BASE LAWS OF ROTATIONAL MOTION OF A FREE BODY©

A.Yu. Zhivotov, Yu.G. Zhivotov
"Yuzhnoye" State Design Office

This paper is dedicated to the analysis of the inertial moments of the inertia forces emerging during rotation of subassemblies and devices of rotary machines and to the basic laws of their interaction. In the present paper, the authors establish the interrelation between the rotary momentum of a subassembly and the inertial moments being capable to affect the dynamics of rotary machines and mechanisms. Starting from a relation for the momentum, the authors derive in a step by step manner the vector equations defining the inertial moment, the impulse of the inertial moment, the kinetic energy of the rotating subassembly, the work and the power being produced by the inertial moments. The method of definition of the rotary momentum transfer is proven in the paper. It forms a basis for both the momentum and impulse conservation laws.

Several kinds of machines (and specifically the rotary machines) involve various subassemblies performing rotary motion. Change of a speed of rotation of assemblies is accompanied by occurrence of the inertial moments of inertia forces opposing this change [1]. Inertia of body is considered in the theory of oscillations being widely applied in machinery [2], in the shock theory and in the dynamics of machines [3]. The existing concept of the inertia forces is stated in a most definitive way in the paper [4]. According to the concept, the inertia forces are basically considered in inertial reference systems. The inertial approach to studying of the rotary dynamics has led to the new equations describing rotation [5]. However, these equations do not take into account the transients resulting from the change of the speed of rotation at which the inertial properties of rotating subassemblies are manifesting themselves. It is a consequence of a lack of the systematized analysis of the occurrence of body's moments of inertia, fragmentation of sources and, at times, unavailability of the equations allowing carrying-out a detailed analysis of all modes of rotation of subassemblies and parts with a due regard to their inertial properties.

The analysis of the conditions of the driving torque occurrences is put as a primary goal of this study together with such task as obtaining and ranking of the basic patterns of relationships and dependences for definition of a degree of influence of inertial properties of subassemblies on the rotary machine dynamics.

A body's motion is a complex motion consisting of the center of gravity translation along a trajectory, and a rotation of a body around of the center of gravity. Rectilinear inertial motion of a body is the simplest component or elementary motion of the translational motion.

The free rotation of a body around its own axis is the second type of the elementary motions of a body. An attempt to counteract a free rotation of a body results in the event of the body's inertial properties such as an occurrence of the rotary inertial moments, which possess a size, a direction. They characterize the feature of a body's rotation. The rotary inertial moment of a body is equal to zero at the quiescent state.

The free rotation of a body is characterized by the fact that all particles of a body are rotating around the principal axis of inertia, whose position in the reference system remains unchanged. Each particle of a body as it is mechanically connected to other particles of a body is keeping a certain face in a direction of and moves around an axis along a circle at the certain radial distance. To change a rotation speed and to not force a body to move rectilinearly, it is necessary to affect a body by a force couple or by a moment of force.

Imagine a body consisting of only two particles of identical mass located on the distance r from each other and connected between themselves with a weightless rigid connection. Assume that both particles are in a quiescent state. We can force one of the particles (for example the second one) to move in a direction being perpendicular to the rigid connection.

According to the first law of Newton, the second particle will get a rectilinear momentum and will start to move rectilinearly. The first particle will counteract the motion of the second particle. Inertia forces of the first particle will affect the second particle through the connection. The influence of the first particle will lead to a change of the second particle velocity, and to occurrence of the inertia force, which will influence upon the first particle through the connection. The first particle is compelled to begin its motion, but it will be done in a direction of force, which is transferred to it from the second particle through the mechanical connection. As a result, both particles start moving in the orthogonal directions. Mutual effect of the particles will lead to the state when a body's motion will consist of rectilinear and rotary motion.

