recommended background capabilities

general aptitude: recognising patterns, and interpolation and extrapolation

This section starts at topic {M2. Fractions and Decimal Notation}, and the absence of questions or problems associated with {M0. Patterns} and {M1. Arithmetic} is not an error.

M2. Fractions and Decimal Notation

How to convert the recurring decimal 0.545454<u>54</u>... (where the underlined digits repeat indefinitely) to a fraction.

First, we multiply the original number by 100, which gives us 54.5454...

Then we subtract the original number, leaving us with 99 times the original number:

$$54.545454... - 0.545454... = 54$$

That means the original number must be the fraction 54 divided by 99.

$$0.545454... = {}^{54}/_{99} = {}^{18}/_{33} = {}^{6}/_{11}$$

\# 1. Convert the recurring decimal 0.108108<u>108</u>... (where the underlined digits repeat indefinitely) to a fraction.

Confidence: 0 | 1 | 2 | 3 | 4

M3. Variables and Algebra

1. We can use letters to represent numbers that we have not established the value of yet. For example, if we multiply something by 5 to obtain 40, we can write x for "something", and write $x \times 5 = 40$.

   To establish ("solve for") the value for x, we work backwards: if both sides of the equation are equal, then they will remain equal after we perform the same operation on both of them.

   In this case we divide both sides by 5, and find that $x = 8$.

   Notice that instead of writing the multiplication sign, we can instead just write the number in front of the x. The multiplying number is called a "coefficient" when written this way.

$$\begin{aligned} x \times 5 &= 40 \\ 5x &= 40 \\ (5x) \div 5 &= (40) \div 5 \\ x &= 8 \end{aligned}$$

\# 1. Find the value of x that would make the equation below true.

$$4 + x = 13$$

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. Find the value of y that would make the equation below true.

$$3y - 2 = 7$$

Confidence: 0 | 1 | 2 | 3 | 4

2. When we add or subtract "terms", we can combine coefficients, if they are multiplying numbers for the same thing. A "term" refers simply to something that we add or subtract.

   The example below shows how to combine the coefficients in $5x + 2 + 2y - 2x + y - xy + 1$.

$$\begin{aligned} 5x + 2 + 2y - 2x + y - xy + 1 &= (5 - 2)x + (2 + 1)y - xy + (2 + 1) \\ &= 3x + 3y - xy + 3 \end{aligned}$$

\# 1. Subtract $2x$ from both sides to find the value of x that would make the equation below true.

$$4x + 3 = 2x + 9$$

Confidence: 0 | 1 | 2 | 3 | 4

3. The following pattern is called the "distributive law".

$$a(b + c) = ab + ac$$

\# 1. Apply the distributive law to the following.

$$7(x + 2) =$$

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. Apply the distributive law in reverse, to the following (this is often called "factorising").

$$3y - 6 =$$

Confidence: 0 | 1 | 2 | 3 | 4

\# 3. Factorise (apply the distributive law in reverse to) the following, in two steps.

$$xy - 2x + 3y - 6 =$$

Confidence: 0 | 1 | 2 | 3 | 4

4. We can consider groups of equations that we need to solve as a group (at the same time, "simultaneously").

$$\begin{aligned} 3x + y &= 10 \\ x + 2y &= 0 \end{aligned}$$

   To do this, we need to isolate x, or y. We can rearrange the equations using the tricks from previously, but we can also combine the equations by adding or subtracting them to or from each other. For example, subtracting the second equation from the first, up to 3 times, gives the following new equations.

$$\begin{aligned} 2x - y &= 10 \\ x - 3y &= 10 \\ -5y &= 10 \end{aligned}$$

   Now we have isolated y from x, we can solve for y, then use that value of y to solve for x.

$$\begin{aligned} y &= -2 \\ x + 2y &= 0 \\ x &= 4 \end{aligned}$$

\# 1. Solve the following simultaneous equations.

$$\begin{aligned} 3x + 2y &= 20 \\ 2x + 5y &= 39 \end{aligned}$$

Confidence: 0 | 1 | 2 | 3 | 4

5. "Quadratic" expressions contain terms that are "squared", in the following case, x multiplied by itself. Instead of writing xx, we usually write a superscript 2, as shown.

$$xx + 2x + 3 = x^2 + 2x + 3$$

It is relatively straightforward to solve for x in quadratic equations, and we will not go into too much detail here, but the resulting pattern is as follows. If we have

$$ax^2 + bx + c = 0$$

, then the solutions are given by the "quadratic formula" below.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

\# 1. Solve for p.

$$2p^2 + 5x - 12 = 0$$

Confidence: 0 | 1 | 2 | 3 | 4

M4. Representation and Notation

1. When we use numbers to describe the real world, there is a limit to how precisely our descriptions match the reality. We should avoid giving more detail than we are sure of, because doing so would falsely imply confidence in the details we give.

