12th IFToMM World Congress, Besançon (France), June18-21, 2007

New Theory of Rotor Dynamics: Dynamics of Jeffcott Rotor with Moment Unbalance

A. Y. Zhivotov*
"Yuzhnoye" State Design Office
Dniepropetrovsk, Ukraina

Abstract—*The dynamics of the rotor with a moment unbalance was examined for the first time relatively to a rotor with a solid shaft in resilient supports. These researches made possible to describe the physics of a rotor rotation process and define a rotor position with respect to a rotation axis. The system of equations obtained makes possible to determine rotational variables of a rotor, forces and moments affecting a rotor under different rotation conditions.*

An important practical parameter, which obtained a mathematical expression, is a critical velocity. The critical velocity depends on the rotor moments of inertia, distance between supports and coefficient of elasticity of supports. With that a coefficient of elasticity of supports defines a proportion between the restoring elastic force and radial displacement of a shaft section in a support. However, application of this formula in determining a critical velocity of a rotor fixed on a flexible shaft, gives a result, which differs by half as much again as a minimum from an actual critical velocity of a rotor. This is related with a principally different suspension/mounting of a rotor in a machine. Such specific rotor attachment on a flexible shaft requires conducting additional researches of the rotor dynamics for obtaining new equations describing the rotor dynamics, which has a moment unbalance.

In this paper the bases are given on causes of the lack of coincidence of critical velocities of rotors with their different suspension type in a rotor machine. The principal difference in rotational features of a rotor is noted, which is related with a rotor position with respect to a rotation axis. The known equation of the rotor dynamics with a dynamic unbalance is analyzed for the purpose of accounting a distinction revealed. New parameter affecting the rotor dynamics is introduced. Transformations of the known dynamic equation are performed. New system of equations is composed. An analysis of the new system of equations is conducted, which describe the rotor dynamics. Researches of the new system of equations are conducted in relation to the disk and cylindrical rotor at sub-critical, transfer and super-critical rotation conditions.

These researches show that from the qualitative point of view the rotor dynamics with a flexible shaft does not differ from the rotor dynamics with a solid shaft, fixed in resilient supports. However, the equations obtained cause essential qualitative distinctions in determination of critical velocities, forces and moments acting on a rotor.

The results obtained show that the rotor dynamics with a quasi-static or dynamic unbalance moment or dynamic unbalance cannot be described using the single system of equations.

For describing the rotor dynamics with a flexible shaft, the system of equations of the rotor dynamics with a static unbalance is required to be composed as well as the system of equations of the rotor dynamics with a moment unbalance. With that an interaction of these systems of equations is required to be established

Keywords: dynamics, rotor, critical velocity

*E-mail: a-zhivotov@mail.ru

I. Introduction

The rotor behavior under effect of a static unbalance is mainly researched in the vibration theory of the rotor dynamics [1]. The problems of a rotor rotating under effect of the so-called dynamic unbalance are under consideration more seldom [2]. Rotation of a rotor with a moment unbalance is mentioned as the important problem [3]. However, the theory of rotor dynamics with a moment unbalance is absent. This is related with that the vibration theory takes into consideration the oscillations of a rotor center of mass with respect to a neutral axis. If for example, the Jeffcott rotor has only a moment unbalance, then oscillations of a center of mass are absent. In this connection the vibration rotor dynamics cannot describe the rotor dynamics with a moment unbalance.

