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НОВАЯ ТЕОРИЯ ДИНАМИКИ РОТОРА: ДИНАМИКА КОНСОЛЬНОГО РОТОРА С КВАЗИСТАТИЧЕСКОЙ НЕУРАВНОВЕШЕННОСТЬЮ

THE NEW THEORY OF ROTOR DYNAMICS: DYNAMICS OF AN OUTBOARD ROTOR WITH A QUASI-STATIC UNBALANCE

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Abstract: *In given paper the new inertial theory is applied for the description of dynamics of an outboard rotor with quasi-static unbalance. Forces and moments acting on a rotor and determining its dynamics on a various speeds of rotation are considered. The new basic system of the equations of rotor dynamics is received and solved. The analysis of new system of the equations is executed for a cylindrical rotor and a disk rotor. Critical speeds of a console rotor are determined. Dependences for definition of forces, the moments and other parameters of rotation on sub-critical, a transient and over-critical rotation mode are received.*
***KEYWORDS**: ROTOR, DYNAMICS, UNBALANCE, SHAFT, SUPPORT.*

1. Introduction

Rotating units and parts fastened on consoles are widely used in the engineering industry. Turbines of pumps and assemblies, drive screws of movers, abrasive discs and others are similarly attached. The rotor unbalance causes the occurrence of vibrations of the rotor machine and unpractical capacity losses of a drive. In order to reduce a vibration level of machines, a preliminary balancing of rotors is made. Such balancing reduces substantially the rotor unbalance. However, even the rotor balanced has the residual unbalance.

Attempts in the vibration rotor dynamics were made to describe the dynamics of the outboard rotor with the static unbalance. The equations obtained did not take into account the shaft bending and moments of rotor inertia. The solution of vibration equations caused the solution, which was identical one with the dynamic equation of the Jeffcott rotor [1]. New inertial theory of the dynamics of rotors solved the dynamics problem of rotors with the static unbalance for the case of a parallel rotor deviation with respect to a rotation axis [2], as well as the dynamics of a vertical rotor with a hinged and elastic support [3]. The rotor dynamic equations take into account the occurrence of the moment unbalance in the case of an occurrence of the static unbalance of a **rotor** with a hinge support. With that a rotor design model supposed the absence of a shaft bending.

Under real conditions the outboard rotors not only deviate from a rotation axis, but also bend a shaft. Bending a shaft causes an additional change of a rotor-tilting angle to a rotation axis [4]. The description of the rotor dynamics with a shaft bended was a problematic one, and the equations obtained did not find a solution [5]. Critical velocities were not defined even in the simplest cases.

Let us consider a solution of this problem for the case when the center of mass is coincided with a rotation axis, i.e., the rotor static self-centering. Let us assume that the rotor displacement and deviation from a rotation axis are small ones. A rotor velocity does not exceed the velocity when the dynamics is defined or is required the more precise definition due to an action of the moment unbalance. We shall investigate the rotor rotation under vacuum and weightlessness conditions.

2. Object of investigation and coordinate system

Let us consider the dynamics of a solid outboard rotor with a mass m (Fig. 1). The equatorial moment of inertia of a rotor equals to I_a. The axial moment of inertia of a rotor equals to I_b. A weightlessness and flexible shaft of an outboard rotor has a rigidity coefficient k and is fixed in a bearing unit (point D).

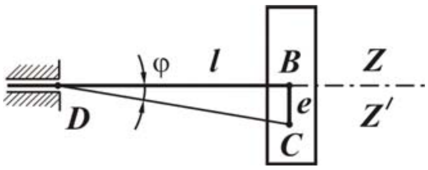

Fig.1. Outboard rotor.

A shaft has a length l. A shaft length exceeds substantially a distance from a shaft attachment point to a rotor to the plane where a rotor center-of-mass is arranged. The static unbalance we set by displacement e of a rotor center-of-mass from a geometric axis. The static unbalance causes the occurrence of the moment unbalance, which is defined by an angle φ.