Mantissa-exponent notation is particularly advantageous for us in two ways: it lets us express large or small numbers in a concise form, and it lets us easily show just how many digits of our number we are sure of.

For example, instead of 0.00000345, we can write 3.45×10^{-6}, or even 3.45e–6.

The "3.45" is the mantissa; the "–6" is the exponent.

Writing numbers in this form allows us to quickly identify the "most significant" digit, which in this case is the 3. All the zeroes to the left of the 3 are not significant. Called "leading" zeroes, they serve only to move the decimal point away and thus adjust the place value of the actual significant digits. In mantissa-exponent notation, it is instead the exponent that performs this function.

This example has 3 significant figures, the 3 digits of the mantissa. Any zeroes in the mantissa (in between other digits, or to the right as "trailing" zeroes) will also be considered significant.

\# 1. How many significant figures are there in 00028900.0?

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. How many significant figures are there in 400?

Confidence: 0 | 1 | 2 | 3 | 4

\# 3. When we multiply or divide numbers from the real world, we should retain the least number of significant figures of any of the numbers that we multiplied or divided.

Calculate 6.0×1.51 and express the answer with the appropriate number of significant figures.

Confidence: 0 | 1 | 2 | 3 | 4

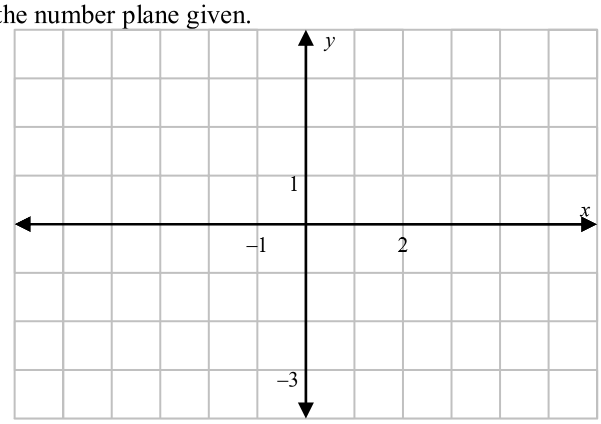
\# 4. When we add numbers from the real world, we need to line them up at the decimal point, and this we do not go by significant figures, but by the largest decimal places we are sure of.

Calculate $2.2 + 1.55 + 0.8 + 9.0 + 0.631$ and express the answer to the correct significance.

Confidence: 0 | 1 | 2 | 3 | 4

2. Plot the points listed below, on the number plane given.

(x,	y)
(0,	3)
(2,	2)
(–2,	1)
(5,	–1)
(0,	0)



Confidence: 0 | 1 | 2 | 3 | 4

M5. Number Theory and Set Theory

1. A counting number, or "natural number", *N* is "divisible" by another natural number *D*, if *N* divided by *D* leaves no remainder. If *N* is divisible by *D*, then we call *D* a "factor" of *N*.

   If *N* is divisible only by two different numbers 1 and *N*, then we call *N* a prime number.

   The first six prime numbers, in order, are 2, 3, 5, 7, 11, 13.

   All natural numbers are either prime numbers, or a unique product of prime numbers ("product" in mathematics means the result of multiplying things together). These other numbers are called "composite".

   This means that for any natural number, we can write a unique "prime factorisation" for that number: simply a list of all the prime numbers we need to multiply together, to obtain that number.

   For example, $24 = 2 \times 2 \times 2 \times 3$. We can use "power notation" to write this more concisely, by writing the number of times a given factor appears, as a superscript. In this way, we would write $24 = 2^3 \times 3^1$.

