‘Revolution’

1900 | 1925

Classical Physics seemed to explain everything, but… | series of startling discoveries → | **Quantum Mechanics**

Resumé of Classical Physics

1. **Matter** particles moving under Newton's laws (1687)
2. **Electromagnetic radiation** obeying Maxwell's eqs. (1864)
   waves

A. Resumé of Classical Physics

Physics up to ~ 1900

Universe consists of two types of things: Matter + Electromagnetic radiation

1. **Matter** particles (mass m) moving under Newton's laws of mechanics (1687)

   [2nd law: $F = \frac{dp}{dt} = m\frac{dv}{dt} = ma$]

→ motion predetermined if the momenta + positions of all particles are known at any instant.

Explains:	sound transmission	Newton
	hydrodynamics	Euler
	kinetic theory of gases	
	thermodynamics (from statistical mechanics)	Boltzmann
	⋮	
	even tried mechanical theory of light !?	

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2. Electromagnetic radiation obeys Maxwell's eqs. (1864)

based on 4 empirical laws

- Coulomb's law F(q q)
- Biot-Savart law F(↑ ↑)
- Faraday's law of induction

line integral of $\vec{E}$ = rate of change of magnetic flux

- Ampère-Maxwell law

$\vec{j} + \frac{\partial \vec{E}}{\partial t}$

line integral of $\vec{B}$ = enclosed $\vec{j}$ + rate of ch. of electric flux

} Maxwell's eqs.

In free space (no charges)

$$\boxed{\begin{aligned}\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} &= \frac{\partial^2\vec{E}}{\partial x^2}\\ \frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2} &- \frac{\partial^2\vec{B}}{\partial x^2}\end{aligned}}$$

Wave eqs.

Elec. + Mag. fields, $\vec{E}$, $\vec{B}$, propagate through space as waves of velocity c $= 3\times10^8 ms^{-1}$ = vel. of light

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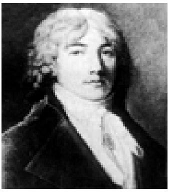
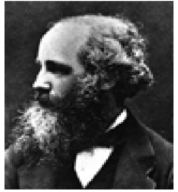
Coulomb 1735-1806
(Franklin, Michell,….)



Biot 1774-1862
(Savart 1791-1841)

Maxwell 1831-1879



Colour perception 1849-70
Saturn rings 1856/7
Kinetic theory of gases 1859-78
(indep. of Boltzmann)
4 Maxwell eqs 1855/6
EM waves have
velocity of light 1862

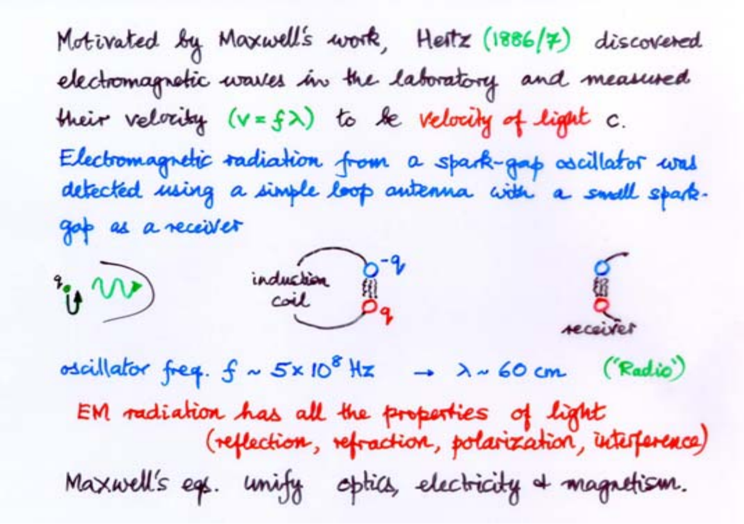
Talkative, with a Scotch brogue

Faraday 1791-1867
★★★

Ampere 1775-1836

A quote from Feynman's Lectures in Physics:

From a long view of the history of mankind, there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electrodynamics. The American Civil War will pale into provincial insignificance in comparison with this important scientific event of the same decade.



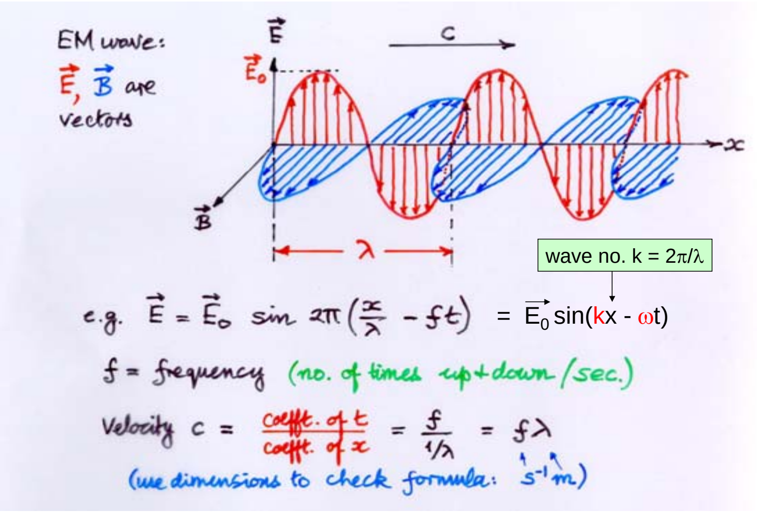
Motivated by Maxwell's work, Hertz (1886/7) discovered electromagnetic waves in the laboratory and measured their velocity ($v = f\lambda$) to be velocity of light c.

Electromagnetic radiation from a spark-gap oscillator was detected using a simple loop antenna with a small spark-gap as a receiver

oscillator freq. $f \sim 5 \times 10^8$ Hz → $\lambda \sim 60$ cm ("Radio")

EM radiation has all the properties of light (reflection, refraction, polarization, interference)

Maxwell's eqs. unify optics, electricity & magnetism.



EM wave:

$\vec{E}$, $\vec{B}$ are vectors

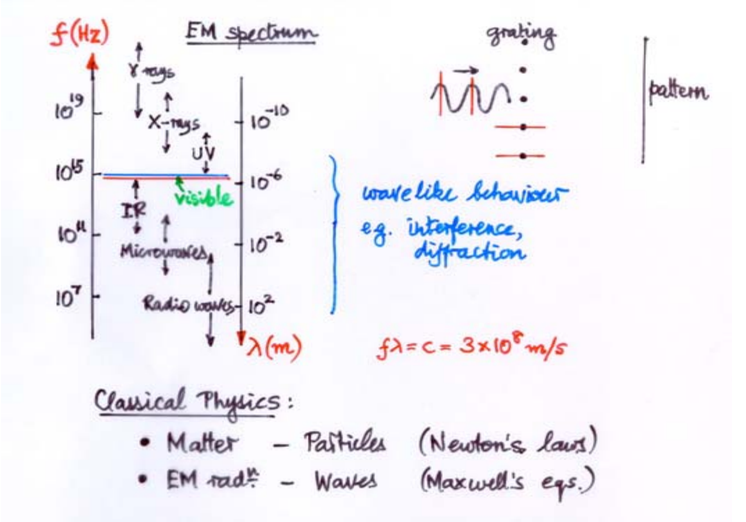
wave no. $k = 2\pi/\lambda$

e.g. $\vec{E} = \vec{E}_0 \sin 2\pi \left(\frac{x}{\lambda} - ft \right) = \vec{E}_0 \sin(kx - \omega t)$

f = frequency (no. of times up + down / sec.)

Velocity $c = \frac{\text{coefft. of } t}{\text{coefft. of } x} = \frac{f}{1/\lambda} = f\lambda$

(use dimensions to check formula: s^{-1} m)



EM spectrum

wavelike behaviour e.g. interference, diffraction

$f\lambda = c = 3 \times 10^8$ m/s

Classical Physics:

- Matter – Particles (Newton's laws)
- EM rad^n – Waves (Maxwell's eqs.)

"In a few years, all the great physical constants will have been approximately estimated, and the only occupation which will then be left to men of science will be to carry these measurements to another place of decimals."

Maxwell's 1871 inaugural lecture at the University of Cambridge….expressing the common view, with which he disagreed

Only measure the next decimal place!

Kind a microscopic questions we wish theory to answer:

- Why sodium vapour emits yellow light?
- Why hydrogen has the chemical properties it has?
- Why an uranium nucleus disintegrates?
- Why silver conducts electricity?
- Why sulphur is an insulator?

Classical Physics does not provide answers.

