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# The Beal's Conjecture and Fermat's Last Theorem (proof)

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**Abstract.** The proof of the Great Beal's conjecture it is reconciled in the given paper. The Pythagorean theorem for a rectangular triangle is put in a basis of the proof. The cosine law and laws of a similarity of triangles are used for the proof also. The mechanism of deriving of the Beal's equation comes to light. It is shown, that the Beal's equation establishes connection of sides of a rectangular triangle. The essence of a Beal's conjecture is explained from positions of geometry and trigonometry. It is shown, that the Beal's equation has no solutions in integers and it confirms validity of a Beal's conjecture. The proof of a Beal's conjecture is the proof of the theorem of Fermat.

## Introduction

"THE BEAL CONJECTURE AND PRIZE

“””Texas millionaire banker Andrew Beal has just offered a major cash prize to the first person to solve "Beal's Problem", a mathematical teaser that has a tantalizing appeal similar to that of Fermat's Last Theorem, which attracted hundreds of professional and amateur mathematicians until its solution three years ago.

Dedicated followers of mathematical gossip will know that Fermat's Last Theorem said that for exponents n greater than 2, the equation

$$z^n = x^n + y^n$$

has no whole number solutions for x, y, and z (apart from trivial answers where one of the unknowns has the value 0).

First proposed in the seventeenth century by the great French mathematician Pierre De Fermat, the "theorem" resisted numerous attempts at solution until British mathematician Andrew Wiles of Princeton University found a proof in 1994. Wiles's achievement was portrayed in a BBC television Horizon documentary "The Proof" last year and described in a small rash of popular books.

Much of the general interest in Fermat's challenge can be attributed to the offer of a cash prize, the Wolfskehl Prize, to the person who first proved the theorem. Established in 1908 by Paul Wolfskehl, a German physician and amateur mathematician, the prize lost much of its value in the German inflation of the 1930s, but was still worth about $50,000 when Wiles collected the award earlier this year.

Beal's Problem is like Fermat's.”””

BEAL'S CONJECTURE: If $z^n = x^m + y^r$, where $z, x, y, n, m, r$ are positive integers and $n, m, r$ are all greater than 2, then $z, x, y$ must have a common prime factor.

THE BEAL PRIZE. The conjecture and prize was announced in the December 1997 issue of the Notices of the American Mathematical Society. Since that time Andy Beal has increased the amount of the prize for his conjecture. The prize is now this: $100,000 for either a proof or a counterexample of his conjecture. The prize money is being held by the American Mathematical Society until it is awarded. In the meantime the interest is being used to fund some AMS activities and the annual Erdos Memorial Lecture.

CONDITIONS FOR WINNING THE PRIZE. The prize will be awarded by the prize committee appointed by the American Mathematical Society. The present committee members are Charles Fefferman, Ron Graham, and Dan Mauldin. The requirements for the award are that in the judgment of the committee, the solution has been recognized by the mathematics community. This includes that either a proof has been given and the result has appeared in a reputable refereed journal or a counterexample has been given and verified.

**1. Statement of a problem.**

It is required to prove Great Beal's conjecture.

The Beal's conjecture say in language of numbers: the equation $z^n = x^m + y^r$, where $n \succ 2$, $m \succ 2$, $r \succ 2$ - the integer, has no solutions in integers $z, x, y$.

For the proof of a Beal's conjecture to show enough**, that one of numbers Beal** $z, x, y$ **is an irrational**.

**For the proof we shall assume, that numbers** $z, x, y$ **are integers.**

**2. Singularities of the Beal's equation**

Let's pay attention to some singularities of the equation. Identical multipliers can be reduced in the equation if they are available in numbers $z, x, y$. The aspect and an essence of the equation do not vary.

Exponents will increase in the equation if numbers $z, x, y$, it is possible to spread out on identical multipliers. However the aspect of the equation does not vary. The Beal's conjecture keeps the essence.

Obviously, it is necessary to produce possible mathematical operations on simplification of the equation and numbers $z, x, y$ before the proof of a Beal's conjecture.

After these mathematical simplifications it is possible to state, that numbers $z, x, y$ have the following properties.

**Property 1.** Numbers Beal $z, x, y$ has no common multipliers.

**Property 2.** Numbers $z, x, y$ cannot be spread out in addition on identical multipliers.

**Property 3.** Each number Beal contains a simple integer from which the root of any degree is not taken.

**The proof**

**3. In the beginning we shall show, that the Beal's equation it is obtained from equation of Pythagorean for a rectangular triangle.**

Really, equation of Pythagorean, which includes integers $c, a, b$, looks like

$$c^2 = a^2 + b^2 \qquad (1)$$

where $c$ - a hypotenuse of a rectangular triangle,
$a, b$ - legs of a rectangular triangle.