\# 1. We can find the prime factorisation of a number by repeatedly dividing by each prime number in turn, until the quotient (result of the division) is no longer divisible by that prime number.

Express 3600 in terms of its prime factors.

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. We can calculate the total number of factors (prime or composite) that a number has, by looking at the powers of the prime factors. Any factor of a number N must contain only prime factors that N has, or else N would not be divisible by the extra prime factor.

We know that 24 has 8 factors, because $24 = 2^3 \times 3^1$. This means that to find a factor of 24, we can include 0, 1, 2, or 3 instances of 2, and independently include 0 or 1 instances of 3. We have 4 options for how many 2s to include, and 2 options for how many 3s, and being independent choices, we multiply 4 by 2 to find the total number of choices.

Therefore, to find the number of factors of N, we add 1 to the power of each prime number, and take the product of all these incremented powers.

How many factors does 3600 have, in total?

Confidence: 0 | 1 | 2 | 3 | 4

2. We can define collections of objects, and call them "sets".

For example, the set of natural numbers less than or equal to 10 would be $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

The cardinality of a set is the number of objects ("elements") in the set. For the set above, that would be 10.

For this task, we define two sets: D_2, natural numbers less than or equal to 10 that are also divisible by 2; and D_3, natural numbers less than or equal to 10 that are also divisible by 3.

\# 1. The "union" of sets is the combination of all elements in any of the sets, without repeats.

What is the cardinality of $D_2 \cup D_3$, where $\cup$ is the set union operator?

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. The "intersection" of sets is the collection of all elements that are in every one of the sets.

What is the cardinality of $D_2 \cap D_3$, where $\cap$ is the set intersection operator?

Confidence: 0 | 1 | 2 | 3 | 4

M6. Exponentials and Logarithms

We have previously used powers as a shorthand way to express many of the same number multiplied together.

We can apply this more generally and call it "exponential notation". We can extend the meaning of the power, as a number of things to multiply together, to fractional powers (which becomes taking power-"roots") and to negative powers (which mean division).

1. When we multiply powers of something, we add the powers together. When we divide powers of something, we subtract one power from the other.

$$\begin{aligned} x^p \times x^q &= x^p x^q \\ &= x^{p+q} \\ x^p \div x^q &= x^{p-q} \end{aligned}$$

When we raise a power of something, to another power, we multiply the powers together.

$$(x^p)^q = x^{pq}$$

\# 1. The number 128 is a simple power of 2 (it is 2 multiplied by itself a certain number of times). What power of 2 is 128 (how many factors of 2 does 128 have)?

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. Hence calculate $2^{10} \div 128$ (avoid using an electronic calculating aid).

Confidence: 0 | 1 | 2 | 3 | 4

\# 3. Calculate $4^{11} \div 8^8$ (avoid using an electronic calculating aid).

Confidence: 0 | 1 | 2 | 3 | 4

2. The square of a number is that number multiplied by itself, making it the 2nd power of that number.

$$x \times x = x^2$$

Inversely, the square root of a number is the number that when squared, becomes the original number. The square root of the original number is the original number to the power of one half.

$$\begin{aligned} \sqrt{x} \times \sqrt{x} &= x \\ \sqrt{x} &= x^{\frac{1}{2}} \end{aligned}$$

Fractional powers are inconvenient to have on the bottom of fractions (as denominators), often because they make it difficult to adjust terms to have a common denominator. To resolve this problem, we can “rationalise” denominators like in the example below.

$$\begin{aligned} \frac{1}{5+\sqrt{3}} &= \frac{1}{5+\sqrt{3}} \frac{5-\sqrt{3}}{5-\sqrt{3}} \\ &= \frac{5-\sqrt{3}}{25+5\sqrt{3}-5\sqrt{3}-3} \\ &= \frac{5-\sqrt{3}}{22} \end{aligned}$$

\# 1. Rationalise the denominator of the fraction below.

$$\frac{\sqrt{5}+1}{3-\sqrt{5}} =$$

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. If, instead of the square (2^{nd}) root, we have a different root, we write which root it is on the left of the "radical" marker. Hence $\sqrt[3]{8}$ means the 3^{rd} root (cube root) of 8, which is 2.