Let us assume, that a rotor rotates steady at velocity ω. Specific rotating features of a rotor we shall consider in the system of coordinates $OXYZ$, which rotates about axis Z. A rotation axis Z passes through a support (point D) with a bearing unit. Plane OXY passes through a rotor center-of-mass and is a measuring plane.

In rotation, due to a shaft bending, a rotor deviates from a rotation axis Z by an angle λ under action of the unbalance (Fig. 2), and turns also about a geometric axis Z^1 by an angel α (Fig. 3). With that a geometric rotor center (point B) is displaced from a rotation axis by distance a.

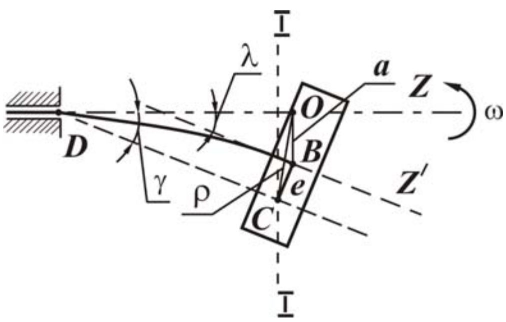

Fig. 2. Schematic drawing of an outboard rotor deviating from rotation axis Z.

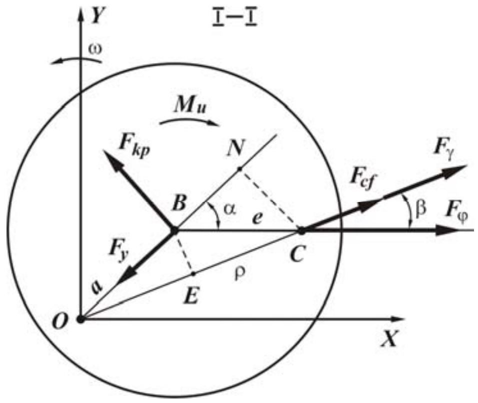

Fig. 3. Schematic drawing of forces and moments acting on a rotor

3. Forces and moments acting on rotor

Forces and moments act on a rotating rotor as shown in Fig. 3.

The centrifugal force of inertia F_{cf} occurs due to a displacement C of a rotor center-of-mass from a rotation axis at distance ρ.

A bending inertia moment is created by a centrifugal rotor moment of inertia, since a line DC is deviated from a geometric rotor axis by an angle φ. A bending inertia moment may be represented as a force F_φ, which is applied to a rotor at point C and acts with respect to a point D in direction to a point B, if $I_a - I_b \succ 0$, or in a n opposite direction, if $I_a - I_b \prec 0$.

Force F_y of an elastic support occurs due to a displacement of a geometric rotor center from a rotation axis.

A bending inertia moment which is created by a centrifugal moment of inertia of a rotor due to a displacement of a line DC from a rotor rotation axis by an angle γ. A bending inertia moment may be represented as a force F_γ, which is applied to a rotor at point C and acts in direction of a point O, if $I_a - I_b \prec 0$, or in an opposite direction, if $I_a - I_b \succ 0$.

A bending inertia moment M_u occurs in deviation of a geometric rotor axis from a rotation axis. It consists of the total sum of separate additional rotating inertia moments of the system; these moments are created because of changing a moment of inertia and centrifugal moment of inertia of a rotor. Value of a centrifugal moment of inertia of a rotor depends on an angle λ. We neglect a rotor turning about its center-of-mass because of the moment unbalance. An additional inertia moment acts about a geometric rotor axis in an opposite way to a rotor rotation.

An additional rotational moment M_{kp} is created by a machine drive for rotating a rotor. An additional rotational moment M_{kp} may be represented as a force F_{kp}, which is applied to a rotor at point B.

