Dynamics of Vertical Rotor with Arbitrary Arrangement of Static and Moment Unbalances

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ABSTRACT

This paper addresses dynamics of the rotor, which is vertically secured in one immovable hinge and one elastic support. There were received and analyzed equations of dynamics for description of disk-shaped and cylindrical rotor with dynamic unbalance. From the overall system of equations, there was received a system of equations for description of dynamics of the rotor under the impact of only static unbalance or only moment unbalance. There were considered cases of stable rotation of the rotor. There were obtained and analyzed dependencies for definition of all forces and moments, which impact the rotor. There were defined conditions at which stability of the rotor's rotation is secured. Studies encompass basic operational modes of the rotor's rotation. Dynamics of the rotor is described by elementary algebraic equations.

KEY WORDS

Critical velocity, inertia, support, vibration

1 INTRODUCTION

There exists a broad class of centrifuges, in which rotors are secured vertically in one immovable hinge support. The hinge support is located either below the rotor or above the rotor. For preservation of the vertical position of the rotor, one usually utilizes an additional elastic support. Enough frequently, the immovable hinge support is made of elastic material. This allows combination in one support of the hinge support and additional elastic support. Owing to the specific suspension, the centrifuge rotor is capable to deviate from the axis of rotation at a certain angle and statically to self align relative to the axis of rotation.

The static self-alignment of the rotor promotes decrease but does not eliminate residual unbalance of the rotor. As it turned out, the vibration amplitude of the centrifuge depends on residual unbalance of the rotor. The vibration theory tried to describe dynamics of such rotors by means of differential equations [1], [2]. And at the same time, an assumption was made that the rotor has only static unbalance. However, the equations obtained did not find mathematical solution and were not able to explain the reasons of residual vibration of the centrifuge.

The inertial theory of rotor dynamics demonstrated that the reason of vibration of the centrifuge [3] is a residual deviation of the rotor's geometrical axis with quasi-static unbalance from the axis of rotation. The aforementioned studies addressed the dynamics of umbrella-type centrifuges, which rotors have static unbalance. They also did not take account of particular qualities associated with the direction of the force of weight at location of the hinge support above the rotor. The difference in schemes of forces that act onto the umbrella-type rotor and vertically suspended rotor impacts significantly the dynamics of these rotors. Furthermore, the rotor dynamics depends substantially on a type of the unbalance. In the general case, the static and moment unbalance have an arbitrary arrangement on a rotor. Therefore, acquisition of correct equations for dynamics of the vertical rotor with dynamic unbalance and their analysis is an important practical problem.

2 OBJECT OF STUDY AND COORDINATE SYSTEM

Let us consider the dynamics of a vertical solid rotor, for which mass is equal to m and weight is equal to P. The equatorial moment of inertia of the rotor is equal to I_a. The axial moment of inertia of the rotor is equal to I_b. A solid shaft of the rotor is mounted on two supports. One end of the shaft is mounted on the hinge support D. Another end of the shaft is mounted in the elastic support. See the figure 1.

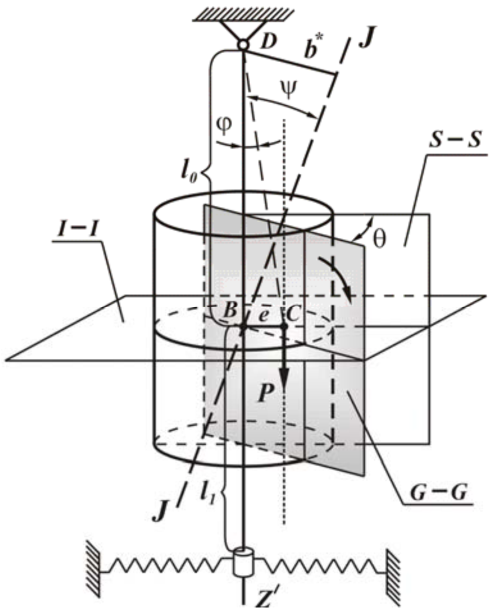

Figure 1: Rotor having a dynamic unbalance

The stiffness coefficient of the elastic support is equal to k. The distance from the hinge support D to the geometrical center B of the rotor is equal to l_0. The distance from a geometric center B to an elastic support equals to l_1. The dynamic unbalance of the rotor is set relative to the rotor's geometrical center B.

