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New Theory of Rotor Dynamics: Disk Rotor Dynamics with Static Unbalance Taking into Account Aerodynamic Drag Forces

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Abstract: This article continues to present principles of a new "inertial" rotor dynamics theory considered any rotor moment of inertia changing (which is caused by rotor shift comparatively rotation axis) as a disturbing factor that resists the rotor rotation. The article analyzes aerodynamic resistance influence on the static unbalance rotor dynamics. Loading diagram of forces and torques with effect on rotor and rotor dynamics equation are presented in the article. Specific research is dedicated to questions related to equation rotor rotation and physical meaning of the rotor rotation process.

Keywords: Dynamics, Rotor, Unbalance, Shaft, Support

1 Introduction

New "inertial" theory absolutely changed view of rotor rotation physics and presented new equations of the dynamics that are the algebraic equations no more than 3rd order [1].

The fundamental dynamics equation of the rotor with static unbalance reflects the rotor rotation under vacuum and weightless conditions. However, most rotors rotate in an atmosphere and suffer the aerodynamic drag effect.

As a result, there is need to consider the rotor dynamics taking into account the aerodynamic drag forces.

2 Study Subject and Coordinate System.

Consider a vertical disc rotor with mass m, mounted to an elastic shaft with stiffness factor k. The rotor is fastened in the middle of the shaft. The shaft is fixed on two rigid bearings. The rotor centroidal moment of inertia exceeds tenfold the equatorial one. The rotor has a center of mass that is at distance e from the geometrical axis, which is common for the rotor and shaft.

Suppose that the rotor rotates in the vertical position.

Consider the features of the rotor rotation in a rotating coordinates $OXYZ$ (Figure 1).

The coordinate system origin O and axis Z are congruent with a rotation axis. The axis X is directed along action of a resultant rotor unbalance vector.

The rotor center of mass lies on the plane OXY. The rotor motion will be studied using the trails of the center of mass (point C), geometrical axis (point B), and rotation axis (point O) in the plane OXY. The figure has not axis Z.

Assume that the rotor is rotating with speed ω so the shaft is deflected and the rotor is turned around the geometrical axis for an angle α.

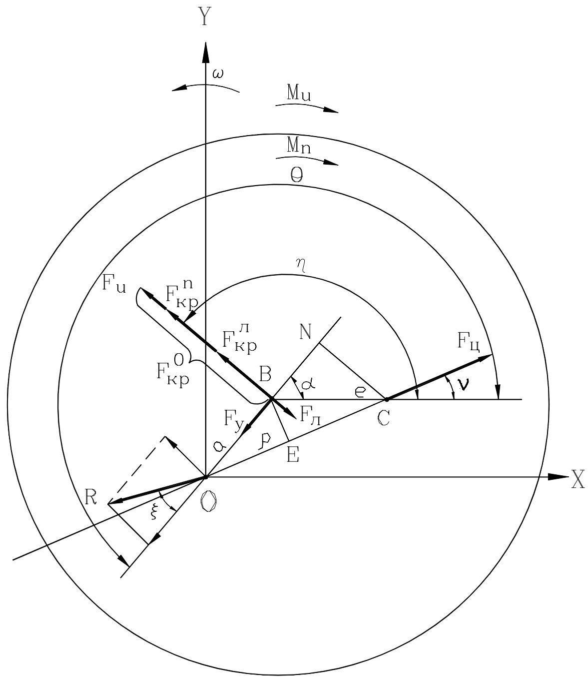

Figure 1. Layout of the forces affecting the rotor.

3 Forces Affecting the Rotor

Consider the forces and moments affecting the rotor. They include:

Centrifugal force $F_ц$ caused by new position of the center of mass in respect to the rotation axis

$$F_ц = m\omega^2\rho \ , \tag{1}$$

where ρ is a distance between O and C points;

Elastic force of the deflected shaft F_y

$$F_y = ka \ , \tag{2}$$

where a is a distance between O and B points.

Head aerodynamic drag $F_л$ applied to point B normal to line OB.

Moment M_n generated by surface aerodynamic drag forces and acting around the rotor geometrical axis.

Force moment M_u resulting from the additional rotor inertia moment ma^2 at the rotor deflection from rotation axis by distance a.