Outstanding problems around 1900

(a) Black body radiation

(b) Photoelectric effect

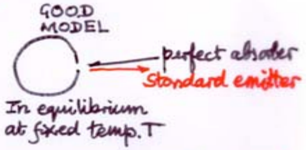
(c) Stability, spectra and size of atoms

(a) Black-body radiation

- What is BB rad^{n}?

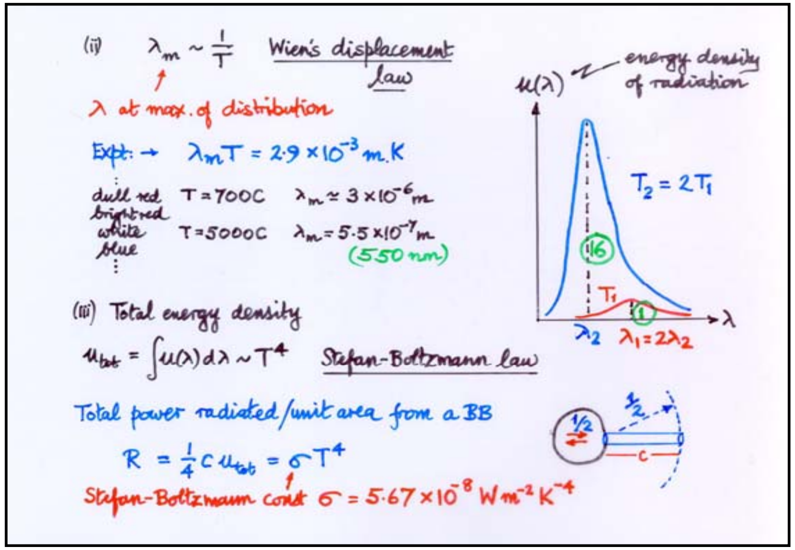
Every body emits + absorbs electromag. radiation of all wavelengths (called 'thermal' radiation)

A black body absorbs all radiation
  - perfect absorber (+ emitter)
  - therefore, a standard



- Empirical observations about spectrum of BB rad^{n} ~ 1890's

(i) Universal spectrum – depends only on T (indep. of shape or material of cavity) (Wedgwood)



(ii) $\lambda_m \sim \frac{1}{T}$ Wien's displacement law

λ at max. of distribution

Expt: → $\lambda_m T = 2{\cdot}9 \times 10^{-3}$ m.K

dull red $T = 700C$ $\lambda_m \simeq 3 \times 10^{-6}$m

bright red

white $T = 5000C$ $\lambda_m = 5{\cdot}5 \times 10^{-7}$m

blue (550 nm)

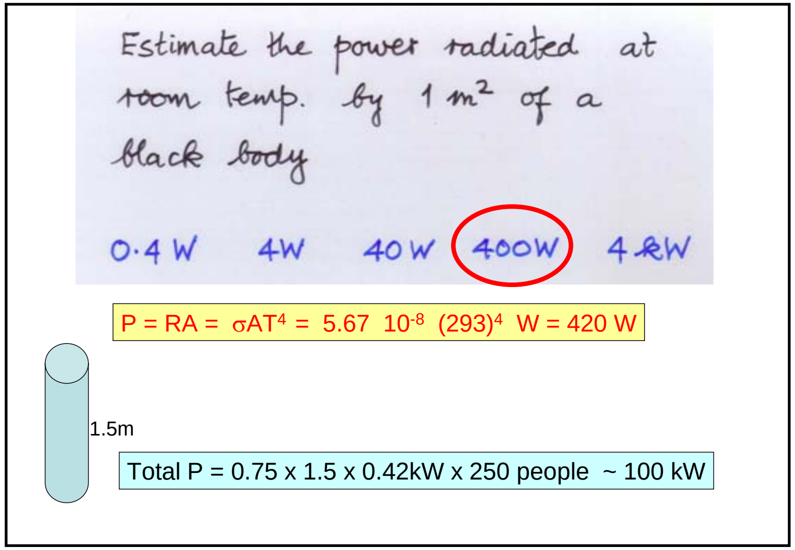
(iii) Total energy density

$u_{tot} = \int u(\lambda)\, d\lambda \sim T^4$ Stefan-Boltzmann law

Total power radiated/unit area from a BB

$R = \frac{1}{4} c\, u_{tot} = \sigma T^4$

Stefan-Boltzmann const $\sigma = 5{\cdot}67 \times 10^{-8}$ W m^{-2} K^{-4}

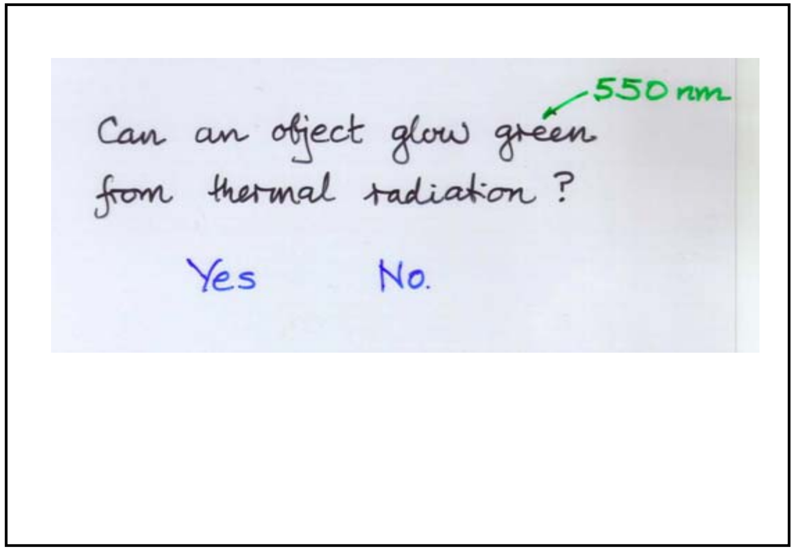


Estimate the power radiated at room temp. by 1 m^2 of a black body

0·4 W   4W   40 W   [✓ 400W]   4 kW

P = RA = σAT^4 = 5.67 10^{-8} $(293)^4$ W = 420 W

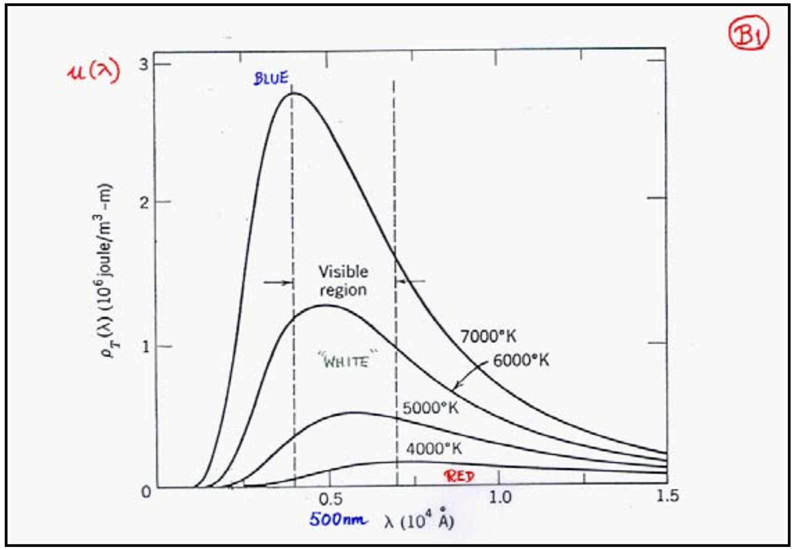
Total P = 0.75 x 1.5 x 0.42kW x 250 people ~ 100 kW

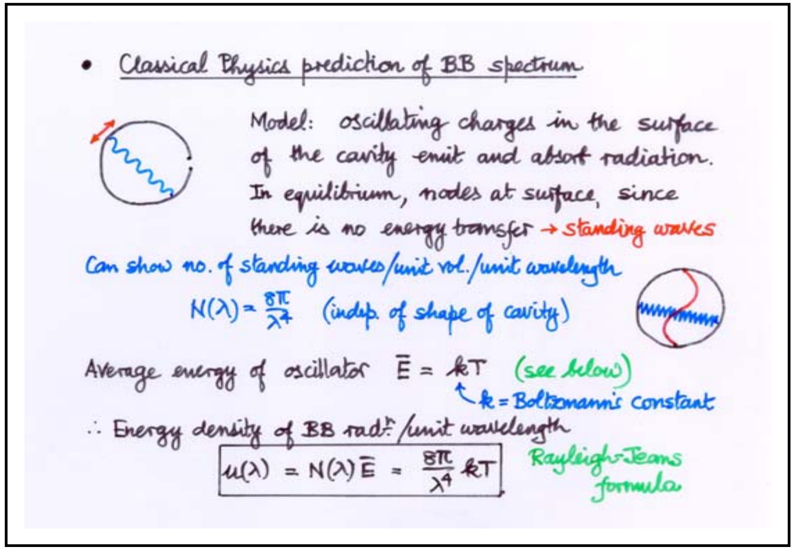


Can an object glow green from thermal radiation ?