The dynamic unbalance consists of static and moment unbalance. The static unbalance of the rotor is set by displacement of the center of mass C from the rotor's geometrical axis Z' at a distance e. The specific feature of this rotor is that the line DC forms the angle φ with the rotor's geometrical axis. The line DC and geometrical axis Z' of the rotor determine an action plane of the principal vector of unbalance – plane $S-S$.

The moment unbalance is set by the deviation angle ψ of the principal axis of inertia $J-J$ of the rotor from the geometrical axis. The distance between points D and axis of inertia $J-J$ is equal to b^*. The action plane $G-G$ of the moment unbalance forms the angle θ with the plane $S-S$. As a positive readout direction of the angle θ from the plane $S-S$, we select the direction opposite to the direction of the rotor's rotational velocity.

We assume that displacements of the rotor from the axis of rotation Z are insignificant relatively to the rotor dimensions. This assumption is agreed with a principle of possible displacements and makes possible to simplify considerably equations of the rotor motion. This assumption meets also the rotation conditions of a natural rotor. The plane $I-I$ perpendicular to the axis of rotation and passing through the center of mass of the rotor is called the measurement plane.

Let us assume that the rotor rotates in vacuum environment around an axis of rotation Z with the angular velocity ω (further – velocity ω). See the figure 2.

Let us assume that the equatorial moment of inertia I_a is bigger than the axial moment of inertia I_b. The direction in which the moment unbalance acts is shown in figure 1 by the arrow in a plane $G-G$. Particular properties of the rotor's rotation let us consider in the rotating coordinate system $OXYZ$. The axis Z of the coordinate system is the axis of rotation. The plane OXY coincides with the measurement plane $I-I$. The axes X and Y in figure 2 are not shown.

During rotation, the geometrical axis Z' of the rotor deviates from the axis of rotation Z at an angle λ. The angle between the line DC and axis of rotation will be defined by the angle γ.

Simultaneously, the rotor turns around the geometrical axis at an angle α. See the figure 3.

The axis Z in figure 3 is depicted by the point O. The distance between points O and C is equal to ρ. The distance between points O and B is equal to a.

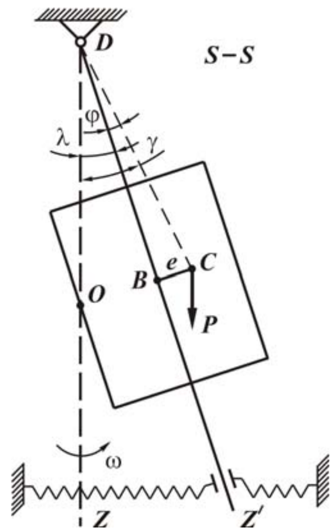

Figure 2: Position of a rotating rotor with a dynamic unbalance

For convenience, let us change the stiffness coefficient of the existing elastic support on the stiffness coefficient k_0 of the presumed elastic support, which conditionally is located in the measurement plane:

$$k_0 = \frac{k\left(l_0 + l_1\right)}{l_0}. \tag{1}$$

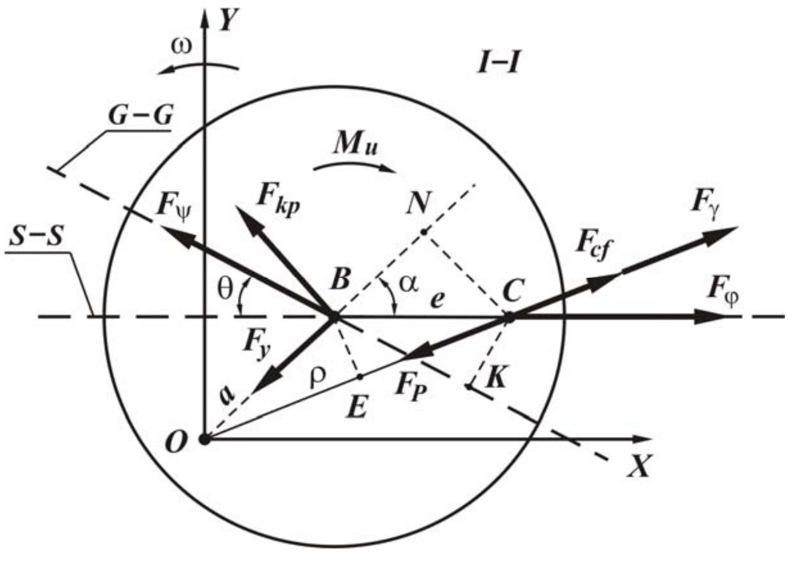

Figure 3: The scheme of forces and the moments acting on a rotor

3 FORCES AND MOMENTS, WHICH ACT ON ROTOR

A rotating rotor is affected by forces and moments so as it is shown in figure 3. The centrifugal force of inertia F_{cf} emerges due to displacement of the center of mass C from the axis of rotation at a distance ρ :

$$F_{cf} = m\omega^2 \rho . \tag{2}$$

The product of inertia of the rotor creates the bending inertial moment since the line DC deviates from the geometrical axis of the rotor at an angle φ .

The bending inertial moment can be presented in the form of a force F_φ, which is applied to the rotor in the point C and acts relative to the point D in the direction of the point B if $I_a - I_b < 0$, or in the opposite direction if $I_a - I_b > 0$:

$$F_\varphi = \frac{(I_a - I_b)\omega^2 \sin\varphi\cos\varphi}{l_0} = \frac{(I_a - I_b)\omega^2 e}{l_0^2}. \quad (3)$$

The force F_y of the elastic support emerges due to displacement of the geometrical center of the rotor from the axis of rotation:

$$F_y = k_0 a. \quad (4)$$

The bending inertial moment, which is created by the product of inertia of the rotor due to deviation of the line DC from the axis of rotation at an angle γ. The bending inertial moment can be presented in the form of a force F_γ, which is applied to the rotor in the point C and acts in the direction of the point O, if $I_a - I_b < 0$, or in the opposite direction if $I_a - I_b > 0$:

$$F_\gamma = \frac{(I_a - I_b)\omega^2 \sin\gamma\cos\gamma}{l_0} = \frac{(I_a - I_b)\omega^2 \rho}{l_0^2}. \quad (5)$$

The bending inertial moment M_u emerges at deviation of the rotor's geometrical axis from the axis of rotation. It consists of the total sum of separate additional rotational inertial moments of the system, which are created due to changes of the moment of inertia and product of inertia of the rotor. The magnitude of the product of inertia depends on the angle λ. The revolution of the rotor around the center of mass due to moment unbalance is ignored. The additional inertial moment acts around the geometrical axis of the rotor opposite to the rotation of the rotor:

$$M_u = m\omega^2 a^2 + \frac{(I_a - I_b)\omega^2 a \sin\lambda\cos\lambda}{l_0} - \frac{Pa^2}{l_0} = m\omega^2 a^2 + \frac{(I_a - I_b)\omega^2 a^2}{l_0^2} - \frac{Pa^2}{l_0}. \quad (6)$$

The rotor's weight creates the force F_p. This occurs due to displacement of the rotor's center of mass from the axis of rotation. The force F_p tends to superpose the line DC with the axis of rotation:

$$F_p = P\sin\gamma = \frac{P\rho}{l_0}. \quad (7)$$

The moment unbalance of the rotor creates a bending moment, which can be represented in the form of a force F_ψ that is applied to the rotor at a point B and acts relative to the point D. The force direction depends on the ratio of equatorial and axial moment of inertia of the rotor:

$$F_\psi = \frac{(I_a - I_b)\omega^2 \sin\psi\cos\psi}{l_0} = \frac{(I_a - I_b)\omega^2 b^*}{l_0^2}. \quad (8)$$

The additional rotational moment M_{kp} is created by the machine's driving gear. The additional rotational moment M_{kp} can be represented by the force F_{kp}, which is applied to the rotor at the point B:

$$M_{kp} = F_{kp} a. \quad (9)$$

4 SYSTEMS OF ROTOR DYNAMIC EQUATIONS WITH UNIFORM ROTATION

For receipt of the system of equations of rotor dynamics, we will utilize a scheme of forces and moments acting onto the rotor if $I_a - I_b > 0$, see the figure 3. Let us set some velocity of a rotor. The rotor's angular velocity is considered constant and, therefore, the rotor occupies a certain position in the rotating coordinate system. For every value of rotor velocity there is also a certain value of an angle α.