The inertial theory of the rotor dynamics has developed a specific condition for solving this problem. In the beginning, from positions of the inertial dynamics, the Jeffcott rotor rotation with a static unbalance was considered [4]. Then a rotation of a rotor with a quasi-static unbalance in resilient supports was considered [5], as well as the dynamics of a console rotor was considered [6]. For the first time the dynamics of a rotor with a moment unbalance was considered using as an example a rotor rotation, a solid shaft of which is fixed in resilient supports [7]. These researches made possible to describe the physics of a rotor rotational process and define a rotor position with respect to a rotation axis. The system of equations obtained made possible to define the rotational parameters of a rotor, forces and moments acting a rotor at different rotational conditions. However, the attempts on using the obtained research results to define the rotational characteristics of the Jeffcott rotor with a moment unbalance revealed the discrepancies. In particular, a critical velocity of the Jeffcott rotor was higher than a rated velocity. In this connection our attention was paid to the structural features of the Jeffcott rotor mounted in solid supports from a rotor fixed to a solid shaft in resilient supports, as well as on the discrepancies in displacement of a shaft attachment point with a rotor to a rotation angle of a rotor cross-section with respect to a rotation axis. These specific features were assumed as a basis of researches given below.

II. Research object and coordinate system

Let us consider the rotation of the Jeffcott rotor with a mass m under zero-g conditions and vacuum. A rotor is made as a body of revolution and has an axial moment of inertia I_z and equatorial moment of inertia $I_э$. Assume that $I_z < I_э$. Rotor is fixed in the middle of a flexible shaft.

The shaft has the elastic properties. Under effect of loads the shaft bends, and when unloaded, it is returned to an original position.

The shaft is mounted in two solid supports. Supports are placed at a distance l_0 from a rotor center of mass. Moment unbalance we set by an inclination angle φ of a main central axis of inertia to a geometrical rotor axis.

The rotation of a rotor we consider in the rotating coordinate system *OXYZ*. An origin *O* of the coordinate system *OXYZ* coincides with a center of mass of a rotor (See Fig. 1).

Axis Z coincides with a rotation axis. At the beginning moment of the rotation an axis X is in a plane coincides with a plane of action of a moment unbalance. Axis Y is not shown in Fig. 1.

To research a rotor motion we choose two measurement planes I-I and II-II. We consider that a distance from a center of mass of a rotor to the measurement planes is also equal to l_0. We shall study the rotor motions in the measurement planes I-I and II-II according to tracks of a rotation axis (points A_1 and A_2), according to tracks of a geometrical axis of a rotor (points B_1 and B_2), according to tracks of a main central axis of inertia of a rotor (points C_1 and C_2).

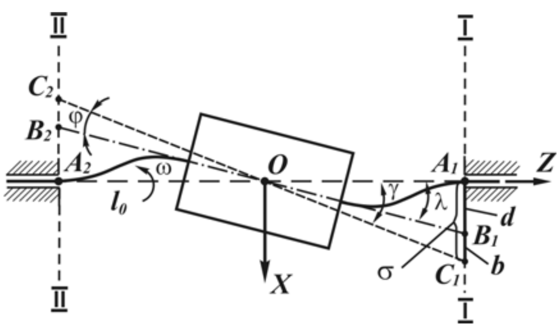

Fig. 1. System of cylindrical rotor – supports

When the rotation begins, a geometrical rotor axis deviates from a rotation axis by an angle λ. An angle γ is formed between a rotation axis and a main central rotor axis of inertia. Angles of the rotor deviation from a rotation axis we take as low.

We also consider that a distance between points A_1 and B_1 (A_2 and B_2) equals to d, a distance between points B_1 and C_1 (B_2 and C_2) equals to b and a distance between points A_1 and C_1 (A_2 and C_2) equals to σ. Positions of the tracks of axes (points A_1, B_1, C_1 and points A_2, B_2, C_2) in the measurement planes I-I and II-II are the same. Therefore in the next we concentrate on the results observed in a plane I-I.

From the beginning of the rotation, a rotor turns about geometrical axis by an angle β in such manner as it is shown in Fig. 2.

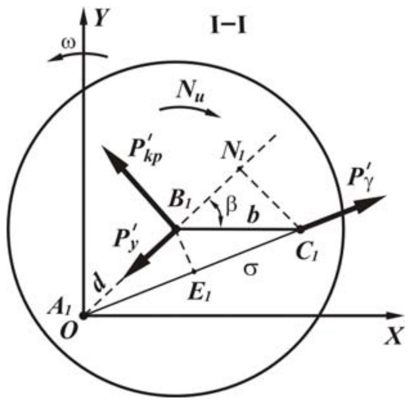

Fig. 2. Schematic layout of forces and moments acting on rotor

The forces and moments presented on the Fig. 2 described below. Equations for their determination are provided below also.