550 nm

Yes   No.





- Classical Physics prediction of BB spectrum

Model: oscillating charges in the surface of the cavity emit and absorb radiation. In equilibrium, nodes at surface, since there is no energy transfer → standing waves

Can show no. of standing waves/unit vol./unit wavelength

$N(\lambda) = \frac{8\pi}{\lambda^4}$ (indep. of shape of cavity)

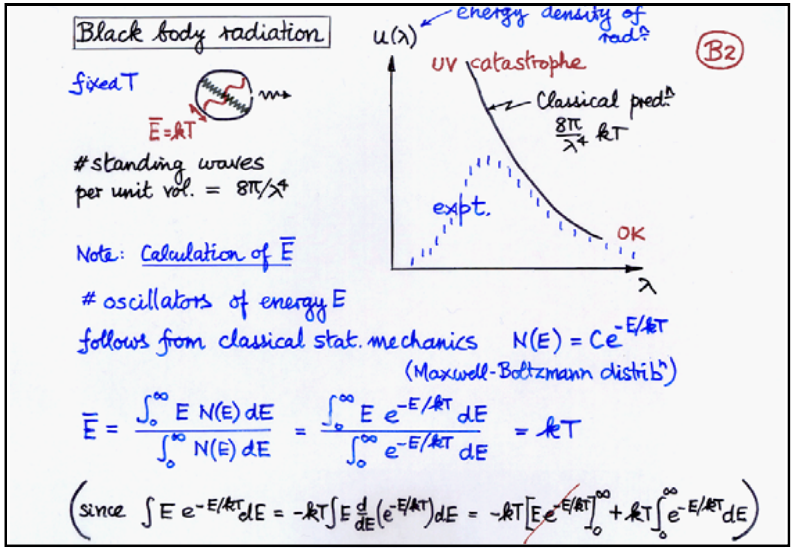
Average energy of oscillator $\bar{E} = kT$ (see below)

k = Boltzmann's constant

∴ Energy density of BB radn/unit wavelength

$$\boxed{u(\lambda) = N(\lambda)\bar{E} = \frac{8\pi}{\lambda^4} kT}$$

Rayleigh-Jeans formula



Black body radiation

Ⓑ2

fixed T

$\bar{E} = kT$

standing waves per unit vol. = $8\pi/\lambda^4$

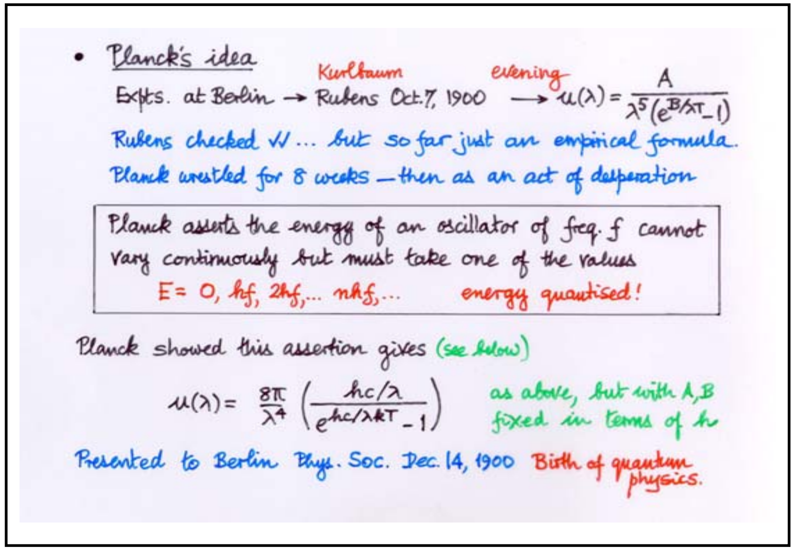
Note: Calculation of $\bar{E}$

oscillators of energy E follows from classical stat. mechanics $N(E) = Ce^{-E/kT}$

(Maxwell-Boltzmann distibn)

$$\bar{E} = \frac{\int_0^\infty E\,N(E)\,dE}{\int_0^\infty N(E)\,dE} = \frac{\int_0^\infty E\,e^{-E/kT}\,dE}{\int_0^\infty e^{-E/kT}\,dE} = kT$$

$$\left(\text{since } \int E\,e^{-E/kT}dE = -kT\int E\frac{d}{dE}\left(e^{-E/kT}\right)dE = -kT\cancel{\left[Ee^{-E/kT}\right]_0^\infty} + kT\int_0^\infty e^{-E/kT}dE\right)$$



- Planck's idea

Expts. at Berlin → Rubens Oct.7, 1900 $\xrightarrow{\text{evening}}$ $u(\lambda) = \frac{A}{\lambda^5(e^{B/\lambda T}-1)}$

Kurlbaum

Rubens checked ✓✓ ... but so far just an empirical formula.

Planck wrestled for 8 weeks — then as an act of desperation

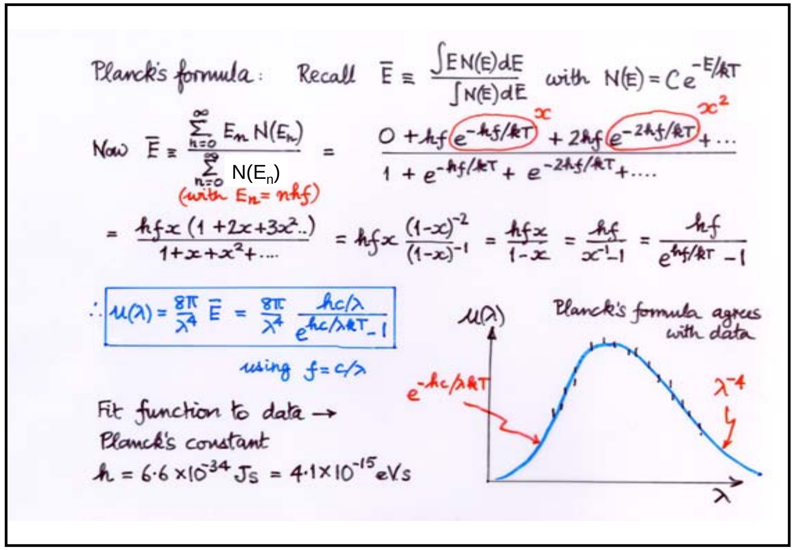
> Planck asserts the energy of an oscillator of freq. f cannot vary continuously but must take one of the values
>
> $E = 0, hf, 2hf, \ldots nhf, \ldots$ energy quantised!

Planck showed this assertion gives (see below)

$$u(\lambda) = \frac{8\pi}{\lambda^4}\left(\frac{hc/\lambda}{e^{hc/\lambda kT}-1}\right)$$

as above, but with A, B fixed in terms of h

Presented to Berlin Phys. Soc. Dec. 14, 1900 Birth of quantum physics.



Planck's formula: Recall $\bar{E} \equiv \frac{\int E\,N(E)\,dE}{\int N(E)\,dE}$ with $N(E) = Ce^{-E/kT}$

$$\text{Now } \bar{E} \equiv \frac{\sum_{n=0}^{\infty} E_n N(E_n)}{\sum_{n=0}^{\infty} N(E_n)} = \frac{0 + hf\,\overset{x}{\boxed{e^{-hf/kT}}} + 2hf\,\overset{x^2}{\boxed{e^{-2hf/kT}}} + \ldots}{1 + e^{-hf/kT} + e^{-2hf/kT} + \ldots}$$