Dependencies defining the forces and moments mentioned are given below.

$$F_{cf} = m\omega^2 \rho \quad (1)$$

$$F_\varphi = \frac{(I_a - I_b)\omega^2 \sin\varphi \cos\varphi}{l} = \frac{(I_a - I_b)\omega^2 e}{l^2} \quad (2)$$

$$F_y = ka \quad (3)$$

$$F_\gamma = \frac{(I_a - I_b)\omega^2 \sin\gamma \cos\gamma}{l} = \frac{3(I_a - I_b)\omega^2 \rho}{2l^2} \quad (4)$$

$$M_u = m\omega^2 a^2 + \frac{(I_a - I_b)\omega^2 a \sin\lambda \cos\lambda}{l} = m\omega^2 a^2 + \frac{3(I_a - I_b)\omega^2 a^2}{2l^2} \quad (5)$$

$$M_{kp} = F_{kp} a . \quad (6)$$

The following remarks are taken into consideration in equations (2), (3), (4) and (5).

An equation (2) includes $\sin\varphi = \frac{e}{l}$ and $\cos\varphi = 1$. An equation (3) includes the coefficient of stiffness $k = \frac{3EJ}{l^3}$. Here EJ – a shaft bending stiffness. According to [4], a value a, under action of a force F, is defined as $a = \frac{Fl^3}{3EJ}$, and a value of an angle λ is defined as $\lambda = \frac{Fl^2}{2EJ}$.

It makes possible to establish the dependence between a bending value of an outboard shaft and a value of an angle of section rotation under action of a force $\lambda = a\frac{3}{2l}$, as well as to obtain a correction slope coefficient of a geometric rotor axis to a rotation axis with a curvilinear shaft bending, which equals to $\tau = \frac{3}{2l}$.

With account of a correction coefficient τ in equations (4) and (5) we have the following: $\sin\lambda = \frac{3a}{2l}$, $\cos\lambda = 1$, $\sin\gamma = \frac{3\rho}{2l}$, $\cos\gamma = 1$.

4. System of rotor dynamic equations

For obtaining the system of dynamic equations of a rotor, we shall use the schematic diagram of forces and moments acting on a rotor if $I_a - I_b \succ 0$ (Fig. 3). A rotor velocity we consider as a constant one and therefore a rotor occupies a definite position in the rotating system of coordinates. Based on the static's laws, the equations of moments of forces about points O, B, C, may be written using the known geometric dependencies [1]

$$BE = \frac{ea\sin\alpha}{\rho} \quad (7)$$

$$\rho = \sqrt{a^2 + e^2 + 2ae\cos\alpha} \quad (8)$$

$$CN = e\sin\alpha \quad (9)$$

$$BN = e\cos\alpha . \quad (10)$$

When we compose the equations of moments of forces about points O, B, C, we obtain the following:

$$\begin{aligned} & F_{kp}a - M_u - F_\varphi a\sin\alpha = 0 \\ & F_{cf}\frac{ae\sin\alpha}{\rho} + F_\gamma \frac{ae\sin\alpha}{\rho} - M_u = 0 . \\ & F_{kp}e\cos\alpha + M_u - F_y e\sin\alpha = 0 \end{aligned} \quad (11)$$

With account of dependencies (7–10) and equations (1–6) from the system of equations (11), we obtain the system of equations describing the dynamics of an outboard rotor.

$$\left[m\omega^2+\frac{5(I_a-I_b)\omega^2}{2l^2}\right](a+e\cos\alpha)- \\ -\frac{(I_a-I_b)\omega^2 a}{l^2}-\kappa a=0 \tag{12}$$

$$\sin\alpha=\frac{a}{e} \tag{13}$$

$$F_{kp}=\left[m\omega^2+\frac{5(I_a-I_b)}{2l^2}\right]a .$$

If $l\to\infty$, then an equation (12) gains the known form [1].

$$m\omega^2(a+e\cos\alpha)-ka=0 .$$

5. Dynamics of cylindrical rotor

For a cylindrical rotor we have $I_b-I_a\prec 0$, $\cos\alpha=1$. In this case, from an equation (12) we have the following:

$$\left[2m\omega^2l^2+5(I_a-I_b)\omega^2\right](a+e)- \\ -2(I_a-I_b)\omega^2a-2\kappa al^2=0 . \tag{14}$$

From an equation (14) we define a bending value of a rotor shaft a:

$$a=\frac{e\left[2m\omega^2l^2+5(I_a-I_b)\omega^2\right]}{2kl^2-2m\omega^2l^2-3(I_a-I_b)\omega^2} .$$