To eliminate a rectilinear component of the motion we shall connect the body with the third particle of the same kind using a rigid weightless connection. We arrange the third and the second particles at the diametrically opposite locations with respect to the first particle at the same distance r from the first particle. Through the first particle, we shall draw an axis of rotation perpendicular to the rigid connections between the particles. We shall choose a plane of motion of the particles perpendicular to the rotation axis. We shall force both the second and third particles to move in this plane with opposite velocities but equal in terms of magnitude, which are directed perpendicularly to the connections.

According to the first law of Newton, the second and the third particle will undertake an attempt of moving rectilinearly in opposite directions. As a result, the second and the third particles will not allow each other to move rectilinearly because of the inertia forces being produced by the particles, which will be equal in magnitude but opposite in terms of direction. The forces of inertia compensate each other and try to break off the connection holding the particles together. As a result, the particle will start to move in a single file along a circle having a radius r with the identical linear velocity producing centrifugal forces of inertia. It is possible to remove the first particle, while connecting the second and third particle by means of a mechanical connection. We shall obtain a body consisting of two particles and having the axis of inertia coinciding with the axis of rotation. The body can consist of several similar particles. In this case, a position of the axis of rotation is those, that the masses of particles are located symmetrically with respect to the axis of rotation.

Each particle of a rotating body possesses a certain momentum. Their sum determines a rotary momentum of a body.

The velocity $\overline{v}$ represents a velocity of a particle along a trajectory and, therefore, it follows a tangent to a circle, which the particle goes along. Taking into account that the velocity of particles is $\overline{v} = \overline{\omega} r$, the direction of linear velocity defines a direction of a particle rotation speed. The vector of a particle linear velocity defines a direction of the vector of angular displacement, which characterizes a direction of a body's rotation and it has modulo, which tends to zero. In view of this note, the rotary momentum of the body consisting of two particles and rotating at a constant rate can be presented as

$$\overline{K_1^B} + \overline{K_2^B} = m_1 \overline{\omega} r + m_2 \overline{\omega} r \; . \tag{1}$$

The equation (1) assumes that $m_1 = m_2$, and $\overline{K_1^B} = \overline{K_2^B}$. The similar equation can be written down also for three particles if they are uniformly allocated on a circle of a radius r. Hence, a symmetric distribution of particles in the rotation plane, which is perpendicular with respect to the rotation axis plays an essential role. The body consists of particles having various masses m_i and being located at various distances r_i from the axis of rotation and in different planes of rotation. Therefore, the equation (1) for the whole body can be written down as

$$\overline{K^B} = \overline{\omega} \sum_1^i m_i r_i \; . \tag{2}$$

Symmetric distribution of masses of a body with respect to the axis of rotation is defined by the following expression

$$D = \sum_1^i m_i r_i \; . \tag{3}$$

Thus, the rotary momentum of a body is defined by the equation

$$\overline{K^B} = D\overline{\omega}. \tag{4}$$

The equation (2) shows that the direction vector of body's rotary momentum coincides with the direction of the rotation speed vector. The rotary momentum is a measure of inertia of a moving body. In a quiescent state the body possesses a quiescent inertia or a rotary momentum being equal to zero. For a free rotating body the axis of rotation simultaneously represents the principal axis of inertia.

The rotary momentum characterizes an ability of a body to have a dynamical influence upon another body. The active body loses rotation speed of rotation because of interaction with other body and gives a quantity of rotary motion.

The passive body increases a rotation speed under an influence of another body and acquires some additional rotary momentum. At the beginning, a passive body might loose a certain amount of a rotary momentum, but finally it acquires some rotary momentum of the opposite direction.

Obviously, the total rotary momentum of the closed system should be equal to the sum of rotary momentum of all bodies. This implies that the vector sum of rotary momentums of all bodies of the system should be equal to zero. However, the rotary momentum of the closed system is defined by the sum of absolute values of rotary momentums of all bodies, which are present in the closed space, i.e. is a scalar and a constant value

$$K_c^B = \sum_1^i m_i v_i = \sum_1^i D_i \omega_i = Const. \tag{5}$$

Rotating bodies can interact among themselves without occurrence of rectilinear motion if forces at their contact are directed along the tangents to contact points. As an example of such forces the friction forces can be cited. We shall consider the friction forces as infinite quantities taking into account that rotating bodies constituting a part of a mechanism could be frequently got into interaction, as an example, through a toothing while their axes of rotation are fixed.

