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BASIC AND OVERALL DRIVING TORQUE FOR SOLID UNBALANCED ROTOR WITH ELASTIC SHAFT

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This paper examines a basic driving torque, which is a component of an overall driving torque for maintaining a uniform induced rotation of a rotor. The overall driving torque takes account of the basic, additional driving torque for maintaining the rotor's position relative to the rotational axis, driving torque for overcoming the impact of dissipative forces and driving torque for performance of a functional problem.

One pays attention to that for rotation of the unbalanced rotor with a constant velocity it is required to apply the basic driving torque with an objective to overcome the inertial moment generated by the unsymmetrical distribution of mass. During this study, one takes account of that the rotor-supports' system during bending of the shaft becomes the system with alternating moments of inertia. It is shown that displacement and deflection of the rotor shaft from the rotational axis under the impact of static, moment and dynamic unbalance leads to an increase of the basic driving torque. The obtained equations of the basic driving torque take into consideration a phenomenon of self-alignment of the rotor.

For all considered systems, one adduces dependencies that allow definition of inertial moments impeding rotation. Differential equations are adduced for definition of the principal driving torque. An equation is adduced for definition of the overall driving torque. A complete equation of the rotor's rotational motion is considered for a statically unbalanced rotor at presence of aerodynamic resistance forces.

1. Introduction.

In the state-of-the-art instrument making industry there are widely applied assemblies and mechanisms, which rotors rotate with high velocities. Unbalance of the rotor leads to generation of vibration. The vibration impacts negatively reliability and durability of an assembly. Besides, the unbalance of the rotor leads to unproductive losses of the driving gear output.

If the rotation velocity of assemblies is insignificant, rotation of the rotor is described by the differential equation of angular motion of a body [1].

It is assumed that balancing of the rotor secures ideal conditions, and the driving torque at absence of external and internal resistance is equal to zero.

The phenomenon of shaft bending and self-alignment of a Laval's turbine revealed new particular qualities of dynamics of a flexible-shaft rotor. It turned out that uniform rotation of of the rotor with an offset center of mass requires application of the driving torque even at absence of moments generated by dissipative forces [2]. There exist also other equations for definition of the driving torque required for maintenance of the rotor uniform velocity [3], [4]. It is assumed that the driving torque determined per these equations takes account of the impact of dissipative forces, and is sufficient for maintenance of uniform rotation of the rotor. However, the dissipative forces are not included into these equations neither as independent members nor in an implicit form.

Besides, the well-known equation, for the case of coincidence of the rotor center of mass with the rotational axis, loses any meaning. Therefore, these equations are not applicable for the rotor with a moment-unbalanced state.

Moreover, the inertial theory of rotor dynamics [5 - 7] revealed existence of an additional driving torque. The additional driving torque secures maintenance of a certain status of the rotor relative to the rotational axis at its uniform velocity and overcoming the driving inertial torque, which opposes to rotation of the rotor. The driving inertial torque does not exclude existence of another one – the basic driving torque. However, the well-known equations do not consider the overall driving torque as a sum of basic and additional driving torques. The well-known equations of the overall driving torque allow only account of moments on overcoming the resisting dissipative forces.

These contradictions confirm the presence of a problem on definition of the overall driving torque.

Topicality to resolve this problem raises no doubts since losses of the driving gear output because of the significant unbalance of the rotor amount to 36%[8].

Taking into consideration the identified contradictions, solution of the problem of the overall driving torque shall be commenced from receiving an equation for definition of the overall driving torque with subsequent account of additional driving torques.

2. Basic driving torque for rotation of balanced solid rotor.

It is widely known that rotation of a free body proceeds around the principal axis of inertia. At the same time, the body rotates during arbitrarily long time without energy supply. Similarly, the planets revolve around their own principal axes of inertia. In order to change the velocity of rotation, one has to apply to a body a driving torque.

Rotors are not free bodies since they are secured in the supports of machines. However, using engineering arrangements, one can achieve a high-quality superposition of the rotor's principal axis of inertia with the rotation axis. During small velocities of the rotor, the high-quality superposition of the inertia with the axis of rotation allows achievement of conditions similar to the conditions of rotation of a free body. From this it follows that a well-balanced rotor uniformly rotates around a fixed axis and does not require application of a driving torque if no forces act onto it. Accordingly, it was assumed that if a rotor is affected by a pair of forces then the rotor rotates around a fixed axis uniformly accelerated or uniformly retarded.