In a transient rotating mode, a rotor turning about geometric axis by an angle α is taken into consideration. An equation (12) has the following form:

$$\left[2m\omega^2l^2+5(I_a-I_b)\omega^2\right]\left(a+\sqrt{e^2-a^2}\right)- \\ -2(I_a-I_b)\omega^2a-2\kappa al^2=0 . \tag{15}$$

By increasing a rotation velocity, a rotor turns about geometric axis by some angle α with respect to an initial position. If an angle becomes equal to 90^o, then a rotor achieves a critical velocity, which meets the frequency of natural oscillations of a non-rotating rotor. In this case a term $\sqrt{e^2-a^2}$ is absent in the equation, and a critical velocity is defined by the following equation:

$$\omega_{kp}=\sqrt{\frac{2kl^2}{2ml^2+3(I_a-I_b)}} . \tag{16}$$

It follows from this equation, that by increasing an equatorial moment of inertia, the critical velocity reduces. The equation (16) shows that if $l\to\infty$, then

$$\omega_{kp}=\sqrt{\frac{k}{m}} . \tag{17}$$

Comparison of equations (16) and (17) shows that the critical velocity of a cylindrical outboard rotor is always lower than the critical velocity of the Jeffcott rotor with the same coefficients of elasticity of a shaft.

An equation (15) may be represented in the following form:

$$\left[2l^2m\omega^2+5(I_a-I_b)\omega^2\right](1+ctg\alpha)- \\ -2(I_a-I_b)\omega^2-2\kappa l^2=0 . \tag{18}$$

An equation (18) shows that a rotor turning by an angle α does not depend on a value of the unbalance.

Under supercritical rotation mode, the $\cos\alpha=-1$. An equation (12) in this case has the following form:

$$\left[2m\omega^2l^2+5(I_a-I_b)\omega^2\right](a-e)- \\ -2(I_a-I_b)\omega^2a-2\kappa al^2=0 . \tag{19}$$

From equation (19) it is easy to define a:

$$a=\frac{e\left[2m\omega^2l^2+5(I_a-I_b)\omega^2\right]}{2m\omega^2l^2+3(I_a-I_b)\omega^2-2kl^2} .$$

If $\omega\to\infty$, then we obtain the following equation:

$$a=\frac{e\left[2ml^2+5(I_a-I_b)\right]}{2ml^2+3(I_a-I_b)} .$$

A rotor self-centering does not take place at supercritical velocities and a main axis of rotor inertia does not coincide with a rotation axis.

6. Dynamics of disk rotor

For a disk rotor we have $I_b-I_a\succ 0$, $\cos\alpha=1$. Therefore an equation (12) is convenient to represent in the following form:

$$\left[2m\omega^2l^2-5(I_b-I_a)\omega^2\right](a+e)+ \\ +2(I_b-I_a)\omega^2a-2\kappa al^2=0 . \tag{20}$$

Equation (20) makes possible to define a.

$$a=\frac{e\left[2m\omega^2l^2-5(I_b-I_a)\omega^2\right]}{2kl^2-2m\omega^2l^2+3(I_b-I_a)\omega^2} . \tag{21}$$

Dependence (21) shows that there are conditions under which a shaft bending in a rotor zone is absent:

$$l=\sqrt{\frac{5(I_b-I_a)}{2m}} .$$

In transition rotation mode of the rotor, a rotor turns about its geometric axis by an angle α.
Equation (12) may be represented in the following form:

$$\left[2m\omega^2l^2-5(I_b-I_a)\omega^2\right]\left(a+\sqrt{e^2-a^2}\right)+ \\ +2(I_b-I_a)\omega^2a-2\kappa al^2=0 . \tag{22}$$