In general, rotating bodies contact in the manner when the mutual arrangement of their main planes of rotation is set at some angle with respect to each other. Moreover, the points of contact of the bodies can lay in planes that are parallel to the main planes of rotation. We consider that the rotation of a body in a plane of rotation occurs clockwise or counter-clockwise.

In case of tangential interaction of bodies, the rotation speed is changed in both a magnitude and a direction. Variation of the body's rotation speed can be defined through the variation of its linear velocity. The final inertial rotary momentum of a body depends only on its final speed of rotation.

The increment of the passive body's rotation speed having a mass distribution D_1 in the range of rotation speed from $\overline{\omega_{1^0}}$ to $\overline{\omega_{1^1}}$ results in increase of the rotary momentum by the value $\Delta\overline{K_1^B}$

$$\Delta\overline{K_1^B} = D_1\overline{\omega_{1^1}} - D_1\overline{\omega_{1^0}} = D_1\Delta\overline{\omega_1}. \tag{6}$$

Increment of rotary momentum is directed from the end of a vector $D_1\overline{\omega_{1^0}}$ to the end of a vector $D_1\overline{\omega_{1^1}}$ and forms an acute angle with a direction of the rotation speed. This direction of a body's rotary momentum variation should be considered as a positive one. A decrement of the rotation speed of an active body having the mass distribution D_2 in the range of rotation speed from $\overline{\omega_{2^0}}$ to $\overline{\omega_{2^0}}$ results in reduction of rotary momentum by the value $\Delta\overline{K_2^B}$

$$\Delta\overline{K_2^B} = D_2\overline{\omega_{2^1}} - D_2\overline{\omega_{2^0}} = -D_2\Delta\overline{\omega_2}. \tag{7}$$

The variation of a rotary momentum is directed from the end of a vector $D_1\overline{\omega_{1^1}}$ to the end of a vector $D_1\overline{\omega_{1^0}}$ and forms an obtuse angle with a direction of the rotation speed. This direction of a body's rotary momentum variation should be considered as a negative one.

The equations (6) and (7) show that the directions of rotary momentum variations for active and passive bodies are opposite. The direction of a rotary momentum variation vector generally does not coincide with a direction of the body's initial or final rotation speed. In special cases the direction of a vector $\overline{\Delta K^B}$ can coincide with a direction of initial or final rotation speed of a body, however it can also have an opposite direction.

If the rotary momentum of a passive body having the mass distribution D_1 increases uniformly during certain time interval, then it can be stated that

$$\frac{D_1\overline{\omega_{1^1}} - D\overline{\omega_{1^0}}}{\Delta t} = D_1\frac{\overline{\omega_{1^1}} - \overline{\omega_{1^0}}}{\Delta t} = D_1\overline{\zeta_1}\,, \tag{8}$$

where $\overline{\zeta_1}$ - is the angular acceleration of a passive body which is defined by the equation

$$\overline{\zeta_1} = \frac{\overline{\omega_{1^1}} - \overline{\omega_{1^0}}}{\Delta t} = \frac{\overline{a_{1^i}}}{r_{1^i}}\,, \tag{9}$$

where a_{1^i} - is an acceleration of i-th particle of a passive body, r_{1^i} - is a distance of i-th particle of a passive body up to an axis of rotation.

From the equations (8) and (9) follows, that

$$D_1\overline{\zeta_1} = D_1\frac{\Delta\overline{\omega_1}}{\Delta t} = D_1\frac{\overline{a_{1^i}}}{r_{1^i}} = \sum_1^i m_{1^i}\overline{a_{1^i}} = \sum_1^i \overline{F_{2^i}}\,. \tag{11}$$

In the equation (11) the forces $\overline{F_{2^i}}$ are directed in a direction of linear velocity of a particle motion. They can be recanted into the form of two resultant forces, which will make a force couple directed tangentially to the same circle opposing each other.

