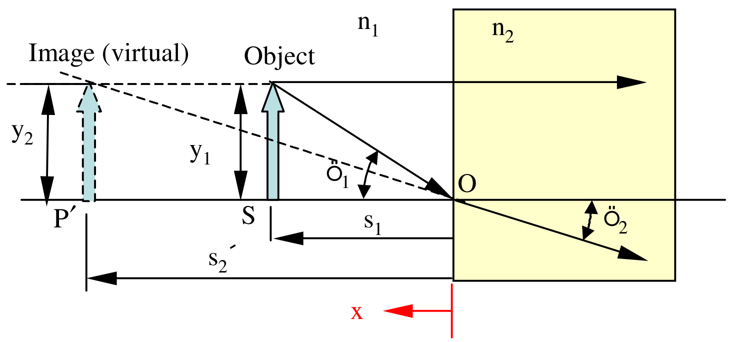
Refraction

Refraction at a plane surface



According to Snell's law,

$$n_1 \sin\phi_1 = n_2 \sin\phi_2$$

$$\tan\phi_1 = \frac{y_1}{s_1}$$

As the image is formed on the rhs of the refracting surface,
i.e. on the negative x axis,

$$\tan\phi_2 = \frac{y_2}{-s_2'}$$

When ϕ_1 is small, ϕ_2 is also small. Hence,

$$\tan\phi_1 \approx \sin\phi_1 \text{ and } \tan\phi_2 \approx \sin\phi_2$$

$$n_1\frac{y_1}{s_1} = -n_2\frac{y_2}{s_2'}$$

Example

Let the object height y_1 be 1 mm and s_1 be 10 mm.
The rays pass from air ($n_1 = 1$) to the plain glass with $n_2 = 1.5$

$$1\frac{1}{10} = -\ 1.5\frac{y_2}{s_2'}$$

For plane surface, $y_2 = y_1$. Therefore,

$$s_2' = -1.5\frac{10}{1} = -15\,mm$$

Paraxial rays

Rays that arrive at a shallow angle with respect to the optical axis are known as paraxial rays.

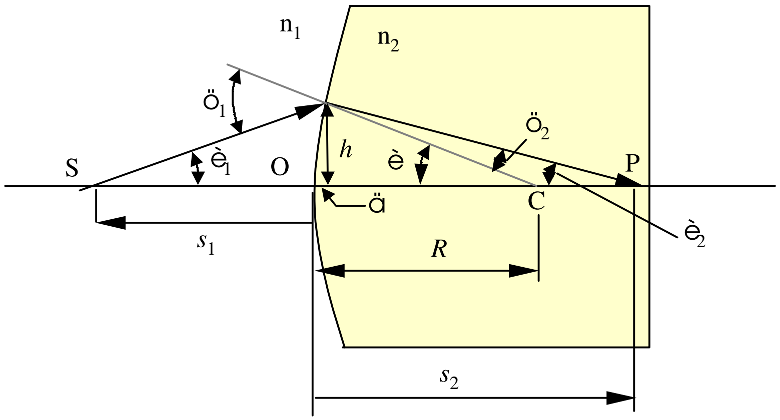
Sign Convention for refracting surfaces

s_1	+ if S is left of O
s_2	+ if P is right of O
R	+ if C is right of O

R is the radius of curvature centered at C.
For plane surface, $R \to \infty$

Spherical Surfaces

Refraction at a spherical surface



R is the radius of curvature centered at C.
O is vertex of the surface.
s_1 is the object distance (OS) . S is a point source.
The axis through OS is the optical axis.
s_2 is the image distance (OP).
P is the point at which the ray crosses the optical axis.

From the diagram,

$$\phi_1 = \theta + \theta_1, \;\; \theta = \phi_2 + \theta_2 \qquad (1)$$

$$\tan\theta = \frac{h}{R-\delta}, \; \tan\theta_1 = \frac{h}{s_1+\delta} \text{ and } \tan\theta_2 = \frac{h}{s_2-\delta} \qquad (2)$$

For paraxial rays, äcan be neglected.