Rationalise the denominator of the fraction below. Hint: $a^3 - b^3 = (a - b)(a^2 - ab + b^2)$.

$$\frac{1}{3-\sqrt[3]{5}} =$$

Confidence: 0 | 1 | 2 | 3 | 4

3. Finding the "logarithm" is finding the power to which a number is raised. For example, the logarithm (in base 3) of 81 is 4. We can write this as below.

$$3^4 = 81$$

$$\log_3 81 = 4$$

Combining this with the rules that apply to powers, we obtain new rules in terms of logarithms.

When we multiply numbers together, we add their logarithms (in the same base).

$$\log_a xy = \log_a x + \log_a y$$

When we raise a number to a power, we multiply its logarithm by that power.

$$\log_a x^p = p\log_a x$$

To find the logarithm of a number in a certain base, we can divide the logarithm of the number in some other base, by the logarithm (in that other base) of our target base.

$$\log_b x = \log_a x \div \log_a b$$

\# 1. Calculate $\log_3 360 - \log_3 1000 + \log_3 25$.

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. Calculate $\log_8 16 - \log_{16} 8$.

Confidence: 0 | 1 | 2 | 3 | 4

M7. Iteration and Recursion

1. "Halflife" refers to how long it takes for something to lose half its value. Consider an exponential in x, such as 2^{-x}. Every time x increases by 1, the value of 2^{-x} halves, so the halflife in this case is 1 unit (of x).

\# 1. What is the halflife of 3^{-x}?

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. A particular exponential in x has a halflife of 0.6 units. For the same exponential in x, how many units (of x) would correspond to a drop to $^1/_{10}$ in value?

Confidence: 0 | 1 | 2 | 3 | 4

2. Consider the following infinite series. We shall call it S for convenience.

$$\begin{aligned} S &= \sum_{n=0}^{\infty}\left(-\frac{3}{4}\right)^n \\ &= 1-\frac{3}{4}+\frac{9}{16}-\frac{27}{64}+\cdots \end{aligned}$$

\# 1. Multiply S by $-\frac{3}{4}$, and write the first few terms, like above.

$$\left(-\frac{3}{4}\right)S \quad =$$

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. Subtract this from S.

$$S-\left(-\frac{3}{4}\right)S \quad =$$

Confidence: 0 | 1 | 2 | 3 | 4

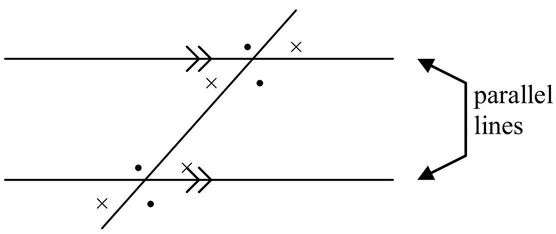
\# 3. Factorise S out, and hence determine the value of S.

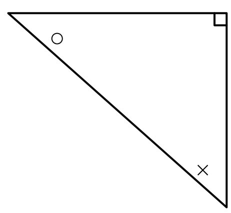
$$S\left(1+\frac{3}{4}\right) \quad =$$

Confidence: 0 | 1 | 2 | 3 | 4

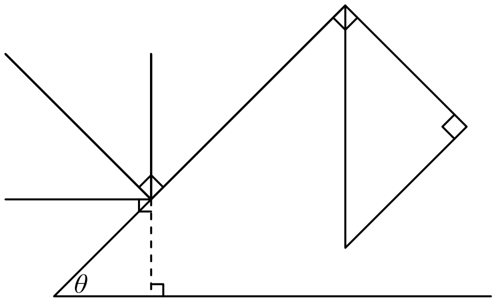
M8. Plane and Solid Geometry

The following diagrams show cases where angles match, or are “complementary” (add to 90°) or “supplementary” (add to 180°). Any angles marked with the same symbol should be the same size as each other. Angles marked × are supplementary to angles marked •, and angles marked ○ are complementary to angles marked •. A right angle (of exactly 90°) is marked as a square. The sum of the angles within a triangle is 180°.