This force couple is not determined in terms of distance between these forces.

Proceeding from a condition of symmetry of the forces being applied to a body in the same direction with respect to an axis the rotation, we shall multiply the left and right sides of the equation (11) by the radius of rotation of each particle measured from the axis of rotation. As a result, we shall obtain

$$\sum_1^i m_{1^1} r_{1^i} r_{1^i}\frac{\Delta\overline{\omega_1}}{\Delta t} = \sum_1^i \overline{F_{2^i}} r_{1^i}\,, \tag{12}$$

or

$$\sum_1^i m_{1^i} r_{1^i}^2\frac{\Delta\overline{\omega_1}}{\Delta t} = \sum_1^i \overline{F_{2^i}} r_{1^i} = \overline{M_2}\,, \tag{13}$$

where $\overline{M_2}$ is the running torque being applied to a passive body from the direction of an active body.

In the examined case, the axis of rotation is at the same time the axis of symmetry of a body. The mass-geometrical characteristic of a body is a scalar value and is referred to as the moment of inertia of a body relative to an axis of symmetry

$$I_{0^1} = \sum_{1}^{i} m_{1^i} r_{1^i}^2 . \qquad (14)$$

The equation (13) can be written in the following form

$$I_{0^1} \frac{\Delta \overline{\omega_1}}{\Delta t} = \overline{M_2} . \qquad (15)$$

The direction of the running torque that increases a rotation speed coincides with a direction of an angular acceleration of a body and forms an acute angle with a direction of rotation speed.

It is obvious that at the interaction of the active body (having a distribution of masses D_2) with a passive body in case of variation of a rotation speed in the range from $\overline{\omega_{2^0}}$ to $\overline{\omega_{2^1}}$, the equation (15) can be written in the form

$$I_{0^2} \frac{\overline{\omega_{2^1}} - \overline{\omega_{2^0}}}{\Delta t} = -I_{0^2} \frac{\Delta \overline{\omega_2}}{\Delta t} = \overline{M_2^u} . \qquad (16)$$

In this case, the running torque $\overline{M_2^u}$ is caused by inertia of an active body. It is directed oppositely to a direction of the rotation speed variation and acts upon an obstacle, which prevents a body from the uniform rotation around its own axis of symmetry. The inertial running torque forms an obtuse angle with a direction of a rotation speed of the body producing this torque.

It could occur in the initial moment that a passive body is braked by the running torque $\overline{M_2}$. In this case for the initial moment of time the passive body is capable to produce an inertial running torque, which is directed against action of the running torque of an active body

$$I_{0^1} \frac{\overline{\omega_{1^1}} - \overline{\omega_{1^0}}}{\Delta t} = -I_{0^1} \frac{\Delta \overline{\omega_1}}{\Delta t} = \overline{M_1^u} . \qquad (17)$$

The passive body, which increases rotation speed under the action of an active body, produces the passive inertial moment, which does not counteract an increment of the passive body's rotation speed. The equation of the passive inertial moment produced under the action of a running torque of an active body in this case can be written down as

$$I_{0^1} \frac{\overline{\omega_{1^1}} - \overline{\omega_{1^0}}}{\Delta t} = -I_{0^1} \frac{\Delta \overline{\omega_1}}{\Delta t} = -\overline{M_2} = \overline{M_1^0} , \qquad (18)$$

where $\overline{M_1^0}$ - is a passive inertial moment of a passive body. It is the inertial moment reacting to an influence of a running torque, and it does not counteract to the rotation.

From the equation (15), it follows that it is possible to change a rotary momentum when acting onto the body by means of a running torque within certain time. In turn, a change of the rotary momentum of a body results in origination of a rotary inertial moment, which has a certain time of existence.