According to Snell's law,

$$n_1 \sin\phi_1 = n_2 \sin\phi_2 \qquad (3)$$

If we assume small values $Ö_1$ and $Ö_2$, (3) becomes,

$$n_1 \phi_1 \approx n_2 \phi_2 \qquad (4)$$

Combining equations (1) and (4) we get,

$$n_1 \theta_1 + n_2 \theta_2 = (n_2 - n_1)\theta$$

Using small angle approximations and (2), we get,

$$\boxed{\frac{n_1}{s_1} + \frac{n_2}{s_2} = \frac{n_2 - n_1}{R}} \qquad (5)$$

The equation (5) is known as first order, paraxial or Gaussian Optics which is the basic theoretical tool for lens design.
The deviation from that of paraxial analysis provides the measure for the quality of an actual optical device

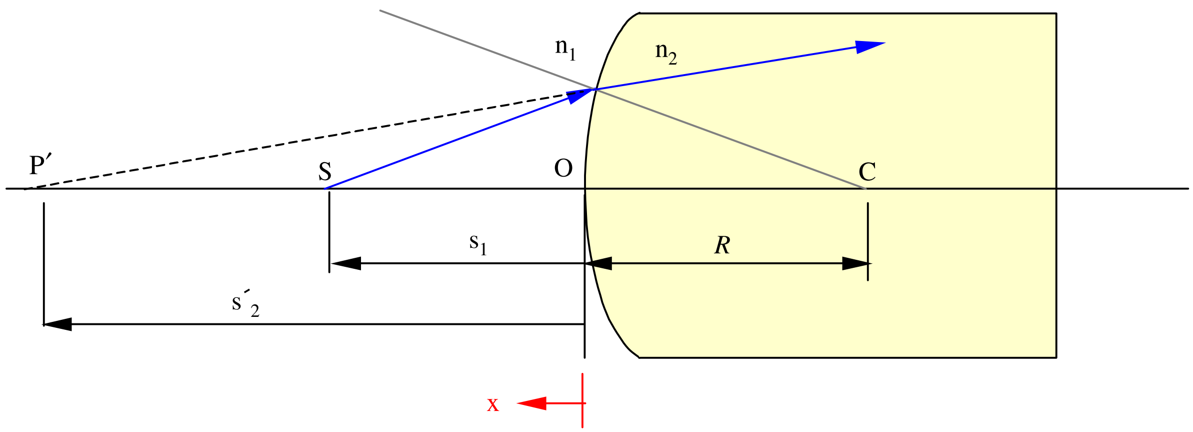
Example

Let R = +20 mm, n_1= 1, n_2 = 1.5, and s_1 = + 80 mm

$$\frac{1}{80} + \frac{1.5}{s_2} = \frac{1.5-1}{20}$$

$$s_2 = +120\,mm$$

Spherical Surfaces



The first order equation for the spherical surface with radius of curvature R centered at C,

$$\frac{n_1}{s_1} + \frac{n_2}{-s_2'} = \frac{n_2 - n_1}{R}$$

where,

s_1 is the object distance (OS) . S is a point source.
s_2´ is the image distance (OP´). The virtual image is formed at P´.
s_2´ is negative, as the real image is formed on the rhs of the refracting surface, i.e. on the negative x axis.

Lens Equation

Equation for the first lens surface (with center at C_1),

$$\frac{n_1}{s_{11}} + \frac{n_2}{-s'_{21}} = \frac{n_2 - n_1}{R_1} \qquad (4)$$

Virtual image at P´ acts as the object for image at P.