Force moment M_u acts around the rotor geometrical axis also.

$$M_u = m\omega^2 a^2 , \quad (3)$$

$$F_u = m\omega^2 a \ . \quad (4)$$

Additional torque $M_{кр}$ is required to overcome as disturbing factors as the rotor aerodynamic drag and additional moment of inertia. Additional torque can be represented by force $F^0_{кр}$ applied to the geometrical axis at point *B* as normal to the line *OB*.

$$F^0_{кр} = F_u + F^n_{кр} + F^л_{кр} , \quad (5)$$

where F_u - is a force acting on arm *a*, by witch additional torque M_u required to overcome the disturbing factor can be replaced.

$F^n_{кр}$ - is a force acting on arm *a*, by witch the additional torque M_n required to overcome the disturbing factor can be replaced.

$F^л_{кр}$ - is a force acting on arm *a*, by witch the additional torque $M_л$ required to overcome the disturbing factor can be replaced.

Forces $F^л_{кр}$ and $F_л$ have no influence upon the rotor transformation due to $F^л_{кр} = F_л$. Therefore, to derive the rotor dynamics equation the force $F_{кр}$ will be used.

$$F_{кр} = F_u + F^n_{кр} \ . \quad (6)$$

Note that in the article there are no dependences to determine $F_л$ because these dependences presented in any course of aerodynamics.

Taking into account that the rotor holds a strictly defined position in the coordinates *OXYZ* and rotor-deflected shaft system takes a stable state at a constant speed, we can form a system of moment equations in respect to *O, B* and *C* points.

4 Rotor Dynamics Equations

To constitute the equation system of the moments about *O*, *B*, *C* points, it is enough to use the geometrical proportions:

$$BE = \frac{ea\sin\alpha}{\rho} , \quad (7)$$

$$CN = e\sin\alpha , \quad (8)$$

$$BN = e\cos\alpha \ . \quad (9)$$

Based on the classical laws of static and taking into account the proportion after series of transformation the equations of moment about *O*, *B*, *C* points will take the following form

$$F_{кр} \cdot a - M_u - M_n = 0 , \quad (10)$$

$$m\omega^2 ea\sin\alpha - M_u - M_n = 0 , \quad (11)$$

$$M_u + M_n + F_{кр} e\cos\alpha - kae\sin\alpha = 0 . \quad (12)$$

From equations (10) and (11), we have

(3)

$$F_{кр} = m\omega^2 e\sin\alpha \ , \quad (13)$$

$$M_n = m\omega^2 a(e\sin\alpha - a) , \quad (14)$$

$$F^n_{кр} = m\omega^2 (e\sin\alpha - a) . \quad (15)$$

From equation (12) considering (10) and dependence (13), take the fundamental equation of rotor dynamics

$$m\omega^2 (a + e\cos\alpha) - ka = 0 . \quad (16)$$

The equation (16) correlates such variables as *a, α, ω*. This correlation will enable determining value *a* depending on a speed and calculating at this speed the forces and moments affecting the rotor if α value is known. With, all this going on, note that

$$\rho = \sqrt{a^2 + e^2 + 2ae\cos\alpha} \ . \quad (17)$$

Assume the direction of resultant unbalance vector as origin of angels, so we can determine angular direction of all effective forces. The angle *θ* designates the direction of shaft elastic forces

$$\theta = \alpha + 180^o . \quad (18)$$

The angle *η* designates the direction of forces generated by additional torque

$$\eta = \alpha + 90^o . \quad (19)$$

The angle ν designates the direction of centrifugal force

$$\sin\nu = \frac{a}{\rho}\sin\alpha . \quad (20)$$

For the rotor rotating under weightless conditions, we can determine the response *R* for each bearing

$$R = \frac{1}{2}\sqrt{(F^0_{кр})^2 + F^2_y} . \quad (21)$$

Bearing response direction with respect to the line of the rotor geometrical axis deflection from the rotation axis is determined by the angle ξ

$$tg\xi = \frac{F^0_{кр}}{F_y} . \quad (22)$$

5 Analysis of Equations

5.1 General observations

Equation (16) coincides absolutely with equation of the rotor dynamics under vacuum [1]. New conditions result in change of the rotor behavior. Equation (16) at under-critical ($\cos\alpha = 1$) and over-critical ($\cos\alpha = -1$) modes of rotation was analyzed earlier [1-2].