In case of a rotation speed reduction, the body's rotating the inertial moment tends to support the body's rotation and to remove an obstacle preventing the inertial rotation of a body. An increment of the body's rotation speed means that the body is passive and that it is under effect of an external running torque.

The inertial running torque is the mass moment as in its production participates each particle of a body and, consequently, the resulting inertial running torque of all particles of a body is defined relative to an axis of rotation.

Eventually, it could occur that the body is under the simultaneous action of several running torques. Such an action can be replaced with one resultant running torque because in case of rotary motion the plane where it acts should pass through the body's center of gravity. Thus, the principle of superposition of the moments is maintained similarly to the principle of forces superposition.

Action of the external running torque onto the body during certain time results in production of an impulse of the running torque $\Delta\overline{L_2}$, which is capable of changing the body's rotary momentum

$$\Delta\overline{L_2} = \overline{M}_2\Delta t = \pm I_{0^1}\Delta\overline{\omega_1}\ . \tag{19}$$

A decrement of a rotary momentum of a body is also capable to produce an impulse of the moving body's inertial running torque

$$\Delta\overline{L_2^u} = I_{0^2}\overline{\omega_{2^1}} - I_{0^2}\overline{\omega_{2^2}} = -I_{0^2}\Delta\overline{\omega_2}\ . \tag{20}$$

An impulse of the inertial running torque has a direction opposite to a direction of the body's rotary momentum variation and forms an acute angle with a plane of the body's rotation.

The rotary momentum is not subject to change when the bodies being part of the system are interacting among themselves. Thus, the vector sum of the lost and acquired rotary momentum of interacting bodies should be equal to zero. Considering this sentence, the following vector equation could be derived

$$I_{0^1}\Delta\overline{\omega_1^*} = -I_{0^2}\Delta\overline{\omega_2^*}\ . \tag{21}$$

The asterisk hereinafter specifies that in a general case of bodies' interaction, the variation of a parameter (in our case the rotation speed variation) remains unknown. It follows from the equation that the vectors of the rotary momentum variation of each body are directed oppositely.

The rotary momentum being transmitted from one body to another depends on a position of the contacting bodies' particles with respect to the direction of rotation speeds of their rotary motion. Before interaction, the direction of rotary momentum of each body coincides with a direction of rotation speed. At interaction, each rotating body produces the inertial running torque intended for overcoming of counteraction of another body. The inertial running torque reaches its maximum value when the direction of the inertial running torque coincides with a direction of the rotation speed. However, the point of contact of the rotating body might not lay in a plane of action of the maximal running torque, and a vertical to the body's surface in the point of contact might form an angle with a direction of the moving body's rotation speed. In these cases, the value of the body's inertial running torque will be less than maximal. In addition, the bodies can rotate towards each other or rotate in the same direction. In these cases, the rotary momentums being transferred from one body to another will depend on a set of rotation speeds, which these bodies have at interaction.

The scalar sum of total rotary momentum of the system before interaction is a constant

$$I_{0^1}\Delta\omega_1 - I_{0^1}\Delta\omega_1^* + I_{0^2}\Delta\omega_2 + I_{0^2}\Delta\omega_2^* = I_{0^1}\Delta\omega_1 + I_{0^2}\Delta\omega_2 = Const\ . \tag{22}$$

However, for in each individual body the rotary momentum is varied both in terms of magnitude, and in terms of direction. The active body's rotary momentum is defined as:

$$\overline{L_1^*} = I_{1^0}\,\overline{\omega_1} - I_{1^1}\,\overline{\omega_1^*}\,. \qquad (23)$$

The passive body's rotary momentum is defined as:

$$\overline{L_2^*} = I_{2^1}\,\overline{\omega_2} + I_{2^0}\,\overline{\omega_2^*}\,. \qquad (24)$$

It follows from a conditions of equality of running torque pulses and their opposite directions including ones that has been produced by the inertial moments during interaction of bodies

$$\overline{M_1^*}\Delta t = -\overline{M_2^*}\Delta t\,. \qquad (25)$$

Therefore, bodies interact with each other by means of the moments equal in terms of magnitude and opposite in terms of direction

$$\overline{M_1^*} = -\overline{M_2^*}\,. \qquad (26)$$

These moments can be distinguished as acting and counteracting ones. The active body creates an acting inertial moment, while the passive body creates a counteracting inertial moment.