Based on the laws of statics, one can write equations of forces moments around the points O, B, C using the well-known geometrical dependencies [4]:

$$BE = \frac{ea\sin\alpha}{\rho}, \quad (10)$$

$$\rho = \sqrt{a^2 + e^2 + 2ae\cos\alpha}, \quad (11)$$

$$CN = e\sin\alpha \,, \tag{12}$$

$$BN = e\cos\alpha \,, \tag{13}$$

$$CK = e\sin\theta \,, \tag{14}$$

$$OS = a\sin\left(\alpha+\theta\right). \tag{15}$$

Putting together the equation of forces moments around the points O, B, C, we will receive:

$$\left\{\begin{array}{l} F_{kp}a - M_u - F_\varphi a\sin\alpha + F_\psi a\sin\left(\alpha+\theta\right) = 0 \\ F_{cf}\dfrac{ae\sin\alpha}{\rho} + F_\gamma\dfrac{ae\sin\alpha}{\rho} - F_p\dfrac{ae\sin\alpha}{\rho} - M_u = 0 \\ F_{kp}e\cos\alpha + F_\psi e\sin\theta + M_u - F_y e\sin\alpha = 0 \end{array}\right\}. \tag{16}$$

Taking into consideration the equations (2-15) from the system of equations (16) we will receive the system of equations that describe the rotor's dynamics:

$$\begin{array}{l} \left[m\omega^2 + \dfrac{\left(I_a - I_b\right)\omega^2}{l_0^2} - \dfrac{P}{l_0}\right]\left(a^2 + ea\cos\alpha\right) + \dfrac{\left(I_a - I_b\right)\omega^2}{l_0^2}ea\cos\alpha + \\ + \dfrac{\left(I_a - I_b\right)\omega^2}{l_0}e\sin\psi\left[\sin\theta - \sin\left(\alpha+\theta\right)\cos\alpha\right] - k_0a^2 = 0 \end{array}, \tag{17}$$

$$\sin\alpha = \frac{a}{e}, \tag{18}$$

$$F_{kp} = m\omega^2 a + \frac{2\left(I_a - I_b\right)\omega^2 a}{l_0^2} - \frac{Pa}{l_0} - \frac{\left(I_a - I_b\right)\omega^2}{l_0}\sin\left(\alpha+\theta\right)\sin\psi \,. \tag{19}$$

Let us assume that the rotor's moment unbalance is absent. In this case $\sin\psi = 0$, $\sin\theta = 0$, and the system of equations becomes:

$$\left[m\omega^2 + \frac{\left(I_a - I_b\right)\omega^2}{l_0^2} - \frac{P}{l_0}\right]\left(a + e\cos\alpha\right) + \frac{\left(I_a - I_b\right)\omega^2}{l_0^2}e\cos\alpha - k_0a = 0\,, \tag{20}$$

$$\sin\alpha = \frac{a}{e}, \tag{21}$$

$$F_{kp} = m\omega^2 a + \frac{2\left(I_a - I_b\right)\omega^2 a}{l_0^2} - \frac{Pa}{l_0}. \tag{22}$$

Let us assume that the rotor's static unbalance is absent. In this case $e = 0$, and the system of equations becomes:

$$\left[m\omega^2 + \frac{\left(I_a - I_b\right)\omega^2}{l_0^2} - \frac{P}{l_0}\right] - k_0 = 0\,, \tag{23}$$

$$F_{kp} = m\omega^2 a + \frac{2\left(I_a - I_b\right)\omega^2 a}{l_0^2} - \frac{Pa}{l_0} - \frac{\left(I_a - I_b\right)\omega^2}{l_0}\sin\psi \,. \tag{24}$$

Displacement of the rotor under the impact of moment unbalance occurs in the direction of the impact of a bending moment.