Such rotation of the rotor was described by the differential equation of rotational motion of a body without taking into account extraneous resistance [1]

$$I\frac{d\omega}{dt} = M_{кр} \quad (1)$$

The equation (1) may be also addressed as an equation for definition of the basic driving torque of a solid rotor with a solid shaft.

However, even a high-quality balanced rotor possesses a residual unbalanced status and the rotor's principal axis of inertia never coincides with the axis of rotation. Therefore, rotation of a real solid rotor has to be considered as rotation of a body with unsymmetrical mass distribution relative to its geometrical axis or rotational axis.

If the rotor shaft is solid and mounted in fixed supports, then such rotor-supports' system is a rigid system with invariable unsymmetrical mass distribution. In all other cases, the rotor-supports' system is considered to be a deformable system with variable unsymmetrical mass distribution, or a system with alternating moments of inertia.

Let us consider principal cases of divergence of the main axis of inertia with the rotor rotation axis in the absence thereof extraneous resistance forces and friction forces in bearing mount assemblies. The rotor rotation will be considered in a rotating coordinate system.

3. Basic driving torque for rotation of statically unbalanced solid rotor with solid shaft.

Let us consider rotation of a solid rotor of mass m (Fig.1) with static unbalanced state and location of the principal axis of inertia parallel to the rotational axis. The static unbalance of the rotor is given by the displacement of the rotor center of mass C from the geometrical axis at a distance e. The rotor has a solid imponderable shaft, which axis coincides with the rotor geometrical axis and rotational axis. The rotor shaft is secured in solid supports. The rotor has the axial moment of inertia I_a and is rotated with the constant velocity ω by a driving gear.

Commonly, the dynamics of a rotor with a static unbalanced state is considered in the observation plane $I-I$, which passes through the center of mass perpendicular to the rotational axis.

Since during rotation the center of mass is located at a distance e from the rotational axis, the rotor has also an additional moment of inertia I_e, which may be determined as [5]

$$I_e = me^2 \quad (2)$$

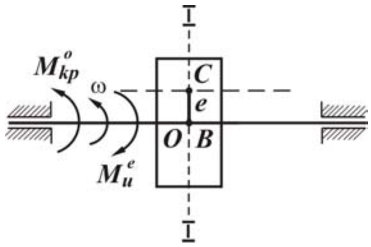

Рис. 1

On the whole, the rotor possesses the overall moment of inertia

$$I^{ob} = I_a + me^2 \quad (3)$$

If the rotor-supports' rigid system rotates uniformly with the velocity M_u^e, then an inertial driving torque emerges, which can be determined per the following dependence [5]

$$M_u^e = m\omega^2 e^2 \quad (4)$$

The inertial driving torque impedes rotation of the rotor.

To overcome the inertial driving torque, one has to apply the basic driving torque M_{kp}^0, which, in this instance, is equal per its magnitude to the inertial driving torque M_u^e.

The driving torque is defined as the basic driving torque since position of the rotor and mass distribution relative to the rotational axis remains invariable at any velocity.

Taking into consideration the overall moment of inertia of the rotor-supports' system and driving inertial torque (4), the differential equation for rotation of the rotor at divergence of the center of mass with the geometrical axis and rotational axis can be written as

$$M_{kp}^0 = \left(I_a + me^2\right)\frac{d\omega}{dt} + m\omega^2 e^2 \quad (5)$$

The equation (5) can also be considered as a differential equation of rotational motion of the rotor. For changing the velocity of rotation, it is required to overcome the overall inertia of the rotor and additional inertial moment.

At a uniform velocity of rotation, the equation (5) becomes as follows

$$M_{kp}^0 = m\omega^2 e^2 \quad (6)$$

The equation (5) shows that at the specified basic driving torque the angular velocity of the rotor at first grows and achieves eventually the maximum value. Further on, the basic driving torque is fully consumed on maintaining the rotor's uniform velocity. The maximum angular velocity of the uniform rotation of the rotor at the specified driving torque can be determined per the following dependence

$$\omega_{max} = \sqrt{\frac{M_{kp}^0}{me^2}} \quad (7)$$

4. Basic driving torque for rotation of solid rotor with moment unbalanced state secured on solid shaft.

Let us consider a case when the rotor's center of mass coincides with the geometrical axis and rotational axis, and the principal axis of inertia of the rotor is deviated from the fixed axis of rotation at a certain angle φ (Figure 2).

