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# New Theory of Rotor Dynamics: Rotor Dynamics with Moment Unbalance

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**The resume**: In the given paper of a basis of the new inertial theory of dynamics of a rotor are declared. The special attention is given to features of behavior of a rotor with moment unbalance. The inertial theory considers any change of the moment of inertia of a rotor as disturbing factor, if change of the moment of inertia of a rotor took place because of displacement or a deviation of a geometrical axis of a rotor from an axis of rotation. According to the theory, the disturbing factor resists with rotation of a rotor. In the given paper of force and the moments, acting on a rotor with moment unbalance, are described. The circuit of forces and the moments, acting on a rotor, is created. On the basis of the circuit of the equation of dynamics of a rotor are received. It shows, that the equation of dynamics of a cylindrical rotor differs from the equation of dynamics of a disk rotor. Accordingly, features of rotation of a cylindrical rotor differ from features of rotation of a disk rotor. The analysis of features of rotation of a disk rotor is executed. The analysis of features of rotation a cylindrical rotor is executed. The special attention addresses on questions of physics of process of rotation of a rotor.

**Keywords:** Dynamics, Rotor, Unbalance, Shaft, Support

## 1 Introduction

From positions of the inertial theory, influence of a static unbalance on dynamics of a rotor is considered[1]. However, the moment unbalance influences dynamics of a rotor also. In general, static unbalance and moment unbalance influence dynamics of a rotor simultaneously. We name this complex kind of a unbalance a dynamic unbalance. However, for the decision of problems of dynamics of a rotor with a complex dynamic unbalance it is important to understand features of rotation a rotor with moment unbalance. The known "oscillatory" theory of dynamics of rotors could not receive the equation of dynamics of a rotor with moment unbalance.

The oscillatory theory allows to receive the equations of dynamics of a rotor only in case there is a displacement of the center of mass from a geometrical axis.

## 2 The Object of a Researches and System of a Coordinates.

Rotation of the rotor having mass *m*, we shall consider in conditions of vacuum and weightlessness. We shall accept, that a rotor - a body of rotation, which has the axial moment of inertia $I_z$ and the equatorial moment of inertia $I_э$. The rotor is established in two elastic supports. The factor of rigidity of each support is equal *k*. Each elastic support is located on distance $l_0$ from the center of mass of a rotor. We shall set moment unbalance with the help of a corner φ of a deviation of the main central axis of inertia of a rotor from a geometrical axis of a rotor.

We shall consider dynamics of a rotor in system of coordinates *OXYZ*, which rotates around of axis *Z* with speed of rotation of a rotor.

The center *O* of systems of coordinates *OXYZ* coincides with the center of mass of rotor (Figure 1). Axis *Z* coincides with an axis of rotation of a rotor. Axis *X* is located in a plane, which coincides with a plane of action of an unbalance in the beginning of rotation of the rotor. Axis *Y* is not shown in figure 1.

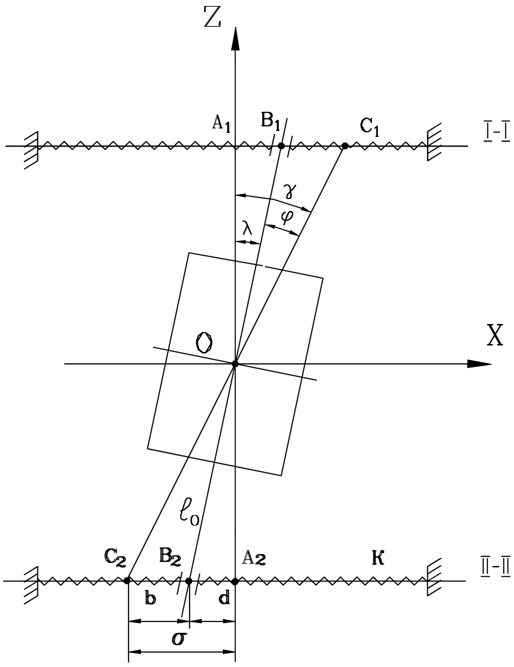


**Figure 1. The System of rotor-supports**

We shall choose two planes of measurement (a plane of measurement I-I and a plane of measurement II-II) for research of dynamics of a rotor. We shall study dynamics of a rotor in planes of measurement, using traces of an axis of rotation (a point $A_1$ and a point $A_2$ ), traces of a geometrical axis (a point $B_1$ and a point $B_2$) and traces of the main central axis of inertia of a rotor (a point $C_1$ and a point $C_2$).