Critical velocity is defined by the following equation:

$$\omega_{kp}=\sqrt{\frac{2kl^2}{2ml^2-3(I_z-I_a)}} . \tag{23}$$

It follows from comparison of equations (17) and (23), that a critical velocity of the rotor is always higher than a critical velocity of the Jeffcott rotor with the same coefficients of shaft elasticity. It is obvious that the following condition shall be observed:

$$l \geq \sqrt{\frac{3(I_b - I_a)}{2m}} .$$

Equation (22) may be also represented in the following form:

$$\begin{aligned}&\left[2l^2 m\omega^2 - 5(I_b - I_a)\omega^2\right](1 + ctg\alpha) + \\ &+2(I_b - I_a)\omega^2 - 2\kappa l^2 = 0\end{aligned} . \quad (24)$$

Equation (24) shows, that a rotor turning by an angle α does not depend on an unbalance value.
At supercritical rotating mode of the rotor $\cos\alpha = -1$.
Equation (12) may be represented in the following form:

$$\begin{aligned}&\left[2m\omega^2 l^2 - 5(I_b - I_a)\omega^2\right](a - e) + \\ &+2(I_b - I_a)\omega^2 a - 2\kappa a l^2 = 0\end{aligned} . \quad (25)$$

From equation (25) we have the following:

$$a = \frac{e\left[2m\omega^2 l^2 - 5(I_b - I_a)\omega^2\right]}{2m\omega^2 l^2 - 2kl^2 - 3(I_b - I_a)\omega^2} .$$

If $\omega \to \infty$, then we have

$$a = \frac{e\left[2ml^2 - 5(I_b - I_a)\right]}{2ml^2 - 3(I_b - I_a)} . \quad (26)$$

Dependence (26) shows that a rotor self-centering does not take place at supercritical velocities.

7. Angles defining force direction

Let us we choose a force direction of a resultant vector of unbalance as the angle origin. In this case we have the following.

Angle $\alpha + 180^\circ$ defines a force direction of an elastic force F_y of a support. Angle $\alpha + 90^\circ$ defines a force direction of a force F_{kp} , which is created by an additional moment of rotation. Angle β defines a force direction of a centrifugal force F_{cf} : $\sin\beta = \frac{a^2}{e\rho}$. Angle β defines also a force direction of a force F_γ for a cylindrical rotor. Angle $\beta + 180^\circ$ defines a force direction of a force F_γ for a disk rotor. The angle equal to 180° , defines a force direction of a force F_φ for a cylindrical rotor. The angle equal to zero, defines a force direction of a force F_φ for a disk rotor.

8. Results and discussion

In comparison with researches performed earlier [1], [5], the new results are obtained at more high level of quality. Values and directions of all forces and moments acting on a rotor are defined. New equations of the dynamics of an outboard cylindrical and disk rotor were obtained with consideration of a shaft bending; these equations describe the position of a rotor and center-of-mass with respect to a rotation **axis** under different rotation modes.

Critical velocities are defined. Absence of a complete rotor self-centering at supercritical velocities is shown, when the rotor dynamics is not defined by the moment unbalance. A rotor self-centering is related with an effect of centripetal force at supercritical velocities.

An additional moment of rotation is defined; this moment is required to maintain a corresponding rotor position relatively to a rotation axis.

Researches have shown a substantial effect of moments of rotor inertia on the rotor dynamics, and, in particular, on a value of critical accelerations. Researches were conducted without consideration of dissipative forces that made possible to detect the real physics of the rotation process, related with an effect of the quasi-static unbalance.

The equations obtained are close by their structure to the structure of known equations [2], [3]. Main feature is using the dependence of a deviation angle of a section on a shaft flexure based on the theory of strength of materials. This new approach made possible to define the rotor dynamics at initial phase – the phase of static self-centering of the rotor, and define the dynamic conditions of a final phase of a rotor rotation under action of the moment unbalance. An effect of the moment unbalance at high supercritical velocities causes a dual shaft bending and is described by other equations.

9. Conclusion

The results obtained of researches may be used for designing the outboard rotors. They may be used for defining the loads acting at bearing units for attaching a rotor shaft and defining the drive capacity losses by rotation of a rotor with quasi-static unbalance at supercritical velocities until beginning an active effect of a moment unbalance.

10. References

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