III. Coefficient of shaft angular rigidity

The shaft has a definite elasticity and under the action of forces and moments it can bend. A shaft bending can take place under the action of a force applied for example to a center of mass of a rotor. In the rotor dynamics with a static unbalance the proportionality is established between this force acting on a rotor and a displacement value of a rotor shaft at the point of force application from a rotation axis. Usually the point of force application is a center of mass of a rotor. A coefficient of proportionality is a coefficient of the rotor rigidity k. A coefficient of the rotor rigidity depends on the details of a shaft fastening in resilient supports and a rotor position with respect to these supports.

In the rotor dynamics with a moment unbalance, when a rotor is fixed on a flexible shaft, a center of mass of a rotor coincides with a geometric center and there is no displacement of a center with respect to a rotation axis. This case excludes the possibility on using the coefficient of a shaft rigidity mentioned.

In spite of the lack of the displacement of a geometric rotor center from a rotation axis, a shaft bending takes place under the action of a moment applied to a rotor for example with respect to its geometric center. Therefore in order to establish the interaction between acting moment and an angular displacement of a rotor cross-section passed through its geometric center, we introduce the concept of an angular rigidity of a shaft, which is characterized by a coefficient of an angular rigidity of a shaft k^a. A rotor cross-section passed through its geometric center, we name as a central cross-section. A coefficient of an angular rigidity of a shaft depends on the fixturing details of a shaft in resilient supports and on position of a rotor with respect to these supports.

With account of a coefficient of an angular rigidity of a shaft, we can write an equation defining the interaction of an acting moment and an angular displacement of a central cross-section:

$$M^d = k^a \lambda = k^a \sin\lambda . \qquad (1)$$

The coefficient of an angular rigidity k^a of a shaft is possible to express using the coefficient of the generally accepted rigidity of a shaft k, related with a displacement of a central cross-section of a rotor from a rotation axis by means of the coefficient of proportionality k^1.

$$k^a = kk^1 . \qquad (2)$$

In particular, for a console rotor the coefficient of proportionality is defined as follows [5]:

$$k^1 = \frac{3}{2l_0} . \qquad (3)$$

IV. Forces acting on rotor

During the rotation, the following forces and moments act on a rotor (See Fig. 2):

– moment N_γ, occurring as a result of deviation of a main central axis of inertia from a rotation axis, which is possible to write in the form:

$$\begin{aligned} N_\gamma &= (I_э - I_z)\omega^2 \sin\gamma \cos\gamma = \\ &= (I_э - I_z)\omega^2 \sin\gamma = (I_э - I_z)\omega^2 \gamma \end{aligned} . \qquad (4)$$

Moment N_γ (4) can be represented as two forces P_γ^I and P_γ^{II}, acting on an arm l_0 about center of mass of a rotor:

$$\begin{aligned} P_\gamma^I = P_\gamma^{II} &= \frac{(I_э - I_z)\omega^2 \sin\gamma \cos\gamma}{2l_0} = \\ &= \frac{(I_э - I_z)\omega^2 \sin\gamma}{2l_0} = \frac{(I_э - I_z)\omega^2 \gamma}{2l_0} \end{aligned} ; \qquad (5)$$

– moment N_y, creating by the shaft elasticity in connection with a deviation of a geometric axis of a rotor from a rotation axis by an angle λ:

$$N_y = k^a \sin\lambda = k^a \lambda . \qquad (6)$$

Moment N_y can be represented as two forces P_y^I and P_y^{II}, acting on an arm l_0 about center of mass of a rotor:

$$P_y^I = P_y^{II} = \frac{k^a \sin\lambda}{2l_0} = \frac{k^a \lambda}{2l_0} ; \qquad (7)$$