From equation (16) follow

$$\omega_{кр} = \sqrt{\frac{k}{m}} \ . \quad (23)$$

The expression shows aerodynamic force take no influence upon value of critical speed of the rotor.

5.2 Transient rotation mode.

Consider the dynamic features of the disk rotor at a transient only where the rotor turn to be taken into account.

From equation (11) accounting dependence (3), take the following

$$\sin\alpha = \frac{M_n + m\omega^2 a^2}{m\omega^2 ea}. \qquad (24)$$

Dependence (24) shows that environment surface aerodynamic drag forces affect behavior of the rotor turn angle α.

When $M_n = 0$, take the well-known dependence[1]

$$\sin\alpha = \frac{a}{e} \; . \qquad (25)$$

To obtain comprehensive information about the rotor dynamics, joint resolution of equation (16) and dependence (24) is need. At that it had to be considered that the moment caused by aerodynamic drag depends on rotation speed

$$M_n = \mu\omega^n, \qquad (26)$$

$$F^n_{кр} = \frac{\mu\omega^n}{a}, \qquad (27)$$

where μ - is a coefficient of proportionality,

n is an exponent.

Relatively low speeds, it is assumed что $n = 1$. At relatively high speeds $n = 2$, etc.

Consider the case when $n = 2$.

Taking into account that at $n = 2$.

$$\sin\alpha = \frac{\mu + ma^2}{mea}, \qquad (28)$$

$$\cos\alpha = \sqrt{1 - \sin^2\alpha} \; . \qquad (29)$$

The equation (16) can be written down as:

$$\begin{aligned} &a^4(k^2 - 2km\omega^2 + 2m^2\omega^4) + \\ &+ a^2(2\mu m\omega^4 - m^2e^2\omega^4) + \mu^2\omega^4 = 0 \end{aligned} \; . \qquad (30)$$

Taking into account dependence (24), we can state that during rotor rotation at under-critical speeds the following condition should be met

$$M_n + m\omega^2 a^2 \le m\omega^2 ea \, . \qquad (31)$$

Condition (31) means that when critical speed passes, a value is always less than e.

Therefore, the aerodynamic drag contributes to “smooth” crossing the critical speed, because provides less rotor deflection from the rotation axis by a resonance time.

Presence of α angle demonstrates that at transient mode trails of geometrical axis, rotation axis and center of mass in plane *OXY* form no one line. At that a distance between trail of center of mass and trail of rotation axis is more than distance between trail of geometrical axis and trail of rotation axis. Trail of geometrical axis as though keeps own position between trails of rotation axis and center of mass.

Resolving this equation (30) with respect to a, take

$$a = \sqrt{\frac{m^2e^2\omega^4 - 2\mu m\omega^4 + \sqrt{m^4e^4\omega^8 + d}}{k^2 - 2km\omega^2 + 2m^2\omega^4}}, \qquad (32)$$

where $d = 4\mu^2\omega^4(2km\omega^2 - k^2 - m^2\omega^4 - m^3e^2\omega^4)$. (33)

Dependence (32) sets required unique dependence a on ω. When $\mu = 0$, dependence (32) is congruent to known [1]

$$a = \frac{me\omega^2}{\sqrt{k^2 - 2km\omega^2 + 2m^2\omega^4}} \, . \qquad (34)$$

Under given rotation conditions and considering dependences (3) and (26), from equation (10) follows

$$M_{кр} = m\omega^2 a^2 + \mu\omega^2 . \qquad (35)$$

Equation (25), at $\mu = 0$, is easy transformed to dimensionless form. For transient mode:

$$\frac{a}{e} = \frac{\dfrac{\omega^2}{\omega^2_{кр}}}{\sqrt{2\dfrac{\omega^4}{\omega^4_{кр}} - 2\dfrac{\omega^2}{\omega^2_{кр}} + 1}} = \sin\alpha \; . \qquad (36)$$

Based on equation (36) we can set a dependence of α on $\dfrac{\omega}{\omega_{кр}}$ (Figure 2).

