define a total algebraic sum of the works have been successively performed by various forces, then the work of the force can be considered as a scalar value.
The work of a force (including an inertia force) per time unit is referred to as a power.
Proceeding from the equation (26), the power can be expressed as a vector equation

$$\Delta\overline{W}=\frac{\Delta\overline{A}}{\Delta t}=\overline{F}\frac{\Delta s}{\Delta t}=\overline{F}\frac{\Delta v}{2}=m\frac{\Delta\overline{v}\Delta v}{2\Delta t}. \qquad (32)$$

Considering the fact that the body's velocity variation when exposed to an action of the force during certain period represents an additional average velocity of the rectilinear motion on the displacement Δs , then the power value is

$$\Delta\overline{W}=\Delta\left(\overline{F}v_{cp}\right). \qquad (33)$$

For the inertia forces, the superposition principle is applicable. The superposition principle is useful when studying a body's motion under a force effect. The law allows representing of the body's motion under a force effect as a complex motion consisting of an inertial motion with a constant velocity and an accelerated motion under effect of the force in the direction of such force application. Each of those motions can be considered separately, and then to be combined into one motion as per vector addition rules. Sometimes it is convenient to avoid considering a uniform motion of a body, while to paid an attention to motion of the body under the force effect being initiated in the quiescent state.
The principal laws of the mechanics lay in the basis of the performed study. The results of study are in consent with an existing theory of the body's rectilinear motion under the effect of forces. Such theory has been proved experimentally and approved through practice. In particular, free oscillations without considering the impedance force occur under effect of a recovering force, which accelerates a body in the direction of the center of oscillation according to the following equation:

$$kx-ma=0, \qquad (34)$$

where k is a proportionality coefficient.
Upon passing an oscillation center, the body performs an inertial motion overcoming the elastic forces resistance.

$$\frac{m_1\overline{v_{1^1}}-m_1\overline{v_{1^0}}}{\Delta t}=-\overline{F_1^u}. \qquad (35)$$

The inertia forces are equal to elastic force in the absolute value. Therefore, the equation of free oscillation can be represented as

$$kx+ma=0. \qquad (36)$$

This equation completely agrees with a classical principal equation of the oscillation theory [2].
The performed study refines the mechanism of the inertia force effect to the rotor dynamics. Obtained equations and dependencies enable more precise calculation of a motion of the movable parts of mechanisms including shock processes and taking into account moments of inertia of such parts. Results of the study form a part of future activities aimed at development of the inertial approach to the solution of practical problems of the mechanics and tasks of the modern instruments, mechanisms, and rotary machines design.

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