The geometrical axis of a rotor deviates on a corner $\lambda$ from the axis of rotation with the beginning of rotation. The corner $\gamma$ also is formed between an axis of rotation and the main central axis of inertia of a rotor. Accordingly, in a plane of measurement I-I of distance between a point $A_1$ and a point $B_1$ become equal $d$ . The distance between a point $B_1$ and a point $C_1$ becomes equal $b$ also. The distance between a point $A_1$ and a point $C_1$ becomes equal $\sigma$ also.

Position of traces of an axis of rotation, a geometrical axis and the main central axis of inertia of a rotor in a plane of measurement I-I it is similar to position of traces of these axes in a plane of measurement II-II.

Therefore, further we shall study rotation of a rotor, using traces of these axes located in a plane of measurement I-I.

Let's pay attention, that the rotor makes turn concerning a geometrical axis on some corner $\beta$ with the beginning of rotation (Figure 2).

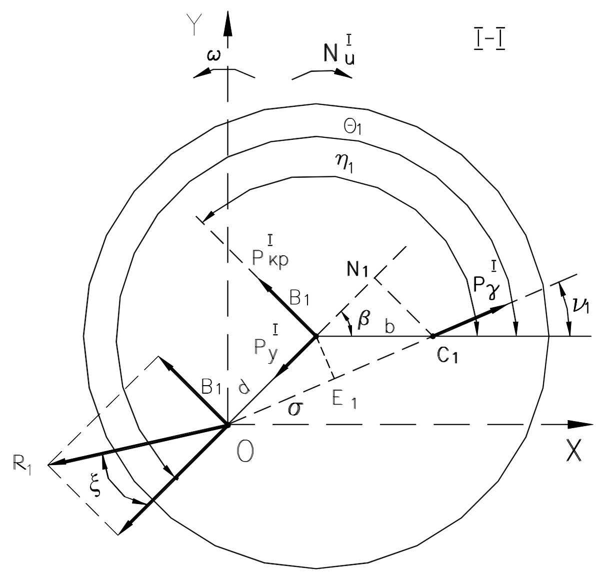


**Figure 2 Circuit of forces and the moments, acting on a rotor in a plane of measurement I-I**

## 3 Forces and Moments, Acting on the Rotor

Let's pay attention to forces and the moments, acting on a rotor.

The moment $N_\gamma$ , arising as a result of a deviation of the main central axis of inertia from an axis of rotation, can be written down as

$$N_\gamma = (I_э - I_z)\omega^2 \sin\gamma \cos\gamma \qquad (1)$$

The moment $N_\gamma$ can be submitted as two forces $P_\gamma^I$ and $P_\gamma^{II}$ **,** acting around of the center of mass of a rotor on a shoulder $l_0$

$$P_\gamma^I = P_\gamma^{II} = \frac{(I_э - I_z)\omega^2 \sin\gamma \cos\gamma}{2l_0} \qquad (2)$$

The moment $N_y$, created by an elastic support as a result of a deviation of a geometrical axis of a rotor from an axis of rotation

$$N_y = 2kdl_0 \qquad (3)$$

Taking info account, that $d = l_0 \sin\lambda$ , we have

$$N_y = 2kl_0^2 \sin\lambda \qquad (4)$$

The moment $N_y$ can be submitted as two forces $P_y^I$ and $P_y^{II}$ , acting around of the center of mass of a rotor on a shoulder $l_0$

$$P_y^I = P_y^{II} = kl_0 \sin\lambda \qquad (5)$$

The rotating moment $N_{кр}$, which can be presented as two forces

$$P_{кр}^I = P_{кр}^{II} = \frac{1}{2}\frac{N_{кр}}{d} \qquad (6)$$

The inertial moment $N_u = N_u^I + N_u^{II}$ , acting around of a geometrical axis in a direction opposite to direction of rotation of a rotor.