Let's assume, that numbers $c^2, a^2, b^2$ can be connected with numbers $z^n, x^m, y^r$ by means of the equations

$$c^2 = z^n,\ a^2 = x^m,\ b^2 = y^r. \qquad (2)$$

Let's substitute numbers $z^n, x^m, y^r$ in equation of Pythagorean instead of numbers $c^2, a^2, b^2$ and we shall receive the Beal's equation.

$$z^n = x^m + y^r \; . \qquad (3)$$

The equation of Beal to become the equation of Fermat, if $n = m = r$

$$z^n = x^n + y^n \qquad (4)$$

**Logic conclusion 1:** the Beal's equation is the equation connecting a hypotenuse with legs of a rectangular triangle. Hence, premises, which have allowed receiving the Beal's equation, can be used for the proof of a Beal's conjecture.

The logic conclusion is fair from positions of geometry. Let's take into account that three algebraic expressions, which enter into the Beal's equation, correspond to three segments. Three segments can be connected among themselves as a triangle, including as the degenerated triangle. The Beal's equation establishes correlation of three numbers, and, hence, establishes correlation of sides of a triangle.

Moreover, the Beal's equation establishes connection of sides of a rectangular triangle. The equation of connection of sides contains four algebraic expressions for any other triangle.

The logic conclusion is fair from positions of trigonometry. The Beal's equation it is possible to present as

$$1 = \frac{x^m}{z^n} + \frac{y^r}{z^n} \; . \qquad (5)$$

The equation, which is known from trigonometry, looks like

$$1 = \sin^2 \alpha + \cos^2 \alpha \; , \qquad (6)$$

where $\sin\alpha, \cos\alpha$ - are defined as the ration of legs to a hypotenuse of a rectangular triangle.

From the equations (5) and (6) follows, that

$$\frac{x^m}{z^n} = \sin^2 \alpha \; , \qquad (7)$$

$$\frac{y^r}{z^n} = \cos^2 \alpha \; . \qquad (8)$$

Thus $\sqrt{z^n}, \sqrt{x^m}, \sqrt{y^r}$ are accordingly a hypotenuse and legs of a rectangular triangle.

**4. Theorem of Zhivotov:** Quadrates of integers $c, a, b$, which define sides of a scalene triangle, it is impossible to decompose on three and more identical whole factors $z, x, y$ so that from segments $z, x, y$ it was possible to make a triangle similar to the basic triangle with sides $c, a, b$.

**The proof.**

The equation of connection of sides of a triangle, according to a cosine law, looks like

$$c^2 = a^2 + b^2 - 2ab\cos\alpha \qquad (9)$$

where $\cos\alpha$ - an angle between sides of a triangle $a$ and $b$.

It is admissible, that integers, which define quadrates of sides of a triangle, it is possible to decompose on various numbers of identical whole factors $z,x,y$, i.e. $c^2 = z^n$ $a^2 = x^m$, $b^2 = y^r$.

In this case the equation (9) can be noted as

$$z^n = x^m + y^r - 2x^{\frac{m}{2}} y^{\frac{r}{2}} \cos\alpha \tag{10}$$

Let's express from the equations (9) and (10) $\cos\alpha$ and we shall receive the equation

$$\frac{a^2 + b^2 - c^2}{ab} = \frac{x^m + y^r - z^n}{x^{\frac{m}{2}} y^{\frac{r}{2}}} \tag{11}$$

or

$$\frac{a}{b} + \frac{b}{a} - \frac{c^2}{ab} = \frac{x^{\frac{m}{2}}}{y^{\frac{r}{2}}} + \frac{y^{\frac{r}{2}}}{x^{\frac{m}{2}}} - \frac{z^n}{x^{\frac{m}{2}} y^{\frac{r}{2}}} \tag{12}$$

The equation is fair, if

$$\frac{a}{b} = \frac{x^{\frac{m}{2}}}{y^{\frac{r}{2}}}, \quad \frac{b}{a} = \frac{y^{\frac{r}{2}}}{x^{\frac{m}{2}}}, \quad \frac{c^2}{ab} = \frac{z^n}{x^{\frac{m}{2}} y^{\frac{r}{2}}} \tag{13}$$

On the other hand numbers $z,x,y$ are a part of numbers $c,a,b$. Segments $z,x,y$ are a part of sides $c,a,b$ of a triangle. Thus the position of segments $z,x,y$ corresponds to a position of sides $c,a,b$ of a triangle. Hence, if from segments $z,x,y$ it is possible to make a triangle this triangle should be similar to a triangle with sides $c,a,b$.