If $l_0 \to \infty$, then the system of equations becomes equivalent to the system of equations, which was received in the paper [4]:

$$m\omega^2\left(a + e\cos\alpha\right) - ka = 0\,. \tag{25}$$

If one changes the direction of the impact of the weight force, then from the equation (20) we will receive the equation:

$$\left[m\omega^2+\frac{(I_a-I_b)\omega^2}{l_0^2}+\frac{P}{l_0}\right](a+e\cos\alpha)+\frac{(I_a-I_b)\omega^2}{l_0^2}e\cos\alpha-k_0a=0\,. \tag{26}$$

The equation correlates to the rotor's equation, which relates to the umbrella-type rotor class [3]. Therefore, dynamics of the umbrella-type rotor with dynamic unbalance is described by the system of equations:

$$\begin{gathered}\left[m\omega^2+\frac{(I_a-I_b)\omega^2}{l_0^2}+\frac{P}{l_0}\right](a^2+ea\cos\alpha)+\frac{(I_a-I_b)\omega^2}{l_0^2}ea\cos\alpha+\\+\frac{(I_a-I_b)\omega^2}{l_0}e\sin\psi\left[\sin\theta-\sin(\alpha+\theta)\cos\alpha\right]-k_0a^2=0\end{gathered}, \tag{27}$$

$$\sin\alpha=\frac{a}{e}, \tag{28}$$

$$F_{kp}=m\omega^2a+\frac{2(I_a-I_b)\omega^2a}{l_0^2}+\frac{Pa}{l_0}-\frac{(I_a-I_b)\omega^2}{l_0}\sin(\alpha+\theta)\sin\psi\,. \tag{29}$$

If the impact of the force of weight is insignificant, the rotor's dynamics is described by the system of equations:

$$\begin{gathered}\left[m\omega^2+\frac{(I_a-I_b)\omega^2}{l_0^2}\right](a^2+ea\cos\alpha)+\frac{(I_a-I_b)\omega^2}{l_0^2}ea\cos\alpha+\\+\frac{(I_a-I_b)\omega^2}{l_0}e\sin\psi\left[\sin\theta-\sin(\alpha+\theta)\cos\alpha\right]-k_0a^2=0\end{gathered}, \tag{30}$$

$$\sin\alpha=\frac{a}{e}, \tag{31}$$

$$F_{kp}=m\omega^2a+\frac{2(I_a-I_b)\omega^2a}{l_0^2}-\frac{(I_a-I_b)\omega^2}{l_0}\sin(\alpha+\theta)\sin\psi\,. \tag{32}$$

5 CRITICAL VELOCITY

The equation for definition of the critical velocity can be obtained from the equation (17) on assumption that $\cos\alpha=0$:

$$\omega_{kp}=\sqrt{\frac{k_0a^2l_0^2+Pa^2l_0}{ma^2l_0^2+(I_a-I_b)a^2+(I_a-I_b)el_0\sin\theta\sin\psi}}\,. \tag{33}$$

As it follows from the equation (33), the value of the critical velocity essentially depends on the value and direction of the moment unbalance impact. The rotors are inoperative if

$$ma^2l_0^2+(I_a-I_b)a^2+(I_a-I_b)el_0\sin\theta\sin\psi<0\,. \tag{34}$$

For a cylindrical rotor we have $(I_a-I_b)>0$. The cylindrical rotor if $\psi>0$ is inoperable when the third member per a module exceeds the sum of the first two members of inequality and angle $\theta>90^\circ$. For a disk-shaped rotor we have $(I_a-I_b)<0$. Operability of a disk-shaped rotor if $\psi>0$ depends on the value of the first member of the inequality if the angle $\theta<90^\circ$.

The rotors are operable if under the specified condition the inequality is fulfilled:

$$ma^2l_0^2+(I_a-I_b)a^2+(I_a-I_b)el_0\sin\theta\sin\psi>0\,. \tag{35}$$

Operating velocities of such rotors cannot exceed the velocities at which the bending moments created by the unbalance exceed the moments created by elastic properties of the support. This remark relates to disk-shaped and cylindrical rotors, and is explained by the ratio of equatorial and axial moments of inertia, as well as by the values $\sin\theta$ and $\sin\psi$.