$$N_u = \frac{d(I_э - I_z)\omega^2 \sin\lambda \cos\lambda}{l_0} \qquad (7)$$

$$N_u^I = \frac{d(I_э - I_z)\omega^2 \sin\lambda \cos\lambda}{2l_0} \qquad (8)$$

## 4 The Equations of Dynamics of the Rotor

Let's accept, that the rotor rotates with constant speed. If speed of a rotor is a constant, a rotor occupies strictly certain position in system of coordinates, which rotates with speed of a rotor. In this case, we can make the equations of the moments concerning points $A_1$, $B_1$, $C_1$.

At drawing up of system of the equations we shall use geometrical dependences

$$B_1E_1 = \frac{db\sin\beta}{\sigma} \qquad (9)$$

$$\sigma = \sqrt{d^2 + b^2 + 2db\cos\beta} \qquad (10)$$

$$C_1N_1 = b\sin\beta \qquad (11)$$

$$B_1N_1 = b\cos\beta \qquad (12)$$

Making system of the equations, after insignificant transformations, we shall receive

$$P^I_{кр} d - \frac{d(I_э - I_z)\omega^2 \sin\lambda \cos\lambda}{2l_0} = 0 \quad (13)$$

$$\frac{(I_э - I_z)\omega^2 db \sin\gamma \cos\gamma \sin\beta}{2l_0\sigma} - N^I_и = 0 \quad (14)$$

$$P^I_{кр} b\cos\beta + N^I_и - kl_0 b\sin\lambda \sin\beta = 0 \quad (15)$$

From the equation (13), we have

$$P^I_{кр} = \frac{(I_э - I_z)\omega^2 \sin\lambda \cos\lambda}{2l_0} \quad (16)$$

From the equation (14), we have

$$\sin\beta = \frac{\sigma \sin\lambda \cos\lambda}{b\sin\gamma \cos\gamma} \quad (17)$$

From the equation (13), we

$$(I_э - I_z)\omega^2 \cos\lambda(d + b\cos\beta) - 2kl_0^2 b\sin\beta = 0 \quad (18)$$

We shall consider dynamics of a rotor, using the theory of small moving. Let's consider, that angular deviations of a rotor from an axis of rotation are small. In this case, dependences (16-18) can be written down as

$$P^1_{кр} = \frac{(I_э - I_z)d\omega^2}{2l_0^2} \quad (19)$$

$$\sin\beta = \frac{d}{b} \quad (20)$$

$$(I_э - I_z)\omega^2(d + b\cos\beta) - 2kl_0^2 d = 0 \quad (21)$$

On the basis of the made assumptions, we shall receive

$$N_y = 2kl_0 d \quad (22)$$

$$P^I_y = kd \quad (23)$$

$$\cos\beta = \sqrt{\frac{b^2 - d^2}{b^2}} \quad (24)$$

The equation (18) can be expressed through corners of deviations of the mentioned axes from an axis of rotation

$$(I_э - I_z)\omega^2(\sin\lambda + \sin\varphi\cos\beta) - 2kl_0^2 \sin\lambda = 0 \quad (25)$$

Let's pay attention to the following. The received basic equation of dynamics is fair for a rotor, established on an elastic shaft. Only it is necessary to take into account, that the factor of rigidity k is equal to half of factor of rigidity of a shaft (the unbalanced rotor bends a shaft in two various directions).

## 5 The Corners Determining the Directions of Action of Forces and the Moments.

We shall choose a direction of action of main the moment of disbalances in a plane of measurements I-I for a sign on a footnote of corners. In this case, it is possible to define directions of all forces, acting on a rotor with the help of corners. The corner $\theta_1$ concerning a direction, accepted for a sign of a footnote, defines a direction of action of elastic force.