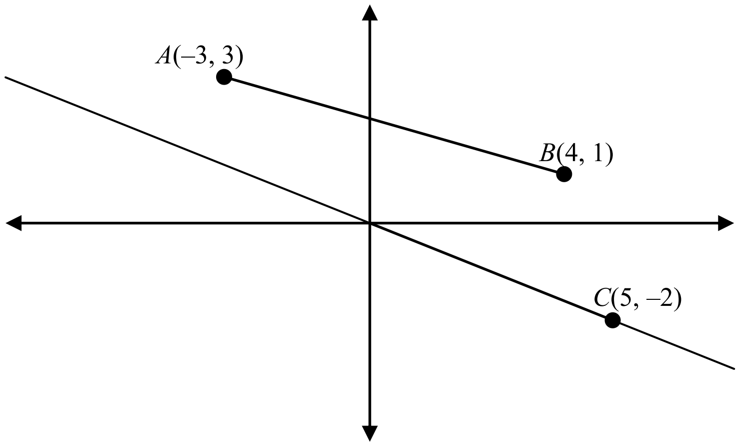
\# 1. Use the θ symbol to mark all angles that must be the same size as the one already marked θ, in this diagram.



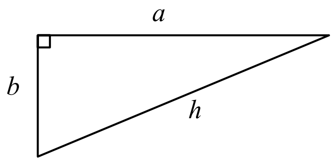
Confidence: 0 | 1 | 2 | 3 | 4

M9. Coordinate Geometry

Here is a coordinate plane in two dimensions (which are described by two variables x and y). It has some points defined on it, which we will use for the following problems.



\# 1. The distance between two points on the coordinate plane is the length of the straight line between them. We can compute this length from the difference in x coordinates and the difference in y coordinates, and matching the corresponding lengths to the sides of a right-angled triangle. There is a rule governing the side lengths of such triangles: the square of the hypotenuse equals the sum of the squares of the other sides.



$$h^2 = a^2 + b^2$$

Calculate the distance between points A and B.

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. The gradient between two points, or of a line passing through two points, is the difference in y coordinates divided by the difference in x coordinates in the same order. Calculate the gradient between points A and B.

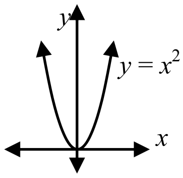
Confidence: 0 | 1 | 2 | 3 | 4

\# 3. Calculate the gradient of the line which is already drawn on the plot passing through point C.

Confidence: 0 | 1 | 2 | 3 | 4

M10. Calculus

We can plot curves on the coordinate plane as well. If the curve corresponds to a function or relation (such as $y = x^2$), then what we draw is a curve that represents every point where the y value is equal to the square of the x value (in this case, a parabola).



Curves have different gradients at different points, and in the parabola above we can see that to the left of the y axis, the parabola moves downwards to the right, but less and less as we proceed to the right. On the left side, this corresponds to a negative gradient that is less and less negative, until reaching the y axis, when the curve is horizontal (and hence has a gradient of 0). To the right of the y axis, the curve moves upwards to the right, and more and more so: it now has a positive gradient that is increasing.

Differential calculus allows us to work out the values of the gradients. For example, we can show that the gradient of $y = x^2$ is $y' = 2x$, which fits with the description above: to the left of the y axis, $x < 0$ and the gradient is negative but becoming closer to 0; at the y axis, the gradient is exactly 0; and to the right of the y axis, the gradient is positive and continues to increase.

Some of the more common patterns in differential calculus are in the table below.

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
C, constant	0	$u(x) + v(x)$	$u'(x) + v'(x)$
x^n	nx^{n-1}	$Cg(x)$	$Cg'(x)$
e^x	e^x	$\ln(x)$	$\frac{1}{x}$
$\sin(x)$	$\cos(x)$	$\cos(x)$	$-\sin(x)$

1. Applying these rules, differentiate the following (for each of the following cases of $f(x)$, determine $f'(x)$).

1. $f(x) = x^4$

$$f'(x) \quad =$$

Confidence: 0 | 1 | 2 | 3 | 4

2. $f(x) = 3x^4 + x$

Confidence: 0 | 1 | 2 | 3 | 4

2. The product rule applies when differentiating two "functions of x", say $f(x)$ and $g(x)$, multiplied together.

$$\begin{aligned}\tfrac{d}{dx} f(x)g(x) &= (f(x)g(x))' \\ &= f'(x)g(x) + f(x)g'(x)\end{aligned}$$

In effect, to differentiate a product of two functions, we make two copies of the product, differentiate a different factor of each copy, and then add the copies together.