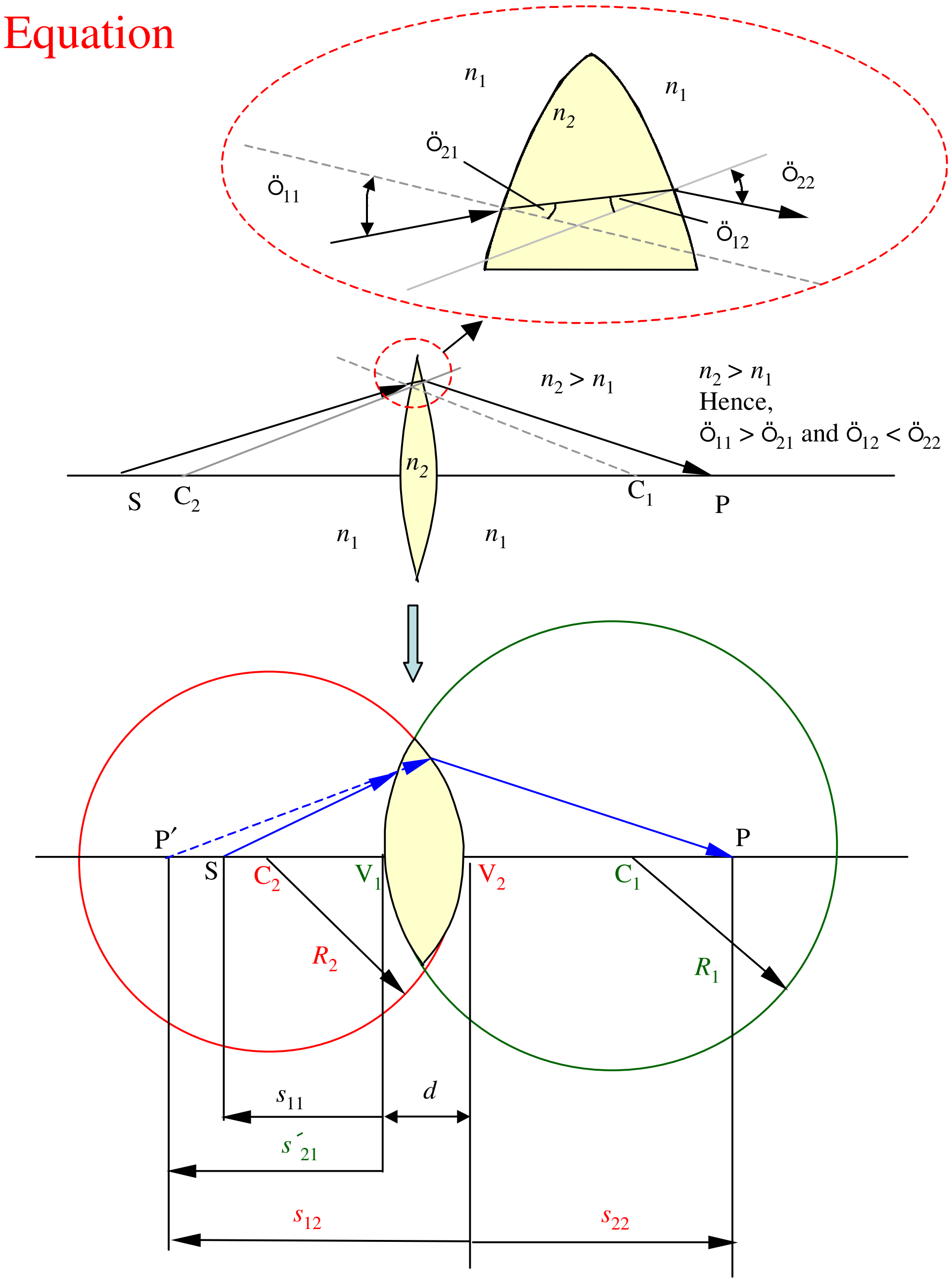
$$s_{12} = |s'_{21}| + d$$

s_{21}´ is negative as the image is formed on the negative axis. Also R_2 is negative for concave lens. The equation for the second lens surface (with center C_2)

$$\frac{n_2}{s'_{21} + d} + \frac{n_1}{s_{22}} = -\frac{n_2 - n_1}{R_2} \qquad (5)$$

Adding equations (4) and (5),. we get,

$$\frac{n_1}{s_{11}} + \frac{n_1}{s_{22}} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{n_2\, d}{(s'_{21} + d)s'_{21}}$$



Thin Lens Equation or Lens Maker's Formula

When the medium is air and d is negligible,

$$\frac{1}{s_{11}} + \frac{1}{s_{22}} = (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

As V_1 and V_2 tend to coalesce as d ◊ 0, $s_{11} = s_1$ and $s_{22} = s_2$.