(with $E_n = nhf$)

$$= \frac{hfx(1+2x+3x^2..)}{1+x+x^2+\ldots} = hfx\frac{(1-x)^{-2}}{(1-x)^{-1}} = \frac{hfx}{1-x} = \frac{hf}{x^{-1}-1} = \frac{hf}{e^{hf/kT}-1}$$

$$\therefore \boxed{u(\lambda) = \frac{8\pi}{\lambda^4}\bar{E} = \frac{8\pi}{\lambda^4}\frac{hc/\lambda}{e^{hc/\lambda kT}-1}}$$

using $f = c/\lambda$

Fit function to data → Planck's constant

$h = 6{\cdot}6 \times 10^{-34}$ Js $= 4{\cdot}1 \times 10^{-15}$ eVs

$h = 4.13 \times 10^{-15}$ eV s

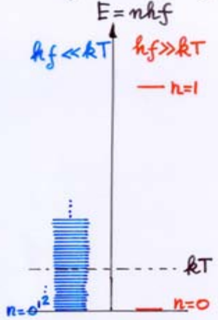
$c = 3 \times 10^{8}$ m s^{-1}

therefore

$hc = 12.4 \times 10^{-7}$ eV m

= 1240 eV nm

- Insight into why Planck's assertion works $\left(f = \frac{c}{\lambda}\right.$



hf ≪ kT (large λ) E's look continuous

$$e^{hf/kT} - 1 = \left(1 + \frac{hf}{kT} + \ldots\right) - 1 \simeq \frac{hf}{kT}$$

$$\bar{E} = \frac{hf}{e^{hf/kT} - 1} \simeq \frac{hf}{hf/kT} = kT$$

the classical result

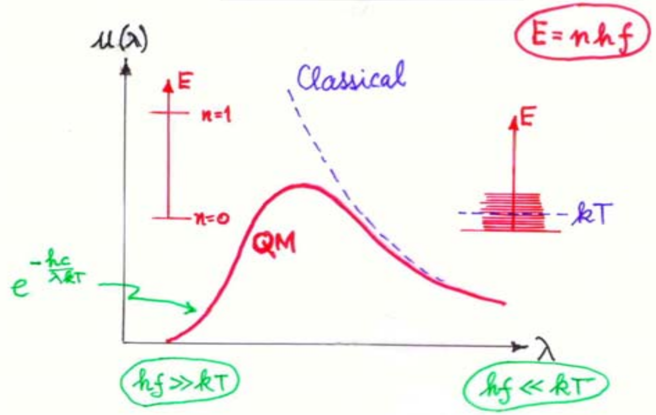
hf ≫ kT (v. small λ)
oscillator mainly in E = 0 state.
The chance of the others is suppressed
by the Boltzmann factor $\exp(-nhf/kT)$.

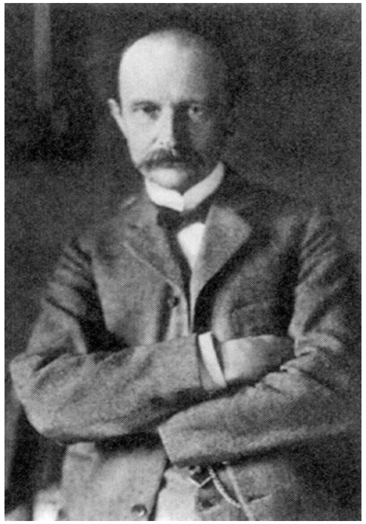
- Can use Planck's spectral distribution to derive

Q5: Stefan-B. law $R = \sigma T^4$ + show $\sigma = \frac{2\pi^5 k^4}{15 c^2 h^3} = 5.67 \times 10^{-8}$ $Wm^{-2} K^{-4}$

Q10: Wien's law $\lambda_m = C/T$ with $C = 2.9 \times 10^{-3}$ mK

$$\boxed{u(\lambda) = \frac{8\pi}{\lambda^4} \frac{hc/\lambda}{e^{hc/\lambda kT} - 1}}$$





Max Planck (1858-1947)

Nobel prize 1918

Q5 Stefan's law

$a \equiv \frac{hc}{kT}$

Energy density of BB $\quad u(\lambda) = 8\pi \frac{hc}{\lambda^5} \frac{1}{e^{a/\lambda}-1}$

$$u_{tot} = \int_0^\infty u(\lambda)\, d\lambda = 8\pi hc \int_0^\infty \frac{1}{\lambda^5} \frac{1}{e^{a/\lambda}-1}\, d\lambda$$

Substitute $x = \frac{a}{\lambda}, \quad \lambda = \frac{a}{x}, \quad d\lambda = -\frac{a}{x^2} dx \qquad \begin{cases} \lambda = 0 \rightarrow x = \infty \\ \lambda = \infty \rightarrow x = 0 \end{cases}$

$$u_{tot} = 8\pi hc \int_0^\infty \frac{x^5}{a^5} \frac{1}{e^x-1} \frac{a}{x^2}\, dx = \frac{8\pi hc}{a^4} \boxed{\int_0^\infty \frac{x^3\, dx}{e^x-1}} \leftarrow \frac{\pi^4}{15}$$

$$\therefore\ R = \frac{c}{4} u_{tot} = \frac{2\pi^5}{15} hc^2 \left(\frac{kT}{hc}\right)^4 = \boxed{\frac{2\pi^5 k^4}{15 c^2 h^3}} T^4$$

Insert the values of k, c, h and show $\sigma = 5{\cdot}67 \times 10^{-8}\ \mathrm{W m^{-2} K^{-4}}$

Now try the exciting Q6 !

Q6

$$u_{tot} = \int_0^\infty u(\lambda)\, d\lambda = 8\pi hc \int_0^\infty \frac{1}{e^{a/\lambda}-1} \boxed{\frac{d\lambda}{\lambda^5}}$$

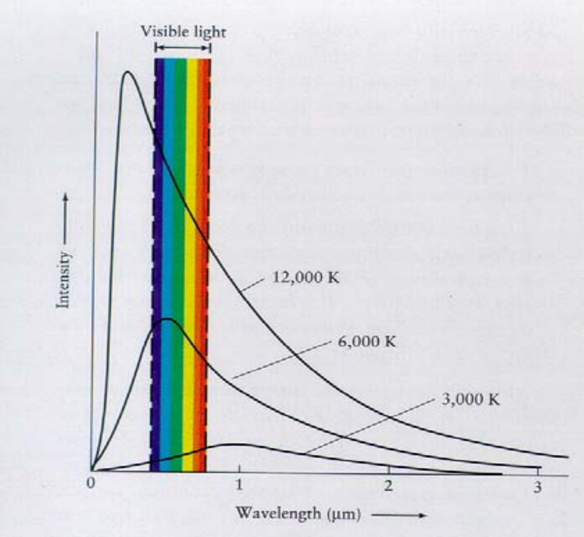
where $a \equiv \frac{hc}{kT}$

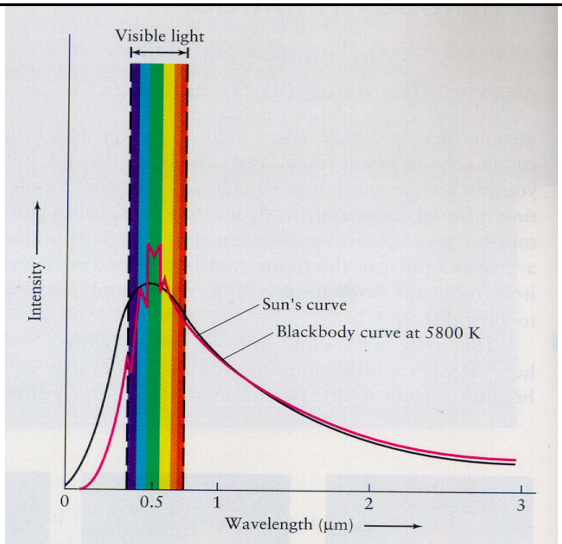
Let $x = \frac{a}{\lambda} \qquad \begin{cases} \lambda = 0 \leftrightarrow x = \infty \\ \lambda = \infty \leftrightarrow x = 0 \end{cases}$

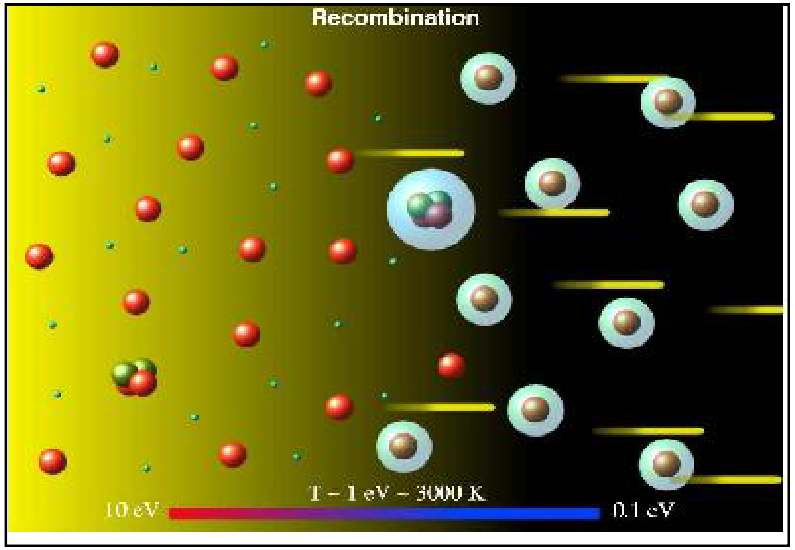
$\frac{dx}{d\lambda} = -\frac{a}{\lambda^2}$ or $\frac{d\lambda}{\lambda^2} = -\frac{dx}{a}$