Corners $\theta_1$ and $\theta_2$ can be determined with the help of dependences

$$\theta_1 = \beta + 180^O \quad (26)$$

$$\theta_2 = \beta \quad (27)$$

The corner $\eta_1$ concerning a direction accepted for a sign of a footnote defines a direction of action of the force created by the rotating moment. Corners $\eta_1$ and $\eta_2$ can be determined with the help of dependences

$$\eta_1 = \beta + 90^o \quad (28)$$

$$\eta_2 = \beta + 270^o \quad (29)$$

The corner $\nu_1$ defines directions of action of force, which is created by the moment. We shall remind, that the moment arises, if the main central axis of inertia deviates from an axis of rotation. Corners $\nu_1$ and $\nu_2$ can be determined with the help of dependences

$$\nu_1 = arcsin(\frac{d}{\sigma} sin\,\beta) \quad (30)$$

$$\nu_2 = arcsin(\frac{d}{\sigma} sin\,\beta) + 180^o \quad (31)$$

The inertial moment $N_и$ counteracts rotation of a rotor. For the rotor rotating in chosen conditions, reactions of supports can be determined with the help of dependences

$$R_1 = R_2 = \sqrt{(P^I_{кр})^2 + (P^I_y)^2} \quad (32)$$

The direction of reaction of support concerning a line of displacement of a point $B_1$ ($B_2$) concerning point $A_1$ ($A_2$) is determining by a corner $\xi_1$ ($\xi_2$)

$$\xi_1 = \text{arctg}\frac{P^I_{кр}}{P^I_y} \quad (33)$$

$$\xi_2 = \text{arctg}\frac{P^I_{кр}}{P^I_y} + 180^o \quad (34)$$

Pay special attention to interrelation of corners λ, γ, β and φ.

The size $d_0$ for any section, located on distance l from the center of mass of a rotor, can be determined with the help of dependence

$$d_0 = l\sin\varphi \quad (35)$$

$$d_0 = b_0 \sin\beta \quad (36)$$

Considering dependences (35) and (36), we have

$$\sin\lambda = \sin\varphi \sin\beta \quad (37)$$

Let's take into account, that

$$\sigma_0 = \sqrt{d_0^2 + b_0^2 + 2d_0 b_0 \cos\beta} \tag{38}$$

In this case, we have

$$\sigma = e\sin\varphi\sqrt{\sin^2\beta + \sin 2\beta} \tag{39}$$

Dependence (39) can be submitted as

$$\sin\gamma = \sin\varphi\sqrt{\sin^2\beta + \sin 2\beta} \tag{40}$$

## 6 The Features of Dynamics of the Cylindrical Rotor.

We shall understand, that a cylindrical rotor - a rotor, for which $I_э > I_z$. Dynamics of rotation of a cylindrical rotor is described by the algebraic equation.

The kind of the equation coincides with the kind of general the equation (18) of dynamics of a rotor.

### 6.1 Subcritical a mode of rotation of a rotor

Under subcritical modes of rotation of a rotor we shall understand some range of speeds. For these speeds with sufficient accuracy it is possible to assert, that $\cos\beta = 1$.

In this case, the equation (21) becomes

$$(I_э - I_z)\omega^2(d + b) - 2kl_0^2 d = 0 \tag{41}$$

Let's take into account, that the rotor is balanced and $b = 0$. In this case, defining $\omega$ of the equation (41), we have

$$\omega_{кр} = l_0\sqrt{\frac{2k}{I_э - I_z}} \tag{42}$$

Dependence (42) confirms existence of special speed. On this speed the system of a rotor-support behaves in strange enough image.

On this speed of a deviation of a rotor from an axis of rotation can grow without restrictions. Usually speak, which this dependence defines frequency of own fluctuations of a rotor.

Let's admit, that $\varphi = 90^o$. In this case, from the equation (18) we shall receive the dependence, similar to dependence (42).

It means, that there is a speed of a rotor, which can be named critical. If the rotor achieves critical speed in system of rotor-support are the resonant phenomena.