– rotational moment $N_{кр}$, which can be represented as two forces:

$$P_{кр}^I = P_{кр}^{II} = \frac{N_{кр}}{2l_0 \sin\lambda} = \frac{N_{кр}}{2l_0 \lambda} ; \qquad (8)$$

– moment of inertia $N_u = N_u^I + N_u^{II}$, acting about a rotation axis in an opposite direction of a rotation:

$$\begin{aligned} N_u &= (I_э - I_z)\omega^2 \sin^2\lambda \cos\lambda = \\ &= (I_э - I_z)\omega^2 \sin^2\lambda = (I_э - I_z)\omega^2 \lambda^2 \end{aligned} , \qquad (9)$$

$$\begin{aligned} N_u^I = N_u^{II} &= \frac{(I_э - I_z)\omega^2 \sin^2\lambda \cos\lambda}{2} = \\ &= \frac{(I_э - I_z)\omega^2 \sin^2\lambda}{2} = \frac{(I_э - I_z)\omega^2 \lambda^2}{2} \end{aligned} . \qquad (10)$$

V. Dynamic equations of rotor

Taking into account that at constant speed of rotation, the system “rotor–flexible shaft–supports” is at unchangeable position in the rotating coordinate system, we can make up the equations of moment with respect to the points A_1, B_1, C_1. In making up the system of equations we shall use the geometric dependencies:

$$d = l_0 \sin\lambda = l_0 \lambda , \qquad (11)$$

$$b = l_0 \sin\varphi = l_0 \varphi , \qquad (12)$$

$$\sigma = l_0 \sin\gamma = l_0 \gamma , \qquad (13)$$

$$B_1E_1 = \frac{db\,sin\,\beta}{\sigma} = \frac{l_0\,sin\,\lambda\,sin\,\varphi\,sin\,\beta}{sin\,\gamma} = \frac{l_0\lambda\varphi\,sin\,\beta}{\gamma}, \quad (14)$$

$$\sigma = \sqrt{d^2 + b^2 + 2db\,cos\,\beta}, \quad (15)$$

$$sin\,\gamma = \sqrt{sin^2\lambda + sin^2\varphi + 2\,sin\,\lambda\,sin\,\varphi\,cos\,\beta}, \quad (16)$$

$$\gamma = \sqrt{\lambda^2 + \varphi^2 + 2\lambda\varphi\,cos\,\beta}, \quad (17)$$

$$C_1N_1 = b\,sin\,\beta = l_0\,sin\,\varphi\,sin\,\beta = l_0\varphi\,sin\,\beta, \quad (18)$$

$$B_1N_1 = b\,cos\,\beta = l_0\,sin\,\varphi\,cos\,\beta = l_0\varphi\,cos\,\beta. \quad (19)$$

Making up the equations of moments with respect to the points A_1, A_2, B_1, B_2, C_1, C_2 we obtain the following:

$$P_{kp}^1 d - N_u^1 = 0, \quad (20)$$

$$P_\gamma^1 B_1E_1 - N_u^1 = 0, \quad (21)$$

$$P_{kp}^1 B_1N_1 + N_u^1 - P_y^1 C_1N_1 = 0. \quad (22)$$

After substitution of values, we obtain the following:

$$N_{kp}\frac{l_0\,sin\,\lambda}{2l_0\,sin\,\lambda} - \frac{(I_э - I_z)\omega^2\,sin^2\lambda}{2} = 0, \quad (23)$$

$$\frac{(I_э - I_z)\omega^2\,sin\,\gamma}{2l_0}\frac{l_0\,sin\,\lambda\,sin\,\varphi}{sin\,\gamma}sin\,\beta - \frac{(I_э - I_z)\omega^2\,sin^2\lambda}{2} = 0, \quad (24)$$

$$N_{kp}\frac{l_0\,sin\,\varphi\,cos\,\beta}{2l_0\,sin\,\lambda} + \frac{(I_э - I_z)\omega^2\,sin^2\lambda}{2} - k^a\frac{l_0\,sin\,\lambda\,sin\,\varphi\,sin\,\beta}{2l_0} = 0. \quad (25)$$