By definition,

$$\lim_{s_2 \to \infty} s_1 \equiv f_1 \text{ and } \lim_{s_1 \to \infty} s_2 \equiv f_2$$

where f_1 and f_2 are object and image focal lengths respectively.

For a thin lens, $f_1 = f_2$. Let $f_1 = f_2 = f$. Substituting we get,

$$\frac{1}{f} = (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{where,} \quad \frac{1}{f} = \frac{1}{s_1} + \frac{1}{s_2}$$

Gaussian Lens Formula

Focal Length (Gaussian lens Formula)

$$\frac{1}{f} = (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

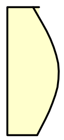
Also

$$\frac{1}{f} = \frac{1}{s_1} + \frac{1}{s_2}$$

Example

Compute the focal length in air of a thin planar-convex lens having a radius of curvature of 50 mm and an index of refraction 1.5

Planar Convex lens



$$R_1 = \infty$$
$$R_2 = -50\,mm$$

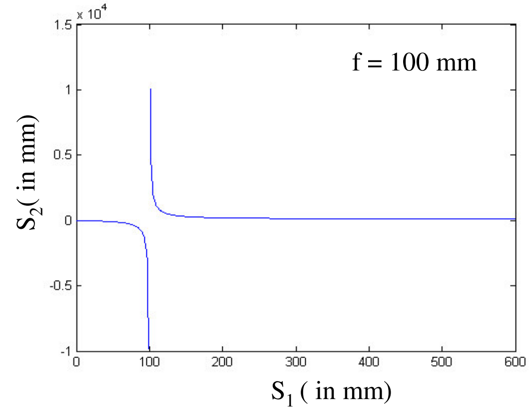
Focal length of thin lens,

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{-50}\right)$$
$$f = 100\,mm$$

If the object is placed at 600 mm from the lens (> f)

$$\frac{1}{100} = \frac{1}{600} + \frac{1}{s_2}$$
$$s_2 = 120\,mm$$

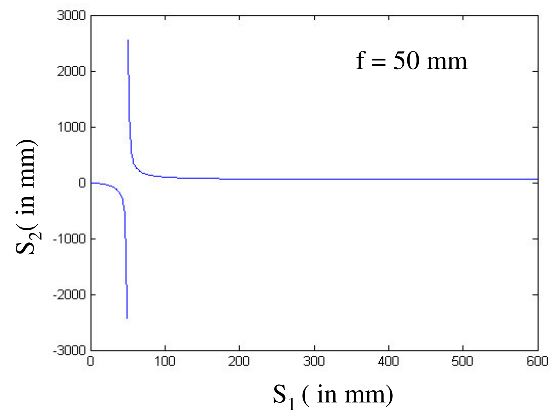
Image Distance, S_2 Vs Object Distance, S_1



where,

$$s_2 = \frac{f\, s_1}{(s_1 - f)}$$

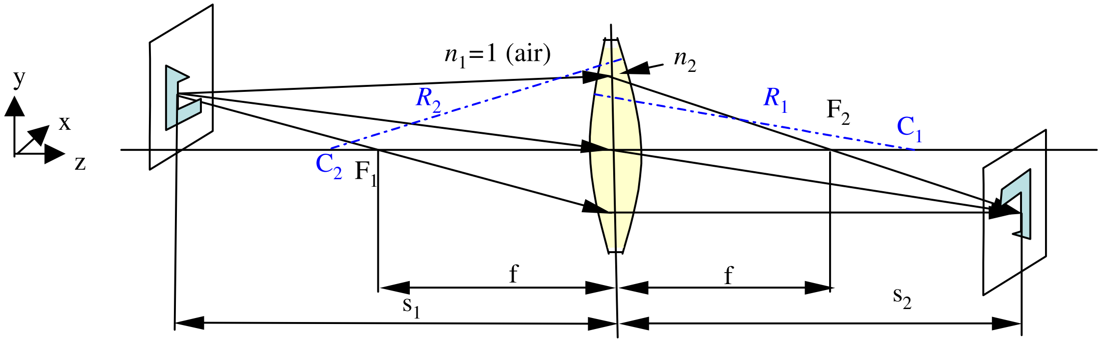
Similarly for Bi-Convex lens ($R_1 = R_2 = 50$ mm)