The worst conditions for self-aligning of the disk-shaped rotor emerge in the situation when the angle $\theta = 90^\circ$. The inequality (35) in this case becomes:

$$ma^2l_0^2 + \left(I_a - I_b\right)a^2 - \left(I_a - I_b\right)el_0 \sin\psi > 0\,. \tag{36}$$

From the inequality it follows that the moment unbalance of the rotor at presence of the static unbalance shall not exceed:

$$\sin\psi < \frac{ma^2l_0^2 + \left(I_a - I_b\right)a^2}{\left(I_a - I_b\right)el_0}\,. \tag{37}$$

At the same time, for a disk-shaped rotor, there shall be implemented the condition:

$$ma^2l_0^2 + \left(I_a - I_b\right)a^2 > 0\,. \tag{38}$$

6 ROTOR DYNAMICS AT OVERCRITICAL VELOCITIES

Dynamics of the rotor after transfer via the overcritical velocity is described by the following system of equations:

$$\left[m\omega^2 + \frac{\left(I_a - I_b\right)\omega^2}{l_0^2} - \frac{P}{l_0}\right]\left(a^2 - ea\right) - \frac{\left(I_a - I_b\right)\omega^2}{l_0^2}ea + k_0a^2 = 0\,, \tag{39}$$

$$F_{kp} = m\omega^2 a + \frac{2\left(I_a - I_b\right)\omega^2 a}{l_0^2} - \frac{Pa}{l_0} + \frac{\left(I_a - I_b\right)\omega^2}{l_0}\sin\theta\sin\psi\,. \tag{40}$$

The system of equations shows that after transfer of the rotor over the critical velocity the impact of moment unbalance is taken up by the solid shaft. Thus, the moment unbalance does not impact dynamics of the rotor at overcritical velocities.

At the velocity tending to infinity since $\cos\alpha = -1$, $\alpha = 180^\circ$, we have

$$\left[m + \frac{\left(I_a - I_b\right)}{l_0^2}\right]a - me - 2\frac{\left(I_a - I_b\right)}{l_0^2}e = 0\,. \tag{41}$$

From the equation (41), one can determine the value of rotor displacement:

$$a = \frac{e\left(ml_0^2 + 2I_a - 2I_b\right)}{ml_0^2 + I_a - I_b}\,. \tag{42}$$

For a cylindrical rotor, shaft displacements a are bigger e at overcritical velocities. Accordingly, for a disk-shaped rotor the shaft displacements a are smaller e.

7 PARTICULAR PROPERTIES OF ROTOR ROTATION IN HINGE SUPPORT

Let us consider a case of rotor rotation if $k_0 = 0$. In this instance, the equation of rotor dynamics becomes:

$$\left[m\omega^2l_0^2 + \left(I_a - I_b\right)\omega^2 - Pl_0\right]\left(a - e\right) - \left(I_a - I_b\right)\omega^2 e = 0\,. \tag{43}$$

For a cylindrical rotor $a - e > 0$. Therefore, the equation (43) for the cylindrical rotor exists if $m\omega^2l_0^2 + \left(I_a - I_b\right)\omega^2 - Pl_0 > 0$.

At the same time, there shall be implemented the inequality:

$$\omega > \sqrt{\frac{Pl_0}{ml_0^2 + I_a - I_b}}\,. \tag{44}$$

Let us draw our attention to the following. The rotor unbalance $\left(I_a - I_b\right)\omega^2 e$ creates conditions for stabilization of rotation. The slightest, random deviation of the rotor from the axis of rotation leads to an increase of the value a, $\left[m\omega^2l_0^2 + \left(I_a - I_b\right)\omega^2 - Pl_0\right]\left(a - e\right)$ at the invariable value $\left(I_a - I_b\right)\omega^2 e$, and consequently, to destabilization of the rotor rotation.

For a disk-shaped rotor, the equation (43) exists if $a-e>0$ and $m\omega^2 l_0^2-\left(I_b-I_a\right)\omega^2-Pl_0<0$. At the same time, the following inequality shall be implemented:

$$\omega<\sqrt{\frac{Pl_0}{ml_0^2-\left(I_b-I_a\right)}}\,. \tag{45}$$

A random decrease of the value a leads to a decrease of $\left[m\omega^2 l_0^2-\left(I_b-I_a\right)\omega^2-Pl_0\right](a-e)$ at the invariable value $\left(I_b-I_a\right)\omega^2 e$, and consequently, to stabilization of the rotor rotation.