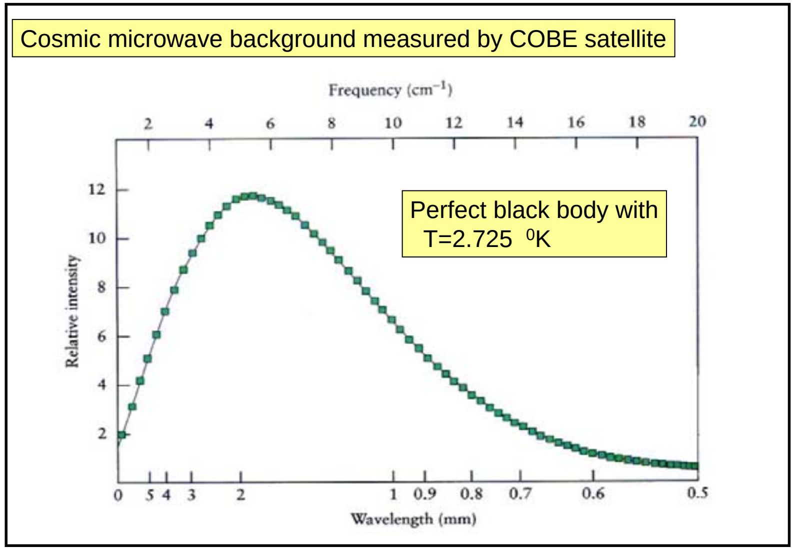
$$u_{tot} = 8\pi hc \int_0^\infty \frac{1}{e^x-1} \underset{\uparrow}{\frac{x^3}{a^3}} \underset{\uparrow}{\frac{dx}{a}} = \frac{8\pi hc}{a^4} \underbrace{\int \frac{x^3\, dx}{e^x-1}}_{\pi^4/15} = \frac{8\pi^5}{15} \frac{(kT)^4}{(hc)^3}$$

$$\boxed{\frac{1}{\lambda^3} \quad -\frac{d\lambda}{\lambda^2}}$$



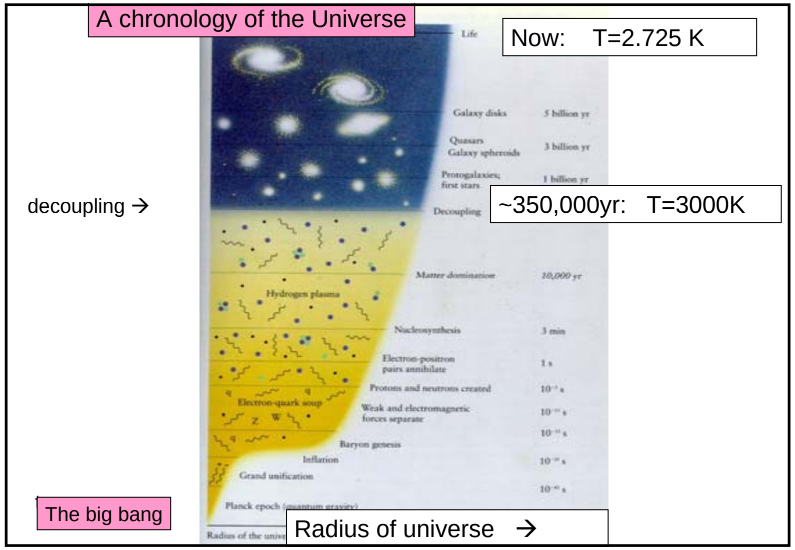






Cosmic microwave background measured by COBE satellite

Perfect black body with
$T=2.725\ {}^0K$



A chronology of the Universe

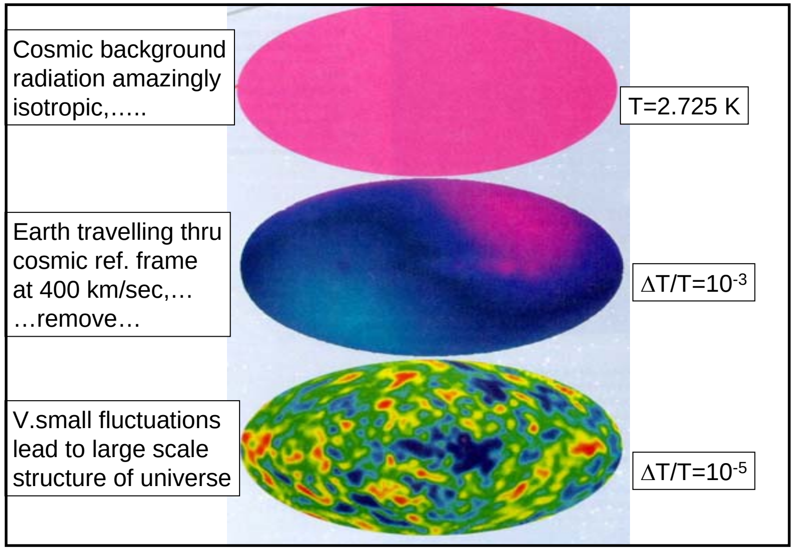
Now: T=2.725 K

decoupling →

~350,000yr: T=3000K

The big bang

Radius of universe →



Cosmic background radiation amazingly isotropic,…..

T=2.725 K

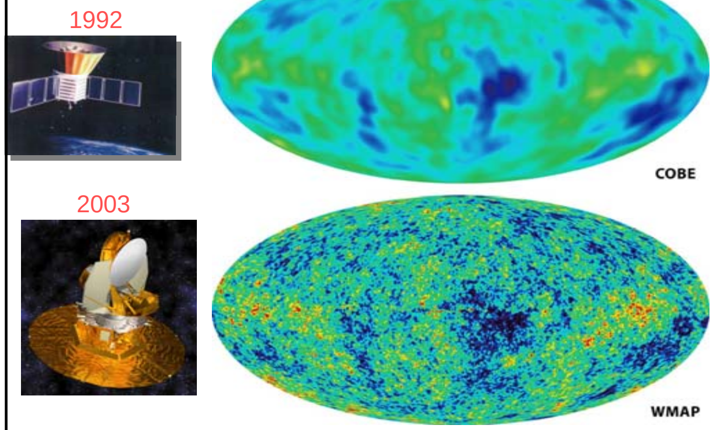
Earth travelling thru cosmic ref. frame at 400 km/sec,…
…remove…

$\Delta T/T=10^{-3}$

V.small fluctuations lead to large scale structure of universe

$\Delta T/T=10^{-5}$

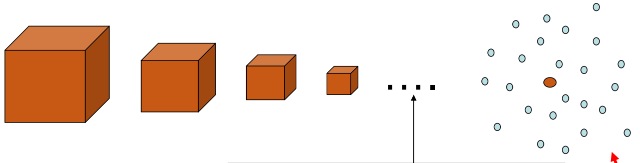
The CMB



A peep ahead…

Each chunk of Cu obeys same classical laws:
same coefft.of expansion,
same conductivity, etc

Cu atom



When quantities of dimension of h, are the order of, or less than h, we must use quantum mechanics

Already prejudiced! We cannot draw this picture of a Cu atom

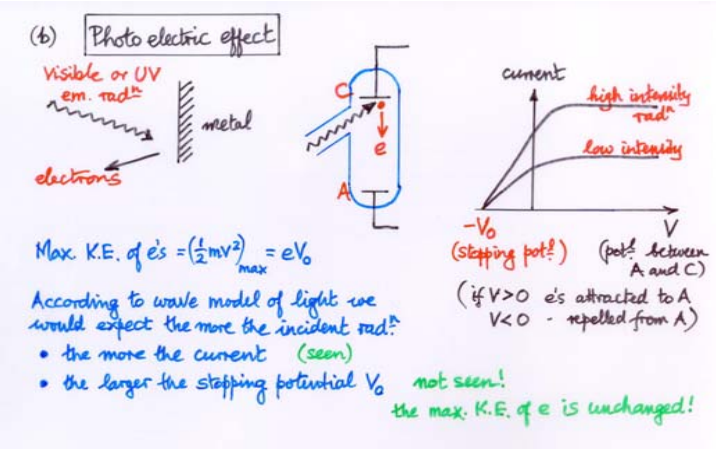
Outstanding problems around 1900

(a) Black body radiation

E(oscillator) = 0, hf, 2hf,…

(b) Photoelectric effect

(c) Stability, spectra and size of atoms



Max. K.E. of e's $= \left(\frac{1}{2}mv^2\right)_{max} = eV_0$

According to wave model of light we would expect the more the incident radn:

- the more the current (seen)
- the larger the stopping potential V_0 not seen!