1. Use the product rule to differentiate $e^x\sin(x)$.

Confidence: 0 | 1 | 2 | 3 | 4

2. Extend the product rule to the differentiation of $f(x)g(x)h(x)$.

Confidence: 0 | 1 | 2 | 3 | 4

3. The chain rule applies when differentiating a function of a function of x, with a pattern $u(v(x))$.

$$\tfrac{d}{dx}\, u(v(x)) \quad = \quad u'(v(x))\, v'(x)$$

We differentiate u "with respect to" v, then we differentiate v with respect to x, and then we multiply these results together to get the final result. For example, we can differentiate $\cos(x^3)$ as follows.

$$\begin{aligned} \tfrac{d}{dx}\cos(x^3) &= -\sin(x^3)\, 3x^2 \\ &= -3x^2\sin(x^3) \end{aligned}$$

\# 1. Differentiate $\sin(2e^x)$.

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. Differentiate $(1 + \ln(x))^3$.

Confidence: 0 | 1 | 2 | 3 | 4

\# 3. Differentiate $\cos(x \ln(x))$.

Confidence: 0 | 1 | 2 | 3 | 4

4. Integration, or integral calculus, involves finding the area under a function curve. The key to integration is that the function acts as the gradient of the area. That is, if we were to differentiate the area under the function curve, we would get the function.

Say we know that differentiating $y = x^2$ gives us $y' = 2x$. Then we can perform the integration below.

$$\int 2x\, dx \quad = \quad x^2 + \text{constant}$$

The extra arbitrary constant term arises because a constant differentiates to zero. It also reflects that this is an "indefinite" integral, where we have not specified exactly where we are measuring the area from.

\# 1. Integrate x.

$$\int x\, dx \quad =$$

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. Integrate $-2\cos(x)$.

$$\int -2\cos(x)\, dx \quad =$$

Confidence: 0 | 1 | 2 | 3 | 4

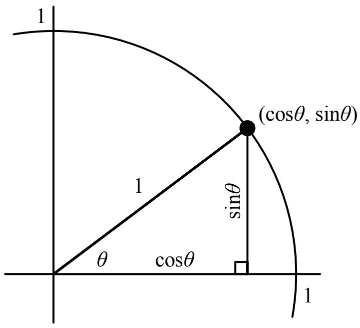
\# 3. Integrate $4x^3\cos(x^4)$.

$$\int 4x^3\cos\left(x^4\right) dx \quad =$$

Confidence: 0 | 1 | 2 | 3 | 4

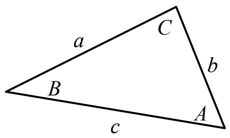
M11. Trigonometry

Trigonometry, of course, comes from the unit circle. The diagram below shows the definition of sine and cosine, as $\sin\theta$ and $\cos\theta$ respectively.



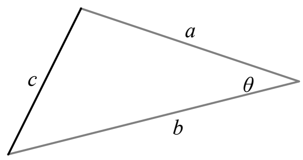
$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

The "sine rule" relates side lengths of a triangle to the sines of the opposite angles.



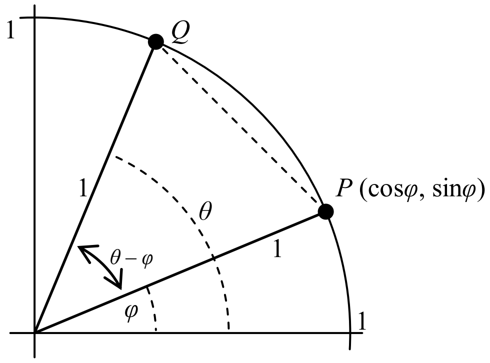
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The "cosine rule" enables us to find the length of a third side of a triangle, if we know the length of two sides and the angle between them.



$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

1. The following diagram shows the difference between two angles, φ and θ. These angles mark out two points on the circle, respectively P and Q.



\# 1. Express the distance between P and Q in terms of $\cos\varphi$, $\sin\varphi$, $\cos\theta$ and $\sin\theta$, using the coordinates of P and Q, and the method of calculating distance from coordinate geometry.

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. Use the cosine rule to express the distance between P and Q in terms of $\cos(\theta - \varphi)$. Remember that this is all on the unit circle, so the other sides of the relevant triangle are 1 unit in length.