where,

$$s_2 = \frac{f\, s_1}{(s_1 - f)}$$

Object and Image Location for a Thin Lens



Gaussian Lens Formula

$$\frac{1}{f} = (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \frac{1}{s_1} + \frac{1}{s_2}$$

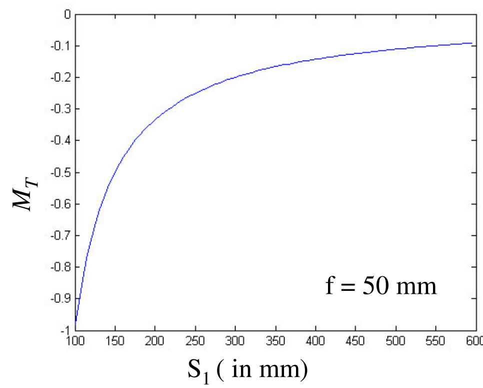
Transverse Magnification (in y axis)

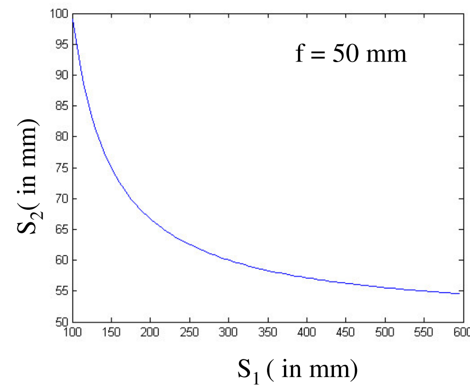
$$M_T = \frac{-s_2}{s_1}$$

Example

Consider Bi-Convex lens ($R_1 = R_2 = 50$ mm)

Case 1: • > s_1 > 2f





Resulting Image

Inverted	Minified

Image distance

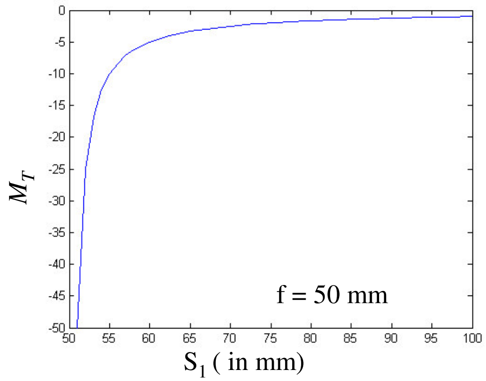
2f < s_2 < f

Object and Image Location for a Thin Lens

Example (contd …)

Bi-Convex lens ($R_1 = R_2 = 50$ mm)