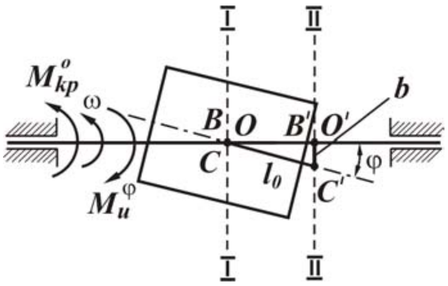

Figure 2

We presume that the rotor possesses apart from the axial moment of inertia I_a, also the product of inertia I_{φ}. The rotor's product of inertia depends upon the angle φ of inclination of the principal axis of inertia to the rotational axis.

For obtaining equations of the rotor dynamics with a moment unbalanced state, one commonly selects the observation plane $II-II$, which does not pass through the center of mass, and is located at a distance l_0 from the center of mass. Motion of the rotor is studied per trails of the rotational axis, geometrical axis and center of mass in the measurement plane $II-II$. Besides, the product of inertia is frequently determined through the axial I_a and equatorial I_b moments of inertia taking into consideration minor transferences per the well-known dependence

$$I_{\varphi} = (I_b - I_a)\sin\varphi\cos\varphi = (I_b - I_a)\frac{b}{l_0} \qquad (8)$$

At the same time, the overall moment of inertia of the rigid rotor-supports' system will be constant per its magnitude because the geometrical axis of the solid shaft coincides with the rotational axis.

$$\begin{aligned} I^{ob} &= I_a + I_{\varphi} = I_a + (I_b - I_a)\sin\varphi\cos\varphi = \\ &= I_a + (I_b - I_a)\frac{b}{l_0} \end{aligned} \qquad (9)$$

The product of inertia impacts the dynamics of a rotating rotor, and during rotation creates the bending inertial moment M^{φ}_{uz}, which at uniform rotation of the rotor can be determined per the following dependence [6]

$$M^{\varphi}_{uz} = I_{\varphi}\omega^2 = (I_b - I_a)\frac{b}{l_0}\omega^2 \qquad (10)$$

However, a part of the bending inertial moment impedes rotation of the rotor – the driving inertial torque, which depends upon the offset ratio b of the principal axis of inertia from the rotational axis in the observation plane and distance between the observation plane and center of mass [6]

$$\begin{aligned} M^{\varphi}_{u} &= \frac{b}{l_0}M^{\varphi}_{uz} = \frac{b}{l_0}I_{\varphi}\omega^2 = \\ &= I_{\varphi}\omega^2\sin\varphi = \omega^2(I_b - I_a)\frac{b^2}{l_0^2} \end{aligned} \qquad (11)$$

Evidently, for overcoming the negative impact of the product of inertia, it is required to apply a certain basic driving torque

$$M^{0}_{kp} = M^{\varphi}_{u} \qquad (12)$$

The obtained equations allow a deduction that a rotor with a moment unbalanced state possesses an overall moment of inertia, which impacts rotation of the rotor around its axis

$$\begin{aligned} I^{ob}_{kp} &= I_a + I_{\varphi}\sin\varphi = I_a + (I_b - I_a)\sin^2\varphi\cos\varphi = \\ &= I_a + (I_b - I_a)\frac{b^2}{l_0^2} \end{aligned} \qquad (13)$$

Taking into consideration the overall moment of inertia I^{ob}_{kp} of the rotor-supports' system and rotational inertial moment (12) of the rotor, the differential equation for definition of the principal driving torque for rotation of the rotor, at deviation of the principal axis of inertia from the geometrical axis and rotational axis, can be written as

$$\begin{aligned} M^{0}_{kp} &= \left(I_a + I_{\varphi}\frac{b}{l_0}\right)\frac{d\omega}{dt} + I_{\varphi}\frac{b}{l_0}\omega^2 = \\ &= \left[I_a + (I_b - I_a)\frac{b^2}{l_0^2}\right]\frac{d\omega}{dt} + (I_b - I_a)\frac{b^2}{l_0^2}\omega^2 \end{aligned} \qquad (14)$$

The equation (14) can be also considered as the differential equation of the rotor's rotational motion.