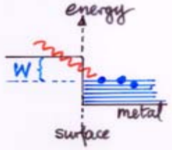
the max. K.E. of e is unchanged!

Einstein's explanation of photoelectric effect (1905)

Energy of EM wave comes in localized bursts of size hf, and such a quantum may be completely transferred to an electron in the metal

$$K.E._{max} = hf - W$$

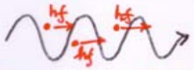
W is min. energy e needs to escape from the metal ≡ work function



Einstein: way ahead of his time

$E = hf - W$ only universally accepted in ~1914 (Millikan established relation – N.Prize '23)

By 1909 Einstein (+ him alone) interpreted quanta as photons – only accepted in 1923



Note: timescale of emission ~10^{-9}s. Multiple absorptions do not happen (except with v. intense lasers)

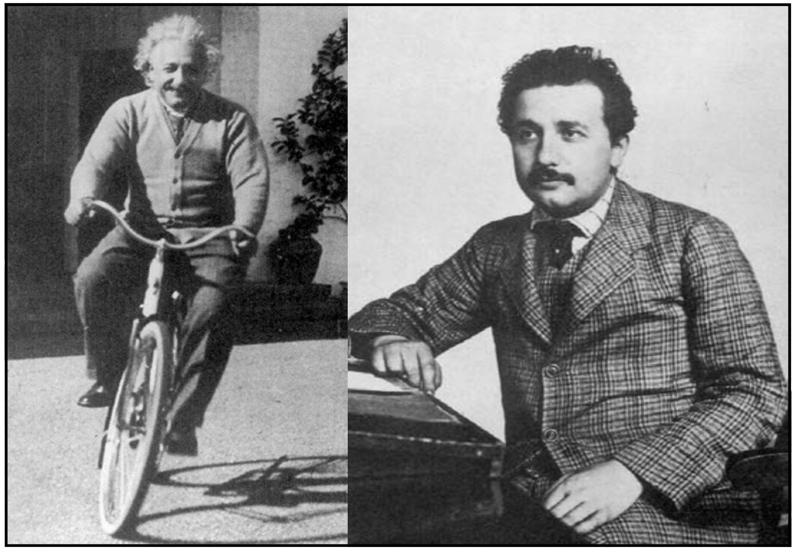
Note: If $hf < W$ then e loses energy in metal, which heats up.

A. Einstein 1905

- Ann. Physik 17 p.132, March 1905
  explained photoelectric effect

N.Prize 1921

- Ann. Physik 17 p.549, May 1905
  explained Brownian motion and provided strong proof of the reality of atoms
- Ann. Physik 17 p.891, June 1905
  invention of the theory of special relativity



Exer: Photoelect. effect E_{max} of e =hf-W

hc=1240 eV nm

When light of wavelength 300nm is incident on potassium, the emitted electrons have maximum K.E. of 2.03 eV.

(a) What is the energy of the photon?

E_γ = hf = hc/λ = 1240/300 = 4.13 eV

(b) What is the work function for potassium?

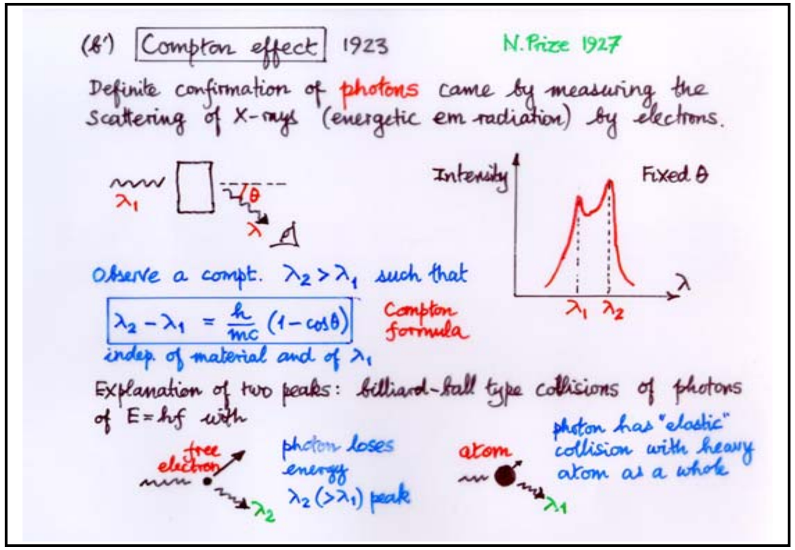
W = E_γ – E_{max} = 4.13 – 2.03 = 2.10 eV

(c) What would be the max. K.E. of the electrons if the incident light had wavelength of 430 nm?

E_{max} = E_γ – W =1240/430 – 2.10 = 0.78 eV

(d) What is the threshold wavelength for the photoelectric effect on potassium?

λ_{th} = hc/W =1240/2.10 = 590 nm



(6') Compton effect 1923

N. Prize 1927

Definite confirmation of photons came by measuring the scattering of X-rays (energetic em radiation) by electrons.

Observe a compt. $\lambda_2 > \lambda_1$ such that

$$\boxed{\lambda_2 - \lambda_1 = \frac{h}{mc}(1-\cos\theta)}$$

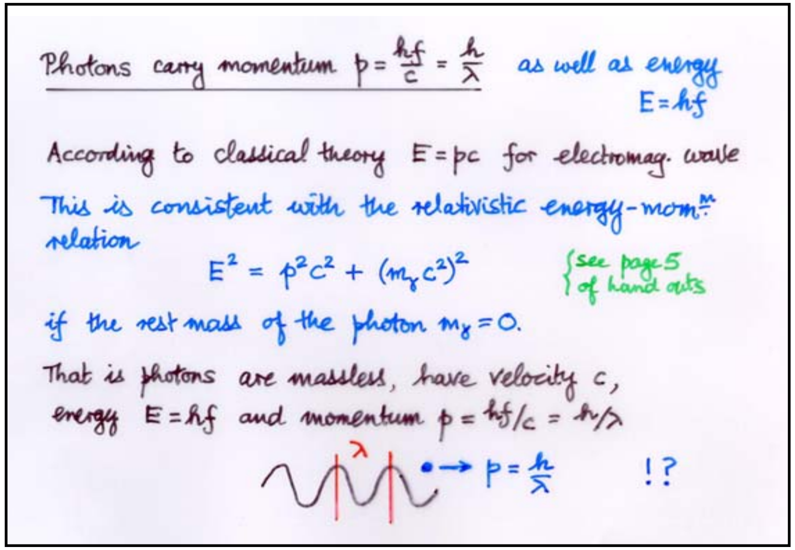
Compton formula

indep. of material and of λ_1

Explanation of two peaks: billiard-ball type collisions of photons of $E = hf$ with

photon loses energy $\lambda_2 (> \lambda_1)$ peak

photon has "elastic" collision with heavy atom as a whole



Photons carry momentum $p = \frac{hf}{c} = \frac{h}{\lambda}$ as well as energy $E = hf$

According to classical theory $E = pc$ for electromag. wave

This is consistent with the relativistic energy-mom^{m} relation

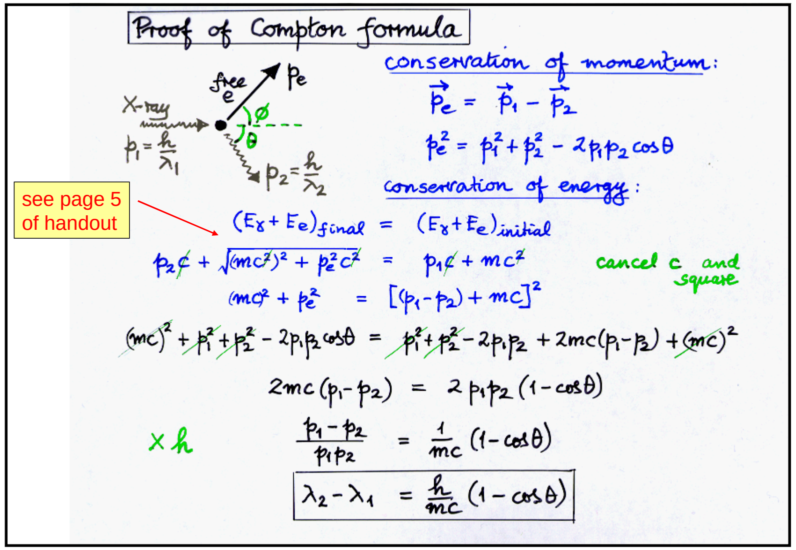
$$E^2 = p^2c^2 + (m_\gamma c^2)^2$$

{see page 5 of hand outs

if the rest mass of the photon $m_\gamma = 0$.