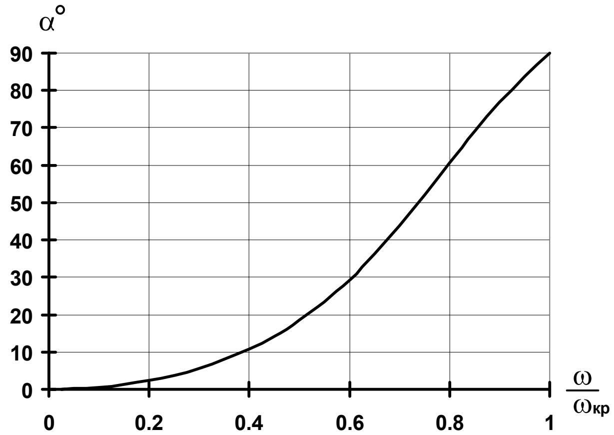

Figure 2. Dependence of angle α on $\dfrac{\omega}{\omega_{кр}}$.

It is easy to see that rotor turn angle values, when aerodynamic drag is present, cannot be lower than obtained curve.

Expression (36) enables setting dependence of $\frac{a}{e}$ on $\frac{\omega}{\omega_{кр}}$ (Figure 3).

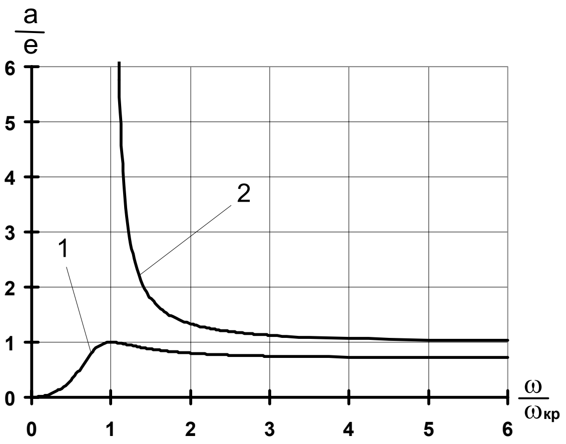

Figure 3. Dependence of $\frac{a}{e}$ on $\frac{\omega}{\omega_{кр}}$.

Curve 1 allows determining value of the rotor motions before resonance effects. When $\mu \neq 0$, values $\frac{a}{e}$ are higher than curve 1.

At over-critical mode the equation (16), when $\cos\alpha = 1$, takes the following form

$$\frac{a}{e} = \frac{\frac{\omega^2}{\omega_{кр}^2}}{\frac{\omega^2}{\omega_{кр}^2} - 1} . \quad (37)$$

Dependence of $\frac{a}{e}$ on $\frac{\omega}{\omega_{кр}}$ as the curve 2 is presented in Figure 3 also.

6 Dynamics Equation of the Rotor on Elastic Bearings

The article considers a rotor fixed on elastic shaft. The shaft, in one's turn, fixed on two absolutely ring supports (bearings). However, there are devices, in which the rotor is attached to rigid shaft, and the shaft is fixed on two elastic supports.

The equations obtained are adequate to descript dynamics of the rotor attached to elastic bearings. For that, it is enough to take into account that the shaft stiffness factor equals a sum of the same stiffness factors of both bearings.

In this case the fundamental rotor dynamics equation takes the following form:

$$m\omega^2(a + e\cos\alpha) - 2k_1 a = 0 , \quad (38)$$

where k_1 - is a stiffness factor of each elastic bearing.

7 Self-Vibration Effect

Note, the rotor dynamics equations point out no possibility of the self-vibration effect, while it exists in fact[2]. It announces that self-vibration effect concerned with not only rotor rotation, but also other features.

However, the rotor dynamics equation and rotation process physics described allow find out and descript the following case of self-vibration effect of a rotary device housing.

In reality, the elastic shaft is fixed into bearing units, which have specific stiffness also. The stiffness factor of bearing units is usually higher than stiffness factor of the shaft.

Consider the rotation features of such rotor. As speed increases, the resonance effect appears that disappears after the rotor crosses the critical speed. At over-critical speeds the rotor self-alignment and rotation stabilization are observed. However, at any speed there are the forces affecting the support and, in this case, the bearing units. Direction of the forces acting is changed along with the rotor rotation speed, and value of ones can be determined using the dependence (21).