At the uniform velocity of rotation, the equation (14) becomes

$$M^{0}_{kp} = I_{\varphi}\frac{b}{l_0}\omega^2 = (I_b - I_a)\frac{b^2}{l_0^2}\omega^2 \qquad (15)$$

The equation (15) shows that at the specified principal driving torque the angular velocity of the rotor at first grows and achieves eventually the maximum value. Further on, the basic driving torque is fully consumed on maintaining the rotor's uniform velocity. The maximum angular velocity of the uniform rotation of the rotor at the specified driving torque can be determined per the following dependence

$$\omega_{max} = \sqrt{\frac{M_{kp}^0 l_0}{I_\varphi b}} = \sqrt{\frac{M_{kp}^0 l_0^2}{(I_b - I_a) b^2}} \qquad (16)$$

5. Basic driving torque for rotation of solid rotor with solid shaft, which principal axis of inertia is displaced and deflected from solid shaft axis.

In a general case, the induced rotation of the rotor around a rotational axis occurs in conditions when the center of mass does not coincide with the fixed rotational axis, and the principal axis of inertia of the body has a slope to the rotational axis (Figure 3).

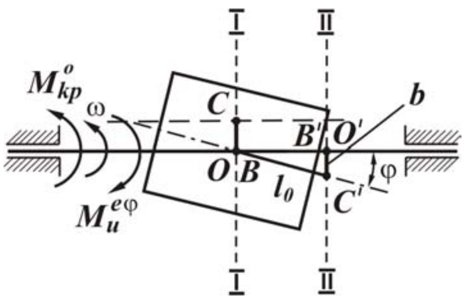

Figure 3

A distinctive feature of the rotor-supports' rigid system is additional moments of inertia (I_e and I_φ) because of displacement and deflection of the rotor from the geometrical axis.

On the whole, the rotor with a static moment unbalanced state has a general moment of inertia, which impacts rotation of the rotor around its axis

$$\begin{aligned} I_{kp}^{ob} &= I_a + me^e + I_\varphi \sin\varphi = \\ &= I_a + me^2 + (I_b - I_a)\sin^2\varphi \cos\varphi = \\ &= I_a + me^2 + (I_b - I_a)\frac{b^2}{l_0^2} \end{aligned} \qquad (17)$$

During rotation of the rotor, these moments of inertia in accordance with the principle of superposition create the rotational inertial moment $M_u^{e\varphi}$, which is equal to

$$\begin{aligned} M_u^{e\varphi} &= me^2\omega^2 + I_\varphi \frac{b}{l_0}\omega^2 = \left(me^2 + I_\varphi \frac{b}{l_0}\right)\omega^2 = \\ &= \left(me^2 + (I_b - I_a)\frac{b^2}{l_0^2}\right)\omega^2 \end{aligned} \qquad (18)$$

The equation for definition of the basic driving torque is a generalization of the equations (5) and (14) and has the appearance

$$\begin{aligned} M_{kp}^0 &= \left(I_a + me^2 + I_\varphi \frac{b}{l_0}\right)\frac{d\omega}{dt} + \left(me^2 + I_\varphi \frac{b}{l_0}\right)\omega^2 = \\ &= \left(I_a + me^2 + (I_b - I_a)\frac{b^2}{l_0^2}\right)\frac{d\omega}{dt} + \\ &+ \left(me^2 + (I_b - I_a)\frac{b^2}{l_0^2}\right)\omega^2 \end{aligned} \qquad (19)$$

The equation (19) can also be considered as a differential equation of rotational motion of the rotor.
During the uniform velocity of rotation, the equation (19) becomes

$$M_{kp}^0 = \left(me^2 + I_\varphi \frac{b}{l_0}\right)\omega^2 = \left(me^2 + (I_b - I_a)\frac{b^2}{l_0^2}\right)\omega^2 \qquad (20)$$

The inertial moments conjointly impede rotation of the rotor. At the uniform rotation of the rotor, from the equation (15) one can determine the maximum angular velocity if the driving torque is known

$$\omega_{max} = \sqrt{\frac{M_{kp}^0}{me^2 + I_\varphi \frac{b}{l_0}}} = \sqrt{\frac{M_{kp}^0}{me^2 + (I_b - I_a)\frac{b^2}{l_0^2}}} \qquad (21)$$

6. Basic driving torque for rotation of solid rotor at bending of shaft and parallel displacement of principal axis of inertia from geometrical axis.