After simple transformations, we have the following:

$$N_{kp} = (I_э - I_z)\omega^2\,sin^2\lambda = (I_э - I_z)\omega^2\lambda^2, \quad (26)$$

$$sin\,\beta = \frac{sin\,\lambda}{sin\,\varphi} = \frac{\lambda}{\varphi}, \quad (27)$$

$$(I_э - I_z)\omega^2(\lambda + \varphi\,cos\,\beta) - k^a\lambda = 0. \quad (28)$$

The dynamics of a rotor we consider with account of small angular deviations of a rotor from a rotation axis. As is known, a critical speed of a rotor is achieved if $cos\,\beta = 0$.

Consequently, we have the following:

$$\omega_{kr} = \sqrt{\frac{k^a}{I_э - I_z}}. \quad (29)$$

There is a critical speed for the rotors where $I_э > I_z$. Such rotors we consider as cylindrical ones. Disk rotors do not have a critical speed, since $I_э < I_z$.

VI. Dynamic features of cylindrical rotor

The gyro-dynamics of a cylindrical a rotor is described by an equation a form of which coincides completely with the general dynamic equation (26).

A. Sub-critical rotation mode

We name the range of velocities as a sub-critical rotation mode, for which we can assume with a sufficient accuracy level, that $cos\,\beta = 1$. In this case an equation (28) takes the form:

$$(I_э - I_z)\omega^2(\lambda + \varphi) - k^a\lambda = 0. \quad (30)$$

Let us assume, that a rotor is balanced and $\varphi = 0$. In this case, from an equation (30), we have:

$$\omega_{ch} = \sqrt{\frac{k^a}{I_э - I_z}}. \quad (31)$$

An equation (31) defines a frequency of natural rotor oscillations ω_{ch}, which coincides with a critical rotor velocity.

As a consequence of this feature, when an unbalanced rotor reaches a critical rotor velocity, the resonance phenomena occur.

Let us note that an equation (30), if $\varphi = 0$, then it is possible to write it keeping a variable λ, in the form:

$$(I_э - I_z)\omega^2\lambda - k^a\lambda = 0. \quad (32)$$

Such form of an equation allows making a conclusion on the stability of this system at random deviation of a rotor by some angle.

If at $\omega = const$, the inequality shall be observed at sub-critical velocities:

$$k^a\lambda > (I_э - I_z)\omega^2\lambda. \quad (33)$$

and then a stable rotor rotation about a rotation axis is possible if

$$\omega < \sqrt{\frac{k^a}{I_э - I_z}} \,. \qquad (34)$$

Defining λ from an equation (30), we have:

$$\lambda = \frac{(I_э - I_z)\omega^2\varphi}{k^a - (I_э - I_z)\omega^2} \,. \qquad (35)$$

B. Transfer rotor mode

Equation (28) can be written in the following form, if $cos\,\beta \neq 1$:

$$(I_э - I_z)\omega^2(\lambda + \sqrt{\varphi^2 - \lambda^2}) - k^a\lambda = 0 \,. \qquad (36)$$

It follows from an equation (36):

$$\lambda = \sqrt{\frac{(I_э - I_z)^2\omega^4\varphi^2}{\left(k^a\right)^2 - 2k^a(I_э - I_z)\omega^2 + 2(I_э - I_z)^2\omega^4}} \,. \qquad (37)$$

We obtain $\lambda = \varphi$ from an equation (37), if $\omega = \omega_{kr}$. If $\omega \to \infty$, then from an equation (37) we have:

$$\lambda = \frac{\varphi}{\sqrt{2}} \,. \qquad (38)$$

The results obtained demonstrate that the resonance phenomenon provides a rotor transfer to a supercritical rotation mode. The resonance phenomenon causes an increasing λ. A value λ becomes higher that a value φ and the regularity described by an equation (27), is violated. This ensured that the conditions occur when a rotor transfers to a supercritical rotation mode and an equation (36) is not fulfilled with increasing a velocity.