That is photons are massless, have velocity c, energy $E = hf$ and momentum $p = hf/c = h/\lambda$

$\bullet \rightarrow p = \frac{h}{\lambda}$!?



Proof of Compton formula

see page 5 of handout

conservation of momentum:

$$\vec{p}_e = \vec{p}_1 - \vec{p}_2$$

$$p_e^2 = p_1^2 + p_2^2 - 2p_1p_2\cos\theta$$

conservation of energy:

$$(E_\gamma + E_e)_{final} = (E_\gamma + E_e)_{initial}$$

$$p_2\cancel{c} + \sqrt{(mc^{\cancel{2}})^2 + p_e^2c^{\cancel{2}}} = p_1\cancel{c} + mc^{\cancel{2}}$$

cancel c and square

$$(mc)^2 + p_e^2 = [(p_1 - p_2) + mc]^2$$

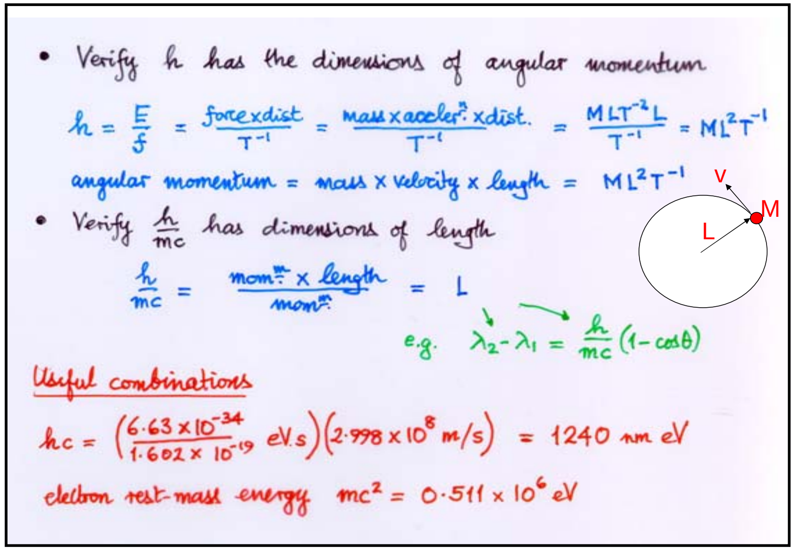
$$\cancel{(mc)^2} + \cancel{p_1^2} + \cancel{p_2^2} - 2p_1p_2\cos\theta = \cancel{p_1^2} + \cancel{p_2^2} - 2p_1p_2 + 2mc(p_1 - p_2) + \cancel{(mc)^2}$$

$$2mc(p_1 - p_2) = 2p_1p_2(1-\cos\theta)$$

$\times h$

$$\frac{p_1 - p_2}{p_1p_2} = \frac{1}{mc}(1-\cos\theta)$$

$$\boxed{\lambda_2 - \lambda_1 = \frac{h}{mc}(1-\cos\theta)}$$



- Verify h has the dimensions of angular momentum

$$h = \frac{E}{f} = \frac{\text{force} \times \text{dist}}{T^{-1}} = \frac{\text{mass} \times \text{acceler}^{n} \times \text{dist.}}{T^{-1}} = \frac{MLT^{-2}L}{T^{-1}} = ML^2T^{-1}$$

angular momentum = mass x velocity x length = ML^2T^{-1}

- Verify $\frac{h}{mc}$ has dimensions of length

$$\frac{h}{mc} = \frac{\text{mom}^{m} \times \text{length}}{\text{mom}^{m}} = L$$

e.g. $\lambda_2 - \lambda_1 = \frac{h}{mc}(1-\cos\theta)$

Useful combinations

$$hc = \left(\frac{6.63 \times 10^{-34}}{1.602 \times 10^{-19}} \text{ eV.s}\right)\left(2.998 \times 10^8 \text{ m/s}\right) = 1240 \text{ nm eV}$$

electron rest-mass energy $mc^2 = 0.511 \times 10^6$ eV

- What is an appropriate wavelength of radiation to show the Compton effect? $\lambda_2-\lambda_1 = \frac{h}{mc}(1-\cos\theta)$

$$\frac{h}{mc} = \frac{hc}{mc^2} = \frac{1240 \text{ eV nm}}{0.511\times10^6 \text{eV}} = 2.43 \text{ pm}$$

$\leftarrow 10^{-12}$m

Want $\underbrace{\lambda_2-\lambda_1}_{\sim10^{-12}\text{m}}$ to be $\gtrsim 0.01\,\lambda_1$, say

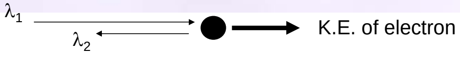
$\sim10^{-12}$m, so $\lambda_1 \lesssim 10^{-10}$m (X-rays)

- If $\lambda_1 = 6$ pm, $\theta = 180°$ then find $\Delta\lambda$ and $(K.E.)_e$

$$\Delta\lambda \equiv \lambda_2-\lambda_1 = \frac{h}{mc}(1-(-1)) = 2\frac{h}{mc} = 4.86 \text{ pm}$$

$$\therefore \lambda_2 = 10.86 \text{ pm}$$

$$(K.E.)_e = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = 1240\left[\frac{1}{6\times10^{-3}} - \frac{1}{10.86\times10^{-3}}\right]\text{eV} = 93 \text{ keV}$$



E=pc=hc/λ

Outstanding problems around 1900

(a) Black body radiation

E(oscillator) = 0, hf, 2hf,…

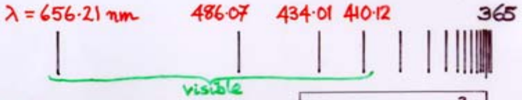
(b) Photoelectric effect

Photon: E = hf, p=hf/c

(c) Stability, spectra and size of atoms

(c) $\boxed{\text{Stability and size of atoms}}$ → Bohr model

- Hydrogen spectrum ~1880's data were collected on the emission of light by atoms in a gas excited by electrical discharge



Balmer (1884) found
Swiss maths teacher

$\boxed{\lambda = 364.56\,\frac{n^2}{n^2-4}}$ nm. $n=3,4,5..$

Special case of a general formula found by Rydberg + Ritz for hydrogen-like atoms (with Z protons in nucleus, but only one electron)

$$\boxed{\begin{array}{l}\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right) \text{ with } n_1>n_2 \\ \text{with } R = 10.97\ \mu m^{-1}\end{array}}$$

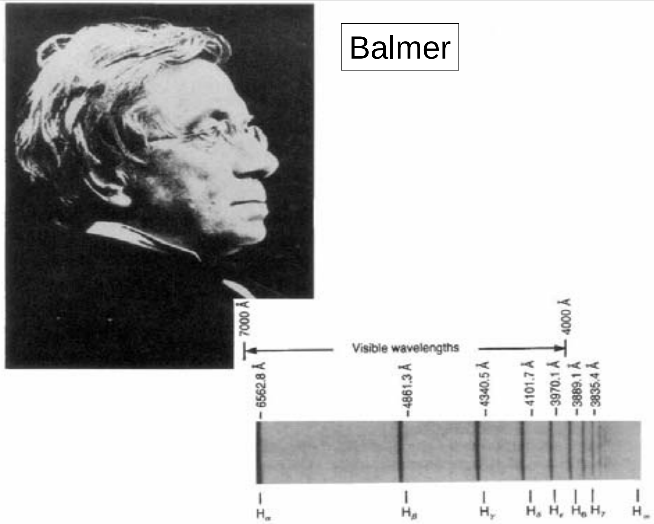
e.g. Balmer formula

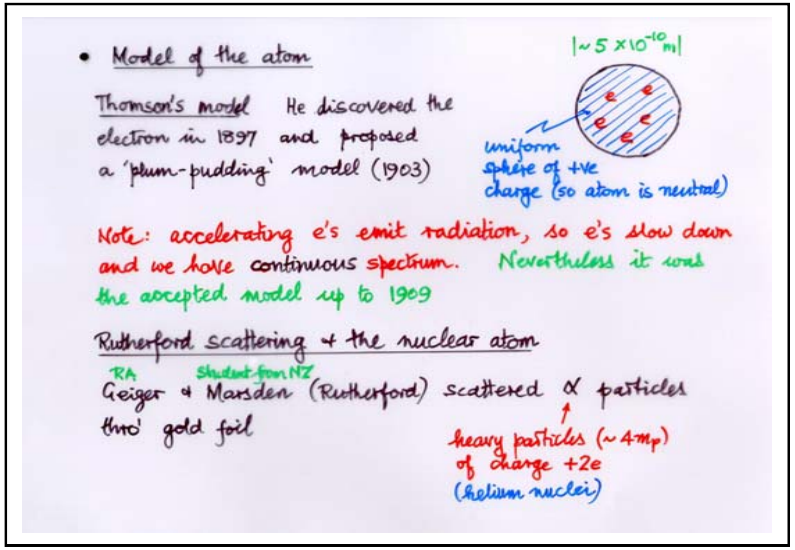
$$\frac{1}{\lambda} = \frac{4}{364.56\text{nm}}\left(\frac{1}{4}-\frac{1}{n^2}\right) = 10.97\mu m^{-1}\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$$

Series of H emission lines

		n_2	n_1
Lyman	UV	1	2,3,....
Balmer	visible	2	3,4,...
Paschen	IR	3	4,5,....
Brackett	IR	4	5,6,....
Pfund	IR	5	6,7,...