If a rotor is considered as a ball, which features $I_b=I_a$, then the equation becomes:

$$m\omega^2 l_0-P=0\,. \tag{46}$$

Therefore, the required condition of rotation of the rotor is

$$\omega<\sqrt{\frac{P}{ml_0}}\,. \tag{47}$$

The obtained result is in compliance with the results of the well-known studies [5].

8 PARTICULAR PROPERTIES OF ROTATION OF BALANCED ROTOR IN HINGE SUPPORT

Let us additionally consider particular qualities of rotation of a disk-shaped rotor if $k_0=0$ и $e=0$, and position of the rotor relative the axis of rotation is determined by the angle λ. See the figure 1.

From the equation (43) it follows:

$$m\omega^2 l_0^2-\left(I_b-I_a\right)\omega^2-Pl_0=0\,. \tag{48}$$

Magnifying both parts of the equation (48) on $\sin\lambda$ we have

$$m\omega^2 l_0^2\sin\lambda-\left(I_b-I_a\right)\omega^2\sin\lambda-Pl_0\sin\lambda=0\,. \tag{49}$$

Let us remove a limitation about minor deviation of the rotor. In this case, the bending moment generated by divergence of the axis of rotation from the geometrical axis of the shaft takes its basic form $\left(I_b-I_a\right)\omega^2\sin\lambda\cos\lambda$. Let us also take into account that $a=l_0\sin\lambda$. Taking into consideration the remarks made, the equation (49) becomes similar to the equation of moments relative to the point D:

$$m\omega^2 a l_0-\left(I_b-I_a\right)\omega^2\sin\lambda\cos\lambda-Pa=0\,. \tag{50}$$

If one takes into account the remarks made and not magnify both parts on $\sin\gamma$, the equation of moments (50) becomes:

$$m\omega^2 l_0^2-\left(I_b-I_a\right)\omega^2\cos\lambda-Pl_0=0\,. \tag{51}$$

The equation (51) makes it possible to find a dependence between the deviation angle of the rotor from the axis of rotation and velocity:

$$\cos\lambda=\frac{Pl_0-m\omega^2 l_0^2}{\left(I_b-I_a\right)\omega^2}\,. \tag{52}$$

The equation (52) shows that with an increase of velocity the deviation angle λ of the rotor from the axis of rotation increases. Let us set out the equation (51) in the form of

$$2m\omega^2 l_0^2\sin\lambda-\left(I_b-I_a\right)\omega^2\sin 2\lambda-2Pl_0\sin\lambda=0\,. \tag{53}$$

In this case, we can find ω:

$$\omega=\sqrt{\frac{2Pl_0\sin\lambda}{2ml_0^2\sin\lambda-\left(I_b-I_a\right)\sin 2\lambda}}\,. \tag{54}$$

Evidently, that $2ml_0^2 \sin\gamma - (I_b - I_a)\sin 2\gamma$ achieves the maximum value if $\sin 2\lambda = 1$, and consequently, $2\lambda = 90^\circ$. In this case, the rotor has a capability to rotate with the maximum velocity. If arises a case that $2\lambda > 90^\circ$, the rotor velocity shall be less then the maximum velocity for preservation of stable rotation.

From this it follows that the allowable angle λ for provision of the rotor's stable rotation shall not exceed 45°.

9 CONCLUSION

The conducted analysis of dynamics of the vertical rotor does not take account of the impact, for example, of aerodynamic resistance forces. Therefore, the obtained equations of dynamics describe the genuine physics of rotation of the unbalanced rotor without the impact of additional disturbing factors. Solution of the algebraic equations does not constitute any particular problems.

Besides, in the equations of rotor dynamics only the additional driving torque is taken into consideration, which is to be applied to the rotor's shaft in order to secure respective position of the rotor relative the axis of rotation. In general, the overall driving torque exceeds the additional driving torque [6].

However, the absence of equations for determination of the overall driving torque does not affect physics of the rotor rotation and does not impede determination of the rotor position relative to the axis of rotation depending on the velocity.

Owing to these particular properties, the obtained equations of rotor dynamics can be used for development of new techniques for identification of rotor unbalances.

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