Let us note that fulfilling an equation (36) with an unlimited increasing a velocity would cause the rotor rotation in the mode, which is not advantageous from the energy point of view.

It follows from equations (27) and (37) that in the transfer rotor rotation mode:

$$sin\,\beta = \frac{\lambda}{\varphi} = \sqrt{\frac{(I_э - I_z)^2\omega^4}{\left(k^a\right)^2 - 2k^a(I_э - I_z)\omega^2 + 2(I_э - I_z)^2\omega^4}} \,. \qquad (39)$$

In the transfer mode a condition (33) of the stable rotor rotation is observed. However, with increasing a velocity, the stability margin decreases. Beginning from some velocity, a random deviation of a rotor from a rotation axis causes an unstable rotation mode, which is characterized by the resonance phenomenon.

C. Supercritical Rotor Mode

The resonance phenomenon causes a violation of regularities described by an equation (36). However, if a rotor passes through a critical velocity, we observe the rotation stabilized.

In stabilization of the rotation, the retrograde precession of a rotor takes place, which exists until an angle β becomes equal to 180^0.

If we assume that $cos\,\beta = -1$, then an equation (28) takes the form:

$$(I_э - I_z)\omega^2(\lambda - \varphi) - k^a\lambda = 0 \,. \qquad (40)$$

This equation exists if $I_э - I_z > 0$ and $\lambda - \varphi > 0$. Defining λ from an equation (40), we have the following:

$$\lambda = \frac{(I_э - I_z)\omega^2\varphi}{(I_э - I_z)\omega^2 - k^a} \,. \qquad (41)$$

Let us assume that $\omega \to \infty$. In this case we obtain $\lambda = \varphi$ from an equation (41).

The results obtained indicate that a rotor self-centering expressed as an approximation of a main central axis of inertia and a rotor rotation axis, is provided. For the supercritical rotation mode a main central axis of inertia is placed between a geometric axis and a rotation axis. With increasing a velocity, a value γ decreases.

A condition (33) at supercritical velocities takes the form:

$$(I_э - I_z)\omega^2\gamma < k^a\gamma \,. \qquad (42)$$

A stability factor of the rotating rotor increases depending on increasing a velocity.

VII. Dynamic equation of disk rotor

As a disk rotor we name a rotor for which $I_z > I_э$.

For a disk rotor a moment unbalance deviates a main central axis of inertia from a geometric axis of a rotor by some angle in the direction where a main moment of unbalances acts (See Fig. 3).

As a result, a rotation axis is placed between a geometric axis and a main central axis of inertia of a rotor. In this case - $\lambda < \varphi$.

In obtaining an equation (28) of the rotor dynamics it was expected that $I_э$ and I_z are arbitrary values. Therefore an equation (28) shall describe the dynamics of any rotor including one for which $I_z > I_э$.

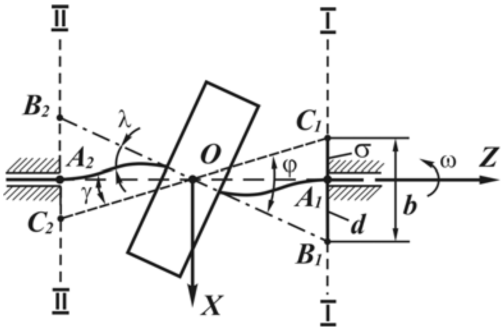

Fig. 3. “Disk Rotor-Supports” System

If $\varphi > \lambda$, then a single equation, which allows that $I_z - I_э > 0$, is an equation, which is given in the form:

$$(I_z - I_э)\omega^2(\varphi - \lambda) - k^a\lambda = 0 \,. \qquad (43)$$

From this a conclusion follows that because of a specific combination of moments of inertia $I_э$ and I_z , a disk rotor rotation meets the super critical conditions.