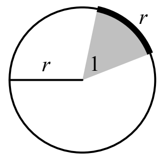
Confidence: 0 | 1 | 2 | 3 | 4

\# 3. The above distances are the length of the same interval (between P and Q), so they must be equal. Matching them in this way, and remembering that $(\cos\theta)^2 + (\sin\theta)^2 = 1$, express $\cos(\theta - \varphi)$ in terms of $\cos\varphi$, $\sin\varphi$, $\cos\theta$ and $\sin\theta$.

Confidence: 0 | 1 | 2 | 3 | 4

2. Common convention defines a complete "revolution", or the angle involved in moving all the way around a circle, as 360°. A more natural definition is the "radian", where we take the radius of the circle, and wrap it around the circumference.

The angle at the centre of the circle, between ends of the wrap-around radius, is 1 radian.



A radius is half of a diameter (which cuts straight across a circle, through its centre), and π is the ratio of circumference to diameter, so we know that we can fit 2π radii around the circumference of a circle.

\# 1. How many degrees corresponds to 1 radian? (Keep the answer as a fraction and in terms of π.)

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. Fill in the table.

degrees	0°	30°	45°	60°	90°
radians	0				

Confidence: 0 | 1 | 2 | 3 | 4

This section ends at topic {M13. Complex Numbers}, and the absence of questions or problems associated with {12. Vectors and Matrices} is not an error.

M13. Complex Numbers

Complex numbers are simple. They are just pairs of "normal" numbers, with the second number in the pair multiplied by i, which has a special property.

$$i^2 = -1$$

We can write complex numbers like $a + ib$, and call a the "real" part, and b the "imaginary" part.

When we add or subtract complex numbers, we add the real and the imaginary parts separately.

$$\begin{aligned}(2 + 3i) + (4 + 5i) &= (2 + 4) + (3 + 5)i \\ &= 6 + 8i\end{aligned}$$

When we multiply complex numbers, we need to invoke the special property of i.

$$\begin{aligned}(2 + 3i) \times (4 + 5i) &= 8 + 10i + 12i + 15i^2 \\ &= 8 + 22i + 15(-1) \\ &= -7 + 22i\end{aligned}$$

The "conjugate" of a complex number is almost the same number, but with an imaginary component that has the opposite sign (positive or negative). For us here, it is useful because when we multiply a complex number by its conjugate, we get just a real number and no imaginary component.

$$\begin{aligned}(2 + 3i) \times (2 - 3i) &= 4 - 6i + 6i - 9i^2 \\ &= 4 - 9(-1) \\ &= 13\end{aligned}$$

\# 1. We can use a trick to divide complex numbers, such as dividing $(1 + i)$ by $(3 - 4i)$. Just multiply the complex number fraction by the conjugate of $(3 - 4i)$, like when we were rationalising denominators.

$$\begin{aligned}\frac{1+i}{3-4i} &= \frac{1+i}{3-4i}\frac{3+4i}{3+4i} \\ &= \end{aligned}$$

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. An elegant and famous equation in mathematics seems quite challenging for Year 12 students, but we should be able to handle it in Year 7 with a little guidance from the right teachers.

\# 1. Differentiate $y = \cos(x) + i\sin(x)$ with respect to x. (Hint: i is just a constant.)

$$y' =$$

Confidence: 0 | 1 | 2 | 3 | 4

\# 2. Some parts of the result will be negative, which makes them just something multiplied by -1. Instead of -1, write these as something multiplied by i^2.

$$y' =$$

Confidence: 0 | 1 | 2 | 3 | 4

\# 3. Show that if we factorise out one i, the other part will turn out to be the y we defined above.

$$y' =$$

Confidence: 0 | 1 | 2 | 3 | 4

\# 4. Differentiate $z = e^{ix}$ with respect to x. (Hint: remember, i is just a constant, and use the chain rule.)

$$z' =$$

Confidence: 0 | 1 | 2 | 3 | 4

\# 5. It seems like we have both $y' = iy$ and $z' = iz$, so they both behave in exactly the same way. What conclusion can we make about $\cos\theta + i\sin\theta$ and $e^{i\theta}$?

$$=$$

Confidence: 0 | 1 | 2 | 3 | 4

END