The condition allows defining the inertial moment of a passive body's reaction, which exists at presence of the acting running torque . Therefore, the inertial moment of a body's reaction does not oppose a motion, while it produces the internal moment affecting particles of a body.

Points of contact of bodies can be located at various distances from the principal axis of symmetry of bodies and consequently the definition of a rotary momentum participating in an exchange requires some additional information.

In order to eliminate an uncertainty of the equation members marked with an asterisk, we proceed from an assumption that the body should spend a minimum of rotary momentum to overcome an obstacle. We also consider that an inertial running torque of a body, which is produced with the purpose of overcoming an obstacle, is decreased proportionally to an angle between a normal to the surface of a body in a point of its contact with another body, and a direction of a rotation speed. It is obvious that the angle between a normal to the surface of an active body in a point of contact and rotation speed of an active body for a transfer of the inertial running torque cannot exceed 90^o. For the bodies having a spherical shape, the normal to a surface passes through the center of masses. The minimum of the rotary momentum, which is transferred during interaction of spherical bodies, depends on rotation speeds. For the definition purposes, it could be assumed that one of the moving bodies possesses a rotation speed equal to zero, while the second body possesses a rotation speed equal to the sum of rotation speeds of both bodies.

A body with the smaller moment of inertia defines the minimal rotary momentum, which is transferred during interaction of spherical bodies with consideration of the geometrical difference of rotation speeds of both bodies, and also an angle of inclination β_a of a normal to the surface of the acting active body at the contact point

$$\Delta K_{\min}^{B} = I_{\min}\left|\left(\overline{\omega_1} - \overline{\omega_2}\right)\right|\cos\beta_a\,. \qquad (27)$$

If the angle β_a is less than 90^o, the body is active and loses a rotary momentum. If the angle β_a is more than 90^o, the body is passive and acquires additional rotary momentum. If the angle β_a is equal to 90^o, the interaction being accompanied with a transfer of rotary momentum between the bodies does not occur. If the angles are identical for both of bodies, then the body with the greater rotation speed is referred to as an active one.

The active body performing an inertial motion is capable to produce impulses of the inertial running torque prior to transition of a body into the quiescent state. At an encounter of an active body with an obstacle, the rotary momentum decreases by the value sufficient for overcoming such an obstacle. The body maintains a residual rotary momentum and remains capable to overcome another obstacle because of possessing a residual rotation speed $\overline{\omega_1}$. In this case, the equation will be valid

$$I_0\left(\overline{\omega_0}-\overline{\omega_1}\right)=\sum_1^i \overline{L_i^u}\,. \tag{28}$$

From this it follows that the vector sum of impulses of the running torques created by an active body cannot exceed a rotary momentum of a body. The running torques can act sequentially. Taking into account an impulse of the running torque by analogy with the equation (28), it is possible to write the following

$$\sum_1^i \overline{L_i}=I_0\overline{\omega_i}-I_0\overline{\omega_0}\,. \tag{29}$$

The equation is valid only for free motion of a body under the action of a running torques applied to the body's center of mass. An impulse of the inertial moment is defined by the value of the inertial running torque and the time of its action. Thus, the body producing an impulse of the inertial moment rotated through the strictly certain angle $\Delta\varphi$ while producing an inertial running torque.