Case 2: $f < s_1 < 2f$



Resulting Image	Inverted	magnified

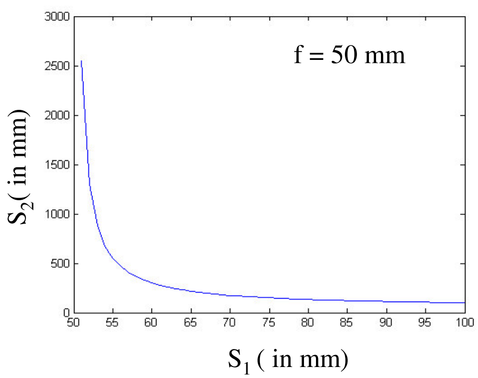


Image distance • > s_2 > 2f

Case 3: $s_1 = 2f = 100$ mm

Results

$s_2 = 2f = 100$ mm

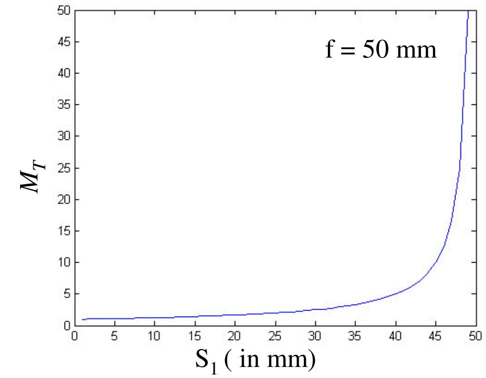
$M_T = -1$

Case 4: $s_1 = f = 50$ mm

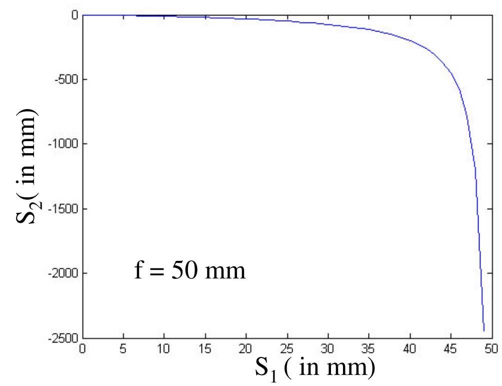
Results

$s_2 =$ •

Case 5: $s_1 < f$

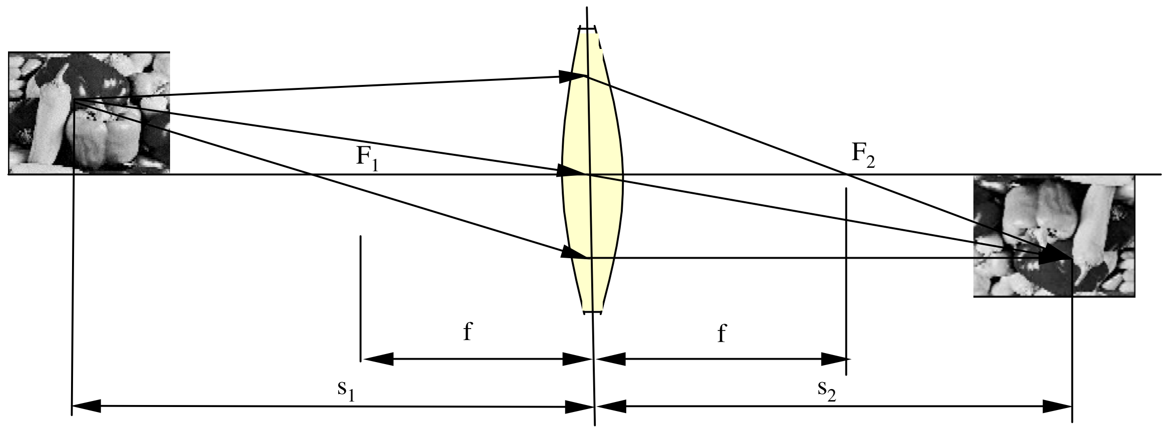


Resulting Image	Erect (Virtual)	magnified



$| s_2 | > s_1$

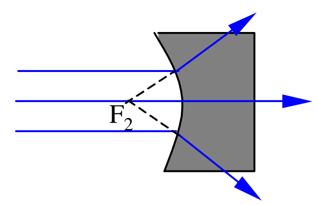
Object and Image Location for a Thin Lens



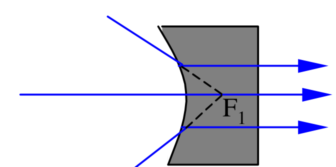
Convex				
Object	**Image**			
Location	Type	Location	Orientation	Relative Size
• > s_1 > 2f	Real	f < s_2 < 2f	Inverted	Minified
s_1 = 2f	Real	S_2 = 2f	Inverted	Same size
f < s_1 < 2f	Real	• > s_2 > 2f	Inverted	Magnified
s_1 = f		± •		
s_1 < f	Virtual	l s_2 l > s_1	Erect	magnified
Concave				
Anywhere	Virtual	l s_2 l < f, l s_2 l < s_1	Erect	Minified