Therefore, it is possible to note for similar triangles with sides $c,a,b$ and $z,x,y$

$$\frac{z}{c} = \frac{x}{a} = \frac{y}{b} \tag{14}$$

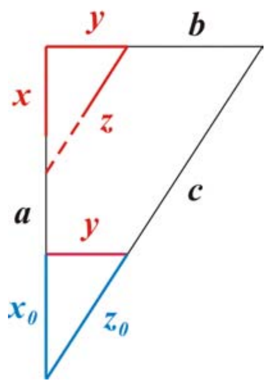
or

$$\frac{a}{b} = \frac{x}{y}, \quad \frac{b}{a} = \frac{y}{x}, \quad \frac{c^2}{ab} = \frac{z^2}{xy} \tag{15}$$

The discordance of the equations (13) and (15) is excluded only in case $a = b = c$, $z = x = y$ and $n = m = r$. The triangle with equal sides refers to as an equilateral triangle.

Thus, **theorem of Zhivotov** is proved.

**5. The proof of an irrationality of one or several numbers of Beal.**




Beal's numbers $z,x,y$ do not organize the three of numbers of Pythagorean and it follows from Zhivotov's theorem.
However each of Beal's numbers $z,x,y$ contains a prime number and consequently for each of Beal's numbers it is possible to select two more numbers which will organize the three of numbers of Pythagorean.
Let such number of Beal is the number $y$, if the exponent $r$ for this number is less than exponents $n,m$ for numbers $z,x$. We shall add number $y$ to such numbers $z_o,x_o$ that they organized the three of numbers of Pythagorean.
It is known, that it is possible to construct a rectangular triangle with sides $c,a,b$, which will be similar to a triangle with sides $z_o,x_o,y$.
Let's choose factor of a similarity, which quadrate can be defined on the equation, for construction of a triangle with sides $c,a,b$.

$$q^2 = \frac{b^2}{y^2} = \frac{y^r}{y^2} \tag{16}$$

In this case quadrates of sides of a similar triangle are equal

$$c^2 = \frac{y^r}{y^2} z_o^2, \qquad a^2 = \frac{y^r}{y^2} x_o^2, \qquad b^2 = y^r \tag{17}$$

Let's show, that the number $z$ and (or) $x$ cannot be integers if number $c^2,a^2$ to replace with numbers $z^n,x^m$. We shall produce replacements of numbers in the equations (17) and we shall receive

$$z = \sqrt[n]{\frac{y^r z_o^2}{y^2}} = \sqrt[n]{y^r}\sqrt[n]{\frac{z_o^2}{y^2}} = \sqrt[n]{y^{r-2}}\sqrt[n]{z_o^2} \tag{18}$$

$$x = \sqrt[m]{\frac{b^2 x_o^2}{y^2}} = \sqrt[m]{\frac{y^r x_o^2}{y^2}} = \sqrt[m]{y^{r-2}}\sqrt[m]{x_o^2} \tag{19}$$

Numbers $\sqrt[n]{y^{r-2}}$, $\sqrt[m]{y^{r-2}}$ are irrationals as the radical with higher index cannot be extracted from number with less high exponent.

The denominator of numbers $\sqrt[n]{\frac{z_o^2}{y^2}}$ $\sqrt[m]{\frac{x_o^2}{y^2}}$ is contained with one simple factor which is not present in numerator of these numbers. Therefore, irrespective of numbers $\sqrt[n]{z_o^2}$ $\sqrt[m]{x_o^2}$, numbers $z, x$ are irrationals.

For an equilateral rectangular triangle $y = x_o$, and $r = m$.

Therefore $x$ also is an integer. However the number $z$ remains an irrational.

**If it so the Beal's conjecture is proved in full for any values $n \succ 2, m \succ 2, r \succ 2$.**

## 6. Connection of the Great Beal's conjecture with the Great theorem of Fermat.

Let's mark, that exponents are equal $n$ the equation of Fermat. Therefore the equations (18) and (19) can be noted as

$$z = \sqrt[n]{\frac{y^n z_o^2}{y^2}} = y\sqrt[n]{\frac{z_o^2}{y^2}} \qquad (20)$$

$$x = \sqrt[n]{\frac{y^n y_o^2}{y^2}} = y\sqrt[n]{\frac{x_o^2}{y^2}} \qquad (21)$$

According to the previous analysis it is possible to state, that from a denominator of numbers $\sqrt[n]{\frac{z_o^2}{y^2}}$, $\sqrt[n]{\frac{x_o^2}{y^2}}$ it is impossible to extract the rational radical of a degree $n \succ 2$.

**If it so the theorem of Fermat is proved in full for any values $n \succ 2$.**

**Literature.**

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