The dynamics of a solid rotor with a flexible shaft significantly differs from the dynamics of a solid rotor with a solid shaft secured in immovable supports. The rotor-supports' system during bending of the shaft ceases to be a rigid system, and becomes a system with alternating moments of inertia. The moments of inertia of the rotor-supports' system vary not only due to bending of the shaft but also at the expense of the rotor's turn around the geometrical axis at a certain angle α. The maximum value of the angle α achieves 180^o at overcritical velocities. At the same time, self-alignment of the rotor is observed. Bending of the shaft leads to a significant change of the moments of inertia of the rotor-supports' system depending on the velocity of rotation of the rotor.

Let us consider rotation of the rotor that has static unbalance with angular velocities at which bending of the shaft occurs and turn of the rotor around its geometrical axis (Figure 4)

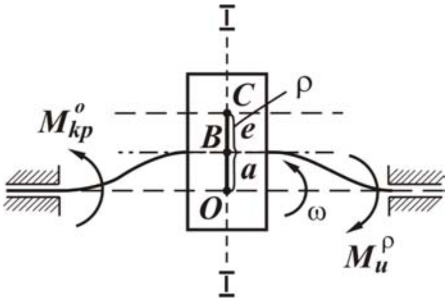

Figure 4

The static unbalance of the rotor is usually specified by dislocation of the center of mass of the rotor e from the geometrical axis. Under the impact of static unbalance, the shaft is bended, and the rotor is dislocated at a distance a from the axis passing through the centers of the supports. This axis is called the rotational axis. Simultaneously, the rotor turns at a certain angle α. As a result, the center of mass of the rotor disposes at a distance ρ from the rotational axis. The values α, a, ρ depend on the velocity of rotation and are variables. These values are interconnected by the equation [5]

$$\rho^2 = \sqrt{a^2 + e^2 + 2ae\cos\alpha} \qquad (22)$$

The rotor, in this case, has the general moment of inertia, which is equal to

$$I^{ob} = I_a + m\left(a^2 + e^2 + 2ae\cos\alpha\right) \qquad (23)$$

The rotational inertial moment that impedes the uniform rotation of the rotor and acts around the axis of rotation is determined as

$$M_u^{\rho} = m\omega^2\rho^2 = m\omega^2\left(a^2 + e^2 + 2ae\cos\alpha\right) \qquad (24)$$

In this case the equation for definition of the basic driving torque can be written as

$$M_{kp}^{0} = \left(I_a + ma^2 + me^2 + 2mae\cos\alpha\right)\frac{d\omega}{dt} + \\ + m\omega^2\left(a^2 + e^2 + 2ae\cos\alpha\right) \qquad (25)$$

It is required to note that the equation (25) is one equation of the rotor dynamics equations' system. This is explained by that dynamics of the rotor with a flexible shaft is described by the equations' system that takes into consideration position of the rotor relative the axis of rotation.

At the uniform velocity of rotation, the equation (25) becomes

$$M_{kp}^{0} = m\omega^2\left(a^2 + e^2 + 2ae\cos\alpha\right) \qquad (26)$$

Under the uniform velocity of rotation of the rotor at overcritical velocities $\cos\alpha = -1$. The equation of the basic driving torque becomes

$$M_{kp}^{0} = m\omega^2\left(a - e\right)^2 \qquad (27)$$

At overcritical velocities, self-alignment of the rotor emerges. At the entire self-alignment of the rotor, the driving torque tends to zero if $a - e \to 0$ at an unlimited increase of the velocity.

7. Basic driving torque for rotation of solid rotor at shaft bending and angular deviation of principal axis of inertia from geometrical axis.

Let us consider rotation of the rotor with angular velocities at which bending of the shaft emerges under the impact of moment unbalance (Figure 5).

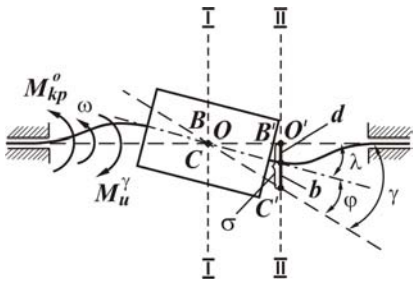

Figure 5

The original moment unbalance is prescribed by the angle φ, which is an angle of deviation of the principal axis of inertia of the rotor from the geometrical axis of the shaft [6]. Such a system primordially has the axial moment of inertia I_a and the product of inertia I_φ. In the observation plane $II - II$, the moment of inertia I_φ is characterized by the value b. Under the impact of moment unbalance, the geometrical axis of the rotor deviates from the rotational axis at an angle λ. Deviation of the geometrical axis from the axis of rotation is characterized by the value d. Simultaneously, the rotor is turned around the geometrical axis at an angle β.