Therefore, a critical velocity can not be achieved at an unlimiting velocity increasing.

It is followed from an equation (43):

$$\lambda = \frac{(I_z - I_э)\omega^2\varphi}{k^a + (I_z - I_э)\omega^2} \,. \qquad (44)$$

Let us assume that $\omega \to \infty$. In this case from an equation (44) we obtain $\lambda = \varphi$.

Consequently, a rotor self-centering takes place, when an inclination angle of a geometric axis to a rotation axis increases. When a velocity increasing is unlimited $\lambda \to \varphi$, and $\gamma \to 0$. The self-centering phenomenon causes an increasing the machine vibrations.

The rotation stability condition has the following form:

$$(I_z - I_э)\omega^2\gamma < k^a\gamma \,. \qquad (45)$$

A condition (45) is observed in rotation of a disk rotor. A rotor rotation is a stable one. According to a condition (43), a stability margin increases depending on increasing a velocity.

VIII. Conclusions

The need for introduction of the “angular rigidity of a shaft” idea was for the first time substantiated to study the rotor dynamics with a moment unbalance.

For the first time the general system of equations of the cylindrical Jeffcott rotor dynamics with a moment unbalance was obtained; this system defines a specific feature of a rotor behavior, forces and moments acting on a rotor. The system of equations describes the rotor dynamics within the whole range of velocities under all rotational conditions.

The system of equations of the Jeffcott rotor dynamics with a moment unbalance is obtained which defines a specific rotor behavior, forces and moments acting on a rotor. The system of equations describes the rotor dynamics within the whole velocity range.

It was shown that a disk rotor motion at any velocities meets the supercritical rotor rotating conditions.

It was shown that the self-centering phenomenon causes decreasing the machine vibrations with a cylindrical rotor and increasing the machine vibrations with a disk rotor.

The research results can be used in designing the rotor machines, in development of the diagnostic systems and systems for balancing the rotors. They are a key element to study the rotor dynamics under effect of the static and moment unbalance.

References

[1] F.M. Dimentberg, K.T. Shatalov, A.A. Gusarov. Oscillations of Machines. – M.: Mashinostroenye,1964. – 308 p.

[2] A.S. Kelzon, Y.P. Tsimansky, V.I. Yakovlev. Dynamics of Rotor in Resilient Supports. – M.: Nauka. Main Editing Staff of Physical-Mathematical Literature, 1983. – 280 p.

[3] R. Gasch, H. Pfuetzner. Rotordynamik Eine Einfuehrung, Springer Verlag Berlin-Heidelberg-New York, 1975, ISBN 3-540-070460-X

[4] A.Y. Zhivotov. New Theory of Rotor Dynamics: Dynamics of Dick Rotor with Static Unbalance // IFToMM Sixth International Conference on Rotor Dynamics. - Sidney, Australia. - Proceedings - Volume 1. - 2002. - p. 1057.

[5] A.Y. Zhivotov, Y.G Zhivotov, Y.G. Brazaluk New Theory of Rotor Dynamics: Dynamics of Ubrella-Type Rotor with Resilient Support // The 2-nd International Symposium on Stability Control of Rotating Machinery. ISCORMA - 2. - Gdansk, Poland. - 2003. - p. 331 - 340.

[6] A.Y. Zhivotov, Y.G. Zhivotov. New Theory of Rotor Dynamics: Dynamics of Outboard Rotor with Quasi-Static Unbalance// Mashinostroenye and Electrotechnology. Mashinintelekt, Sofia, Bulgaria. - № 10-11 – 2005. – p. 11-14.

[7] A.Y Zhivotov. Features of Rotor Dynamics with Moment Unbalance // Instrument-Making Technology. – 1’2000. NITIP, Ukraine, Kharkov.