If the forces rotation speed is closed to the free frequency of the bearing unit, then there appear the device vibrations that called as "self-vibration".

As the disturbing force exist at any speed of rotor rotation, so self-vibrations appeared at one speed exist also at all higher. When the rotor speed decreases lower speed of self-vibration appearance, the self-vibration effect disappears.

In case of elastic supports, the physics of process differs significantly from described above.

The elastic supports in fact transfer no disturbing effects from the rotating rotor to the device housing. The housing mass has enough inertia to damp little disturbs. Therefore, it is necessary to expect that after crossing the critical speed, which depends on elastic support stiffness, the self-vibration effect of the device should not be appeared. However, the support bearings have also a finite stiffness and, at specified speeds, the repeated resonance is possible.

Vibrations of elastic supports disappear after crossing the rotor critical speed determined by bearing stiffness and total mass of the rotor and supports.

8 The Rotor Direct Precession

Some studies [2] show that at high over-critical speeds the rotor rotation speed exceeds the shaft rotation speed.

One of cause of this effect can be explained using dependence that determines additional torque $M_{кр}$. At that, such effect can exist even under vacuum conditions.

Transfer the dependence (35) to dimensionless form, assuming that $\mu = 0$, $\cos\alpha = -1$, and value a is founded from equation (16),

$$\frac{M_{кр}}{ke^2} = \frac{\frac{\omega^6}{\omega_{кр}^6}}{\frac{\omega^4}{\omega_{кр}^4} - 2\frac{\omega^2}{\omega_{кр}^2} + 1} . \quad (39)$$

Dependence of $\frac{M_{кр}}{ke^2}$ on $\frac{\omega}{\omega_{кр}}$ is represented in the Figure 4.

It is easy to make certain that at very high over-critical speeds the torque exceeds significantly the torque at medium over-critical speeds.

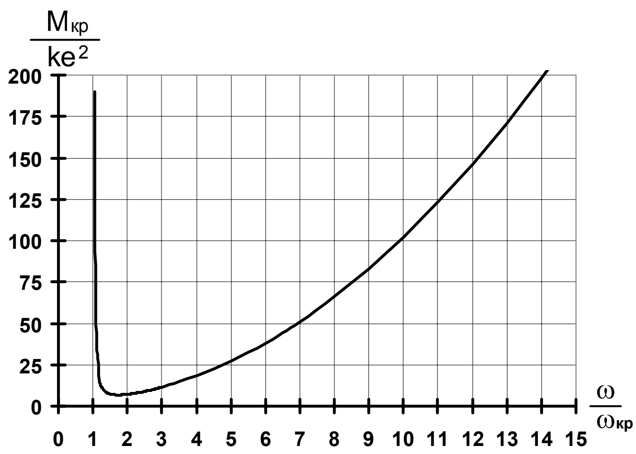

Figure 4. Dependence of $\frac{M_{кр}}{ke^2}$ on $\frac{\omega}{\omega_{кр}}$.

May be due to this the shaft speed ceases increasing, but the rotor speed increases. It is obvious, on the analogy of the gyroscope theory, the dependence to determine the speed within over-critical speeds, which is derived from equation (16), at $\cos\alpha = -1$, we have to write in the following form

$$\omega_P = \frac{ka}{m(a-e)\omega_B}, \quad (40)$$

where ω_p - is a speed of rotor,

ω_B - is a speed of shaft.

It is important to note that equations (36-37) and (39) are in fact dimensionless and can be an analogy parameters that could be applied to rotor dynamic simulation.

9 Summary

(1) The dynamic equation obtained enable determining a parameters of rotation and all forces and moment affecting the rotor taking into account the effect of environmental aerodynamic drag, as well as explaining the features of the rotor rotation over a complete speed range).

(2) The represented hypotheses allow explaining a rotary device self-vibration effect and rotor precession at over-critical speeds.

(3) The possibility of dynamics tasks' solving using the analogy parameters is shown.

Literature:

1. Zhivotov., A.Y. New theory of rotor dynamics: disk rotor dynamics with static unbalance. Proceedings IFTOMM Sixth international Conference of rotor dynamics, 2002.
2. Timoshenko S.P. Vibration problem in engineering. Moscow, The main edition of the physical and mathematical literature, Publishing house "Science", 1967