As a result of deviation of the geometrical axis of the rotor from the axis of rotation and turn of the rotor around the geometrical axis, the product of inertia of the I_γ system emerges, which is characterized by the value σ.

At the uniform rotation of the rotor, the product of inertia of the rotor creates an additional inertial moment, which also impedes rotation of the rotor [6].

The rotor has the overall moment of inertia, which is equal to

$$I^{ob} = I_a + I_\gamma = I_a + \left(I_b - I_a\right) sin\gamma\ cos\gamma = \\ = I_a + \left(I_b - I_a\right)\frac{\sigma}{l_0} = \\ = I_a + \left(I_b - I_a\right)\frac{\sqrt{d^2 + b^2 + 2db\cos\beta}}{l_0} \quad (28)$$

The product of inertia impacts dynamics of the rotating rotor and at uniform rotation of the rotor creates the inertial moment M_{uz}^{γ}, which can be determined by the dependence [6]

$$M_{uz}^{\gamma} = I_\gamma \omega^2 = \left(I_b - I_a\right)\omega^2 sin\gamma\ cos\gamma = \\ = \left(I_b - I_a\right)\omega^2 \frac{\sigma}{l_0} = \\ = \left(I_b - I_a\right)\omega^2 \frac{\sqrt{d^2 + b^2 + 2db\cos\beta}}{l_0} \quad (29)$$

However, the uniform rotation of the rotor is impeded by a component of the centrifugal moment and, consequently, a component of the bending inertial moment impedes rotation – the rotational inertial moment, which depends on the ratio of displacement σ of the principal axis of inertia from the axis of rotation in the observation plane and distance between the observation plane and center of mass [6].

$$M_u^{\gamma} = M_{uz}^{\gamma} sin\gamma = \frac{\sigma}{l_0} M_{uz}^{\gamma} = \frac{\sigma}{l_0} I_\gamma \omega^2 = \\ = \left(I_b - I_a\right)\omega^2 \frac{d^2 + b^2 + 2db\cos\beta}{l_0^2} \quad (30)$$

Evidently, for overcoming the negative impact of the product of inertia at the uniform rotation of the rotor, it is required to apply a certain basic driving torque

$$M_{kp}^{0} = M_u^{\gamma} \quad (31)$$

Evidently, the rotor with the moment unbalance has the overall moment of inertia, which impacts rotation of the rotor around its axis.

$$I_{kp}^{ob} = I_a + I_\gamma\ sin\gamma = I_a + \left(I_b - I_a\right) sin^2\gamma\ cos\gamma = \\ = I_a + \left(I_b - I_a\right)\frac{d^2 + b^2 + 2db\cos\beta}{l_0^2} \quad (32)$$

The equation for determination of the basic driving torque in this case becomes

$$M_{kp}^{0} = \left[I_a + \left(I_b - I_a\right)\frac{d^2 + b^2 + 2db\cos\beta}{l_0^2}\right]\frac{d\omega}{dt} + \\ + \omega^2\left(I_b - I_a\right)\frac{d^2 + b^2 + 2db\cos\beta}{l_0^2} \quad (33)$$

Under the uniform rotation of the rotor, the equation becomes

$$M_{kp}^{0} = \omega^2\left(I_b - I_a\right)\frac{d^2 + b^2 + 2db\cos\beta}{l_0^2} \quad (34)$$

Under the uniform rotation of the rotor at overcritical velocities ($cos\,\beta = -1$), the equation of the basic driving torque becomes

$$M_{kp}^{0} = \omega^2\left(I_b - I_a\right)\frac{\left(d - b\right)^2}{l_0^2} \quad (35)$$

At overcritical velocities, self-centering of the rotor exhibits. At the overall self-alignment of the rotor, the basic driving torque tends to zero, if $d - b \to 0$ at an unlimited increase of the velocity.

8. Basic driving torque for rotation of solid rotor with flexible shaft, which principal axis of inertia is displaced and deviated from solid shaft axis.