Balmer





- Model of the atom

Thomson's model He discovered the electron in 1897 and proposed a 'plum-pudding' model (1903)

$|\sim 5\times10^{-10}\,\mathrm{m}|$

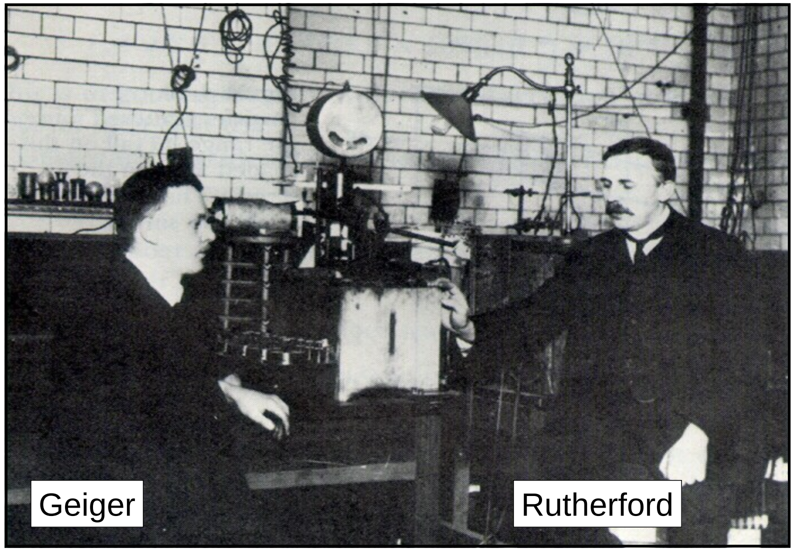
uniform sphere of +ve charge (so atom is neutral)

Note: accelerating e's emit radiation, so e's slow down and we have continuous spectrum. Nevertheless it was the accepted model up to 1909

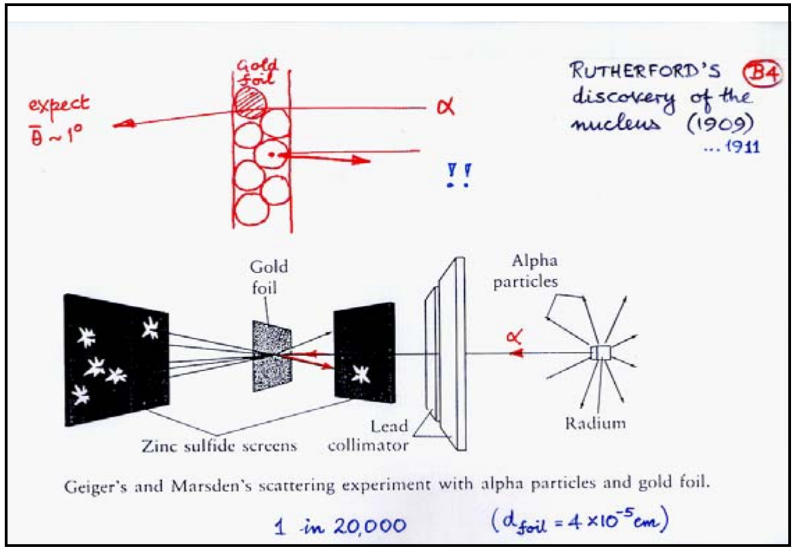
Rutherford scattering + the nuclear atom

Geiger (RA) + Marsden (Student from NZ) (Rutherford) scattered α particles thro' gold foil

α ↑ heavy particles ($\sim 4m_p$) of charge +2e (helium nuclei)

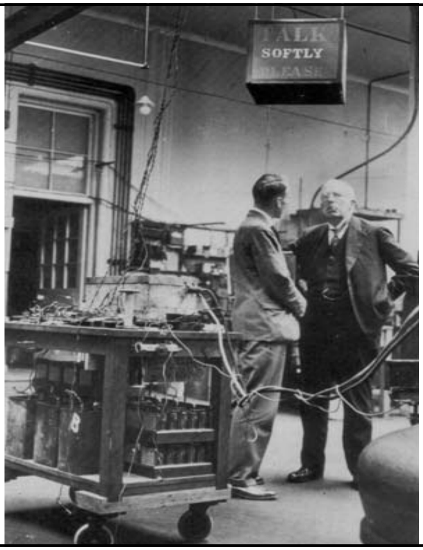


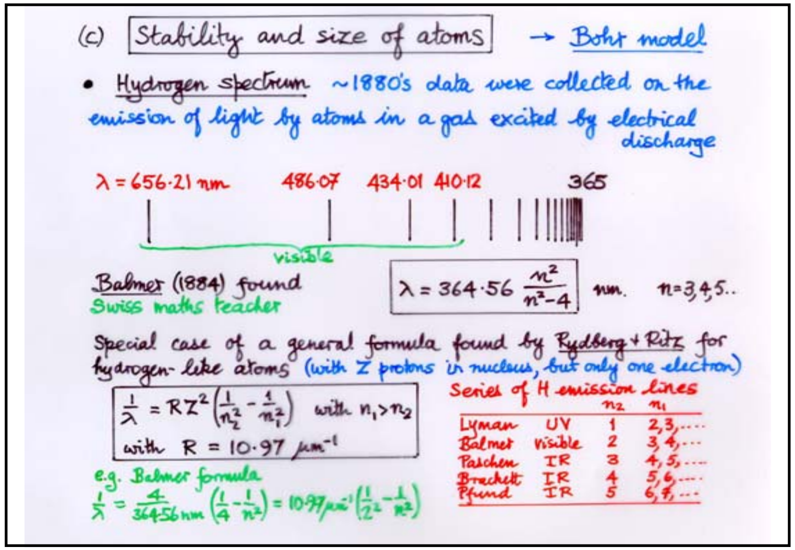
RUTHERFORD'S discovery of the nucleus (1909) ... 1911 Ⓑ4



Geiger's and Marsden's scattering experiment with alpha particles and gold foil.

1 in 20,000 ($d_{foil} = 4\times10^{-5}\,\mathrm{cm}$)





(c) Stability and size of atoms → Bohr model

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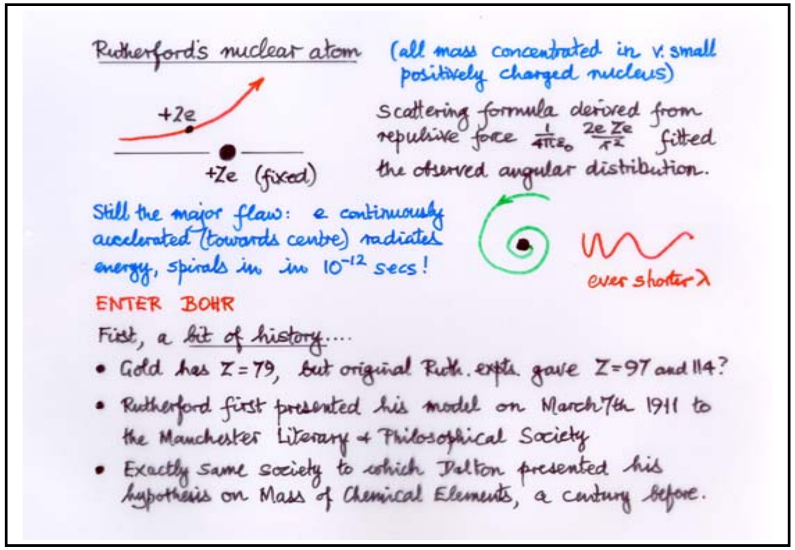


Rutherford's nuclear atom

(all mass concentrated in v. small positively charged nucleus)