Note: Image is virtual, when rays diverge from it

Parameter	Sign	
	+	-
s_1	Real object	Virtual object
s_2	Real image	Virtual image
f	Converging lens	Diverging lens
y_1	Erect object	Inverted object
y_2	Erect image	Inverted image

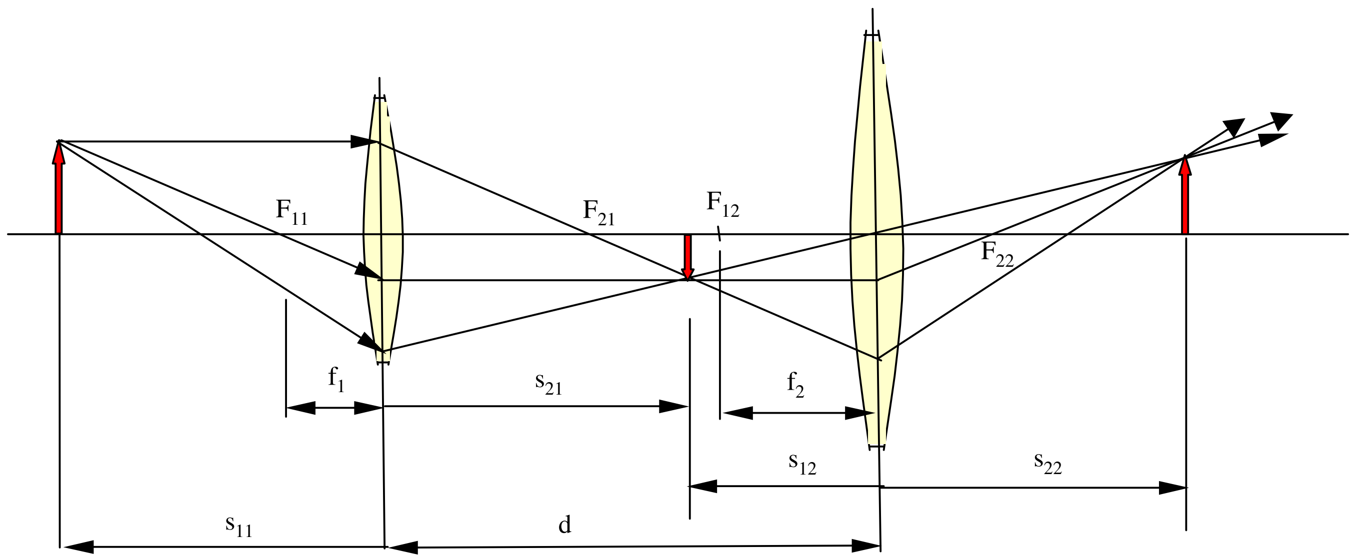


F_2 – Virtual image point



F_1 – Virtual object point

Back Focal Length (b.f.l)



The distance from the last surface of an optical system to the second (image) focal point of that system as a whole is b.f.l.

$$b.f.l = \frac{f_2\left(d - f_1\right)}{d - (f_1 + f_2)}$$

Example

Let f_1 = - 30 cm, f_2 = + 20 cm, d = 20 cm

$$b.f.l = \frac{20\left(10 - (-30)\right)}{10 - (-30 + 20)} = 40\ cm$$

When two lenses are brought into contact, i.e., when d ◊ 0,

$$\frac{1}{b.f.l} = \frac{1}{f_1} + \frac{1}{f_2}$$

For N lenses in contact,

$$\frac{1}{b.f.l} = \frac{1}{f_1} + \frac{1}{f_2} + \Lambda + \frac{1}{f_N}$$

Pinhole Camera