In a general case, a shaft, if a rotor has static and moment unbalance, bends in the direction of unbalance action. At the same time, the planes of static and moment unbalance usually do not coincide. However, the principle of superposition allows definition of the basic driving torque

$$M_{kp}^{0} = \left(I_a + m\rho^2 + I_\gamma\ sin\gamma\right)\frac{d\omega}{dt} + \\ + \left(m\rho^2 + I_\gamma\ sin\gamma\right)\omega^2 \quad (36)$$

or

$$M_{kp}^{0} = \left[\begin{array}{l} I_a + m\left(a^2 + e^2 + 2ae\cos\alpha\right) + \\ + \left(I_b - I_a\right)\dfrac{d^2 + b^2 + 2db\cos\beta}{l_0^2} \end{array}\right]\frac{d\omega}{dt} + \\ + \left[\begin{array}{l} m\left(a^2 + e^2 + 2ae\cos\alpha\right) + \\ + \left(I_b - I_a\right)\dfrac{d^2 + b^2 + 2db\cos\beta}{l_0^2} \end{array}\right]\omega^2 \quad (37)$$

9. Overall driving torque of solid rotor.

The overall driving torque of a rotor with a flexible shaft consists of the basic driving torque and additional driving torque

$$M_{kp}^{n} = M_{kp}^{0} + M_{kp}^{d} \quad (38)$$

The additional driving torque M_{kp}^{d} was received for principal cases of possible kinds of unbalance of the rotor with a flexible shaft [5-7]. The additional driving torque depending on operational conditions can consist of the moment M_{kp}^{p}, which ensures a certain position of the rotor relative to the axis of rotation, moment M_{kp}^{c} ensuring overcome of dissipative forces and loading moment M_{kp}^{z}, etc.

$$M_{kp}^{d} = M_{kp}^{p} + M_{kp}^{c} + M_{kp}^{z} + \ldots \tag{39}$$

For a case of a solid rotor with an absolutely solid valve, the overall driving torque has

$$M_{kp}^{n} = M_{kp}^{0} + M_{kp}^{c} + M_{kp}^{z} + \ldots \tag{40}$$

For the rotor with a flexible shaft in case of static unbalance and parallel displacement of the geometrical axis from the rotational axis in the absence of resistance forces

$$\begin{aligned} M_{kp}^{n} &= \left(I_a + ma^2 + me^2 + 2maeCos\alpha\right)\frac{d\omega}{dt} + \\ &+ m\omega^2\left(2a^2 + e^2 + 2aeCos\alpha\right) \end{aligned} \tag{41}$$

For the rotor with a flexible shaft in case of static unbalance and parallel displacement of the geometrical axis from the rotational axis in the presence of resistance forces proportional to the square of velocity

$$\begin{aligned} M_{kp}^{n} &= \left(I_a + ma^2 + me^2 + 2maeCos\alpha\right)\frac{d\omega}{dt} + \\ &+ m\omega^2\left(2a^2 + e^2 + 2aeCos\alpha\right) + \mu\omega^2 \end{aligned} \tag{42}$$

For the rotor with a flexible shaft in case of static unbalance and angular deflection of the geometrical axis from the rotational axis in the absence of resistance forces

$$\begin{aligned} M_{kp}^{0} &= \left[I_a + \left(I_b - I_a\right)\frac{d^2 + b^2 + 2db\cos\beta}{l_0^2}\right]\frac{d\omega}{dt} + \\ &+ \omega^2\left(I_b - I_a\right)\frac{2d^2 + b^2 + 2db\cos\beta}{l_0^2} \end{aligned} \tag{43}$$

10. Discussion and conclusions

The conducted studies show that the flexible-rotor-shaft –supports' system is a system with alternating moments of inertia and alternating products of inertia, which depend upon the rotational velocity of the rotor. Accordingly, inertial moments emerge in the system depending on the velocity, and which impede rotation of the rotor. The obtained equations are those, which allow determination of the basic driving torque in every specific case. The overall driving torque is defined as a sum of driving torques required for securing rotation of the rotor and performance of the operational cycle.

The results of these studies may be used during engineering of machines and mechanisms.

A comparative analysis with the well-known experimental data [8] showing significant losses of the driving gear's power confirms accuracy of the conducted study. The obtained equations confirm and explain exceeding of the driving torque at bending of the rotor's shaft over the driving torque sufficient for rotation of the rotor in the absence of shaft's flexure.

The equations that take account of the impact of friction forces in bearing assemblies and forces of aerodynamic resistance can be used as guiding equations in computer software for calculations of the required driving torque of the driving gear or processing of the results of experiments.

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