Multiplying the right and left sides of equation (17) by an angle of rotation $\Delta\varphi$, we shall obtain

$$I_0\left(\overline{\omega_1}-\overline{\omega_0}\right)\Delta\varphi=\overline{M^u}\Delta t\Delta\varphi\,, \tag{30}$$

or

$$I_0\left(\overline{\omega_1}-\overline{\omega_0}\right)\frac{\Delta\varphi}{\Delta t}=\overline{M^u}\Delta\varphi\,. \tag{31}$$

The expression $\frac{\Delta\varphi}{\Delta t}$ represents an average rotation speed of a body during time Δt. Therefore the equation (31) can be written down as

$$\frac{I_0\overline{\Delta\omega}\Delta\omega}{2}=\overline{M_u}\Delta\varphi=\overline{M_u}\frac{\Delta\omega}{2}\Delta t\,. \tag{32}$$

The inertial running torque $\overline{M_u}$ has a direction, which is opposite to the variation of a vector of the rotation speed. The passive body, while perceiving an impulse of the inertial moment of an active body changes a direction and increases its rotation speed. In this case the equation looks like

$$\overline{M}\Delta\varphi = I_0 \frac{\left(\overline{\omega_1} - \overline{\omega_0}\right)}{\Delta t}\frac{\Delta\varphi}{\Delta t} = \frac{I_0 \overline{\Delta\omega}\Delta\omega}{2}. \tag{33}$$

In this case, the direction of the running torque coincides with the direction of variation of the rotation speed vector and is directed towards a passive body. The running torques, both external and inertial, performs a work, while turning a body through an angle of $\Delta\varphi$ in a direction of action of the running torque. The work can be determined as

$$\Delta\overline{B} = \overline{M}\Delta\varphi = \overline{M}\frac{\Delta\omega}{2}\Delta t. \tag{34}$$

The active rotating body performs a work in a direction of the action of the inertial running torque, and in a direction, which is opposite to an external running torque

$$\Delta\overline{B_u} = \overline{M_u}\Delta\varphi = -\overline{M}\Delta\varphi. \tag{35}$$

The impulse of the inertial running torque produced by the rotating active body is finite. Its magnitude is limited to the rotary momentum, which the active body possesses. The rotating body produces accordingly the inertial running torque capable of performing a limited work. The maximal work of the inertial running torque of an active body is defined by the dependence

$$\overline{B_u} = \overline{M_u}\varphi = \frac{I_0 \overline{\omega_0}\omega_0}{2}, \tag{36}$$

where φ - is a full turn of a body prior to transition into a quiescent state, $\overline{\omega_0}$ - is an initial rotation speed of a rotating body.

The active body, even when interacting with several bodies is able of maintaining a residual rotation speed of rotary motion. It is obvious that such a case can be described by the equation

$$\frac{I_0 \overline{\omega_0}\omega_0}{2} = \sum_1^i \overline{M_u^i}\varphi_i + \frac{I_0 \overline{\omega_k}\omega_k}{2}. \tag{37}$$

If goal is to define a total algebraic sum of work that has been performed by various running torques in turn, then a work of the running torque can be considered as a scalar value.

A rotating body's kinetic energy direction coincides with a direction of rotation and it is equal to the product of rotary momentum of a body by a half value of the speed

$$\overline{J_0} = \frac{I_0 \overline{\omega_0}\omega_0}{2}. \tag{38}$$

A variation of body's the kinetic energy of rotation can be written as

$$\Delta\overline{J} = \frac{I_0 \Delta\overline{\omega}\Delta\omega}{2}. \tag{39}$$

A variation of the kinetic energy can coincide or not coincide with a direction of the kinetic energy of a body.

There are two events when a kinetic energy of a body's rotation changes.

The active body loses kinetic energy of rotation while interacting with another body or several bodies

$$\frac{I_0 \Delta\overline{\omega_0} \Delta\omega_0}{2} = \sum_1^i \frac{I_0 \Delta\overline{\omega_i} \Delta\omega_i}{2} + \frac{I_0 \Delta\overline{\omega_k} \Delta\omega_k}{2}, \tag{40}$$

i.e. when there is an inertial running torque on the side of a passive body affecting a rotating body.