Let's pay attention, that, if $\cos\beta = 1$, the equation (41), if $b = 0$, it is possible to write down as

$$(I_э - I_z)\omega^2 d - 2kl_0^2 d = 0 \tag{43}$$

Such assumption allows keeping $d$ in the equation (43).

Such kind of the equation allows to making a rating of the stability of rotation of a rotor, if there was a casual deviation of a rotor from an axis of rotation on some a corner.

Obviously, on subcritical speeds the inequality should be observed

$$2kl_0^2 d > (I_э - I_z)\omega^2 d \tag{44}$$

Therefore, steady rotation of a rotor around of an axis of rotation is possible, if

$$\omega < \sqrt{\frac{2k}{I_э - I_z}} \tag{45}$$

Defining $d$ from the equation (41), we have

$$d = \frac{(I_э - I_z)\omega^2 b}{2kl_0^2 - (I_э - I_z)\omega^2} \tag{46}$$

### 6.2 A transitive mode of rotation

The equation (18), if $\cos\beta \neq 1$, it is possible to write down, as

$$(I_э - I_z)\omega^2(d + \sqrt{b^2 - d^2}) - 2kl_0^2 d = 0 \tag{47}$$

From the equation (47) follows

$$d = \frac{(I_э - I_z)\omega^2 b}{\sqrt{4l_0^4k^2 - 4l_0^2k\omega^2(I_э - I_z) + 2\omega^4(I_э - I_z)^2}} \tag{48}$$

From dependences (48) and (49), if $\omega = \omega_{кр}$, we shall receive, that $d = b$.

Having divided the right part of the equation (48) (numerator and a denominator) on $(I_э - I_z)\omega^2$, if $\omega \to \infty$, we shall receive

$$d = \frac{b}{\sqrt{2}} \tag{49}$$

The received results testify, that the phenomenon of a resonance provides transition of a rotor through the critical speed. The further rotation of a rotor occurs on a non-stationary mode of rotation.

The phenomenon of a resonance results in increase $d$. The laws, described by the equation (18) and dependence (20), are broken. The deviation of a rotor from an axis of rotation grows. Rotation of a rotor in such conditions can be counted unprofitable from the point of view of power.

Having in view of dependence (20), it is possible to define

$$\sin\beta = \frac{(I_э - I_z)\omega^2}{\sqrt{4l_0^4k^2 - 4l_0^2k\omega^2(I_э - I_z) + 2\omega^4(I_э - I_z)^2}} \tag{50}$$

On a transitive mode of rotation the condition of stability (44) is observed. However, the stock of stability of rotation of a rotor starts to decrease with increase in speed. Since some speed, the slightest casual deviation of a rotor from an axis of rotation results in a non-stationary rotation. On this speed the phenomenon of a resonance begins.

### 6.3 A supercritical mode of rotation of a rotor

The phenomenon of a resonance provides infringement of the laws described by the equation (18). However, stabilization of rotation of a rotor is observed, if the rotor starts to rotate in supercritical a mode of rotation.

Let's note, that an inverse procession takes place at stabilization of rotation of a rotor.

Inverse processions exist, if value of a corner $\beta$ has not reached value $180^{o}$.

Let's admit, that $\cos\beta = -1$. In this case the equation (41) becomes

$$(I_{э} - I_{z})\omega^{2}(d-b) - 2kl_{0}^{2}d = 0 \quad (51)$$

The equation exists, if $I_{э} - I_{z} > 0$ and $d - b > 0$.

Defining *d* from the equation (51), we have

$$d = \frac{(I_{э}-I_{z})\omega^{2}b}{(I_{э}-I_{z})\omega^{2}-2kl_{0}^{2}} \quad (52)$$

Let's accept, that $\omega \to \infty$. Having divided the right part of dependence (52) (numerator and a denominator) on $(I_{э} - I_{z})\omega^{2}$, we shall receive $d = b$.

The received results testify, that there is a self-centering of a rotor. The self-centering of a rotor will consist in to overlapping a main central axis of inertia with an axis of rotation of a rotor.

On a supercritical mode of rotation the main central axis of inertia of a rotor settles down between a geometrical axis and an axis of rotation.