The passive body increases kinetic energy of rotation under action of the inertial running torque of an active body

$$\frac{I_0 \Delta\overline{\omega_k} \Delta\omega_k}{2} = \frac{I_0 \Delta\overline{\omega_0} \Delta\omega_0}{2} + \sum_1^i \frac{I_0 \Delta\overline{\omega_i} \Delta\omega_i}{2}. \tag{41}$$

Kinetic energy of rotation of a body is defined by the final rotation speed of a body.

At interaction, one body loses a certain amount of its kinetic energy of rotation, while another body acquires a certain amount of the kinetic energy of rotation. However, these amounts of the kinetic energy of rotation are not equal to each other.

Moreover, the work is performed by one of the bodies being part of a system, while the kinetic energy of rotation is acquired by another body. If the body loses its kinetic energy of rotation, then it means that the body performs a work over another body.

The power is defined as the work accomplished by the moment at a unit of time

$$\Delta\overline{W^B} = \frac{\Delta\overline{B}}{\Delta t} = \overline{M}\frac{\Delta\varphi}{\Delta t} = \overline{M}\frac{\Delta\omega}{2} = I_0 \frac{\Delta\overline{\omega}\Delta\omega}{2\Delta t}. \tag{42}$$

A variation of the rotation speed of a body under the action of the running torque within a time interval represents an additional average rotation speed at an angle of rotation $\Delta\varphi$, and consequently it can be written that

$$\Delta\overline{W^B} = \Delta\left(\overline{M}\omega_{cp}\right). \tag{43}$$

All previously mentioned features of the body's rotary motion have been received under condition of equality of the body's moments of inertia relative to any axis of symmetry. Such property possesses, for example the body of spherical shape. However, in most cases, bodies possess different moments of inertia with respect to the main axes of symmetry.

The role of the mass distribution in a body becomes especially important for the rotary motion. Such a body as a disk steadily rotates around an axis, which passes through the center of mass and is perpendicular to the plane of a disk. The body having a shape of a cylinder can theoretically rotate around the axis coinciding with an axis of the cylinder too. However, if for any reason it would emerges a certain deviation of a geometrical axis of the cylinder with respect to an axis of rotation, then a cylinder starts rotating around an axis, which passes through the cylinder's center of mass and is perpendicular to its geometrical axis. Rotation of the cylinder around a new axis of symmetry is attributable to an occurrence of the moment which turns the cylinder around.

Any intermediate axis of rotation between these two positions is perceived by a rotating body as an influence of another body, which opposes an inertial rotation of the body. In this connection, the rotating body produces an inertial running torque to overcome this obstacle. Due to the fact that a real obstacle is absent, the body occupies a new position, at which there is no any moment resulting in a turn of the cylinder around. Moreover, the direction of a vector of a body's rotary momentum maintains the direction.

The principal laws of the rotary motion mechanics lay in the basis of the study has been performed. The carried out researches refine the definition of the origination an influence mechanisms of occurrence and influence of the inertial moments onto the dynamics of rotating subassemblies. The obtained equations and dependences allow the making of the refined calculations of motion of the rotating subassemblies of mechanisms and rotary machines taking into account the moments of inertia of the subassemblies. The results of the study make part of the further activities aimed at the development of the inertial approach to the resolution of practical problems of the mechanics and designing of modern devices, mechanisms and rotary machines.

REFERENCES

1. Тарг С.М. Краткий курс теоретической механики. – М.: Наука. 1964. – 478 с.
2. Тимошенко С.П. Колебания в инженерном деле. – М.: Государственное издательство физико-математической литературы. 1959. – 439 с.
3. Животов А.Ю. Особенности вращения дискового ротора со статической неуравновешенностью. // Вестник Восточноукраинского Национального Университета. – 2001. – 2(36). – С. 218 – 225.
4. Николаев В.И. Силы инерции в общем курсе физики. Ж. Физическое образование в вузах. Т.6, №2, – 2000.
5. Животов А.Ю., Животов Ю.Г. Инерционные особенности вращения ротора. // Техническая механика. Институт технической механики НАНУ и НКАУ. – 2003. – № 1. – С 107 – 117.