The Value of size *d* with growth of speed of a rotor decreases.

$$(I_{э} - I_{z})\omega^{2}\sigma < 2kl_{0}^{2}\sigma \quad (53)$$

The stock of stability of rotation of a rotor starts to increase with growth of speed.

## 7 The Equations of Dynamics of the Disk Rotor

We understand, that a disk rotor - a rotor, for which $I_{z} > I_{э}$.

Moment unbalance of a disk rotor rejects the main central axis of inertia from a geometrical axis of a rotor on some corner in a direction of action of main the moment of disbalance a rotor (Figure 3).

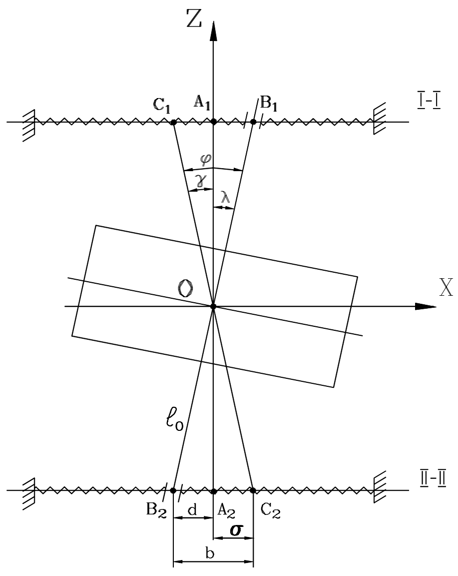


**Figure 3 System a disk rotor-supports**

As a result of it, the axis of rotation occupies position between a geometrical axis and the main central axis of inertia of a rotor.

We did not pay attention to a ratio of the inertial moments at drawing up of the equation (18).

Therefore, the equation (18) should describe dynamics of any rotor, including a rotor, for which $I_{z} > I_{э}$.

Let's pay attention to the equation (51). The equation (51) is the unique equation, which allows taking into account an especial ratio of the moments of inertia of a disk rotor. The equation (51) can be submitted as

$$(I_{z} - I_{э})\omega^{2}(b-d) - 2kl_{0}^{2}d = 0 \quad (54)$$

The equation (54) describes dynamics of a rotor on supercritical speeds. This equation defines, that the rotor is rotated on a supercritical mode. The special combination of the moments of inertia of a rotor is the reason of it. Therefore, critical speed cannot be achieved at unlimited increase in speed.

From the equation (54) follows

$$d = \frac{(I_{z} - I_{э})\omega^{2}b}{2kl_{0}^{2} + (I_{z} - I_{э})\omega^{2}} \quad (55)$$

Let's admit, that $\omega \to \infty$.

Having divided the right part of dependence (55) (numerator and a denominator) on $(I_{z} - I_{э})\omega^{2}$, we shall receive $d = b$.

Hence, there is a self-centering of a rotor. In this case, the increase of a corner $\lambda$, which defines a deviation of a geometrical axis of a rotor from an axis of rotation, occurs.

In unlimited increase in speed the phenomenon of a self-centering of a rotor results in growth of fluctuations of the machine.

The condition of steady rotation of a rotor becomes

$$(I_{z} - I_{э})\omega^{2}\sigma < 2kl_{0}^{2}\sigma \quad (56)$$

The condition (56) is carried out at rotation of a disk rotor. Rotation of a rotor is steady.

According to a condition (56), stock of stability of rotating of a rotor starts to increase with growth of speed.

## 8 Summary

(1) The general equation of dynamics of a rotor with moment unbalance for the first time is received. The equation defines all features of behavior of a rotor and also all forces and the moments, which are acting on a rotor.

(2) The equations of dynamics of a disk rotor for the first time are received. The phenomenon of a self-centering of a disk rotor results in increase in fluctuations of the machine.

**Connections:**


1. Zhivotov. A.Y. The new theory of dynamics of a rotor: dynamics of a disk rotor with static an unbalance. Hearings. IFTOMM. The Sixth International Conference on Rotor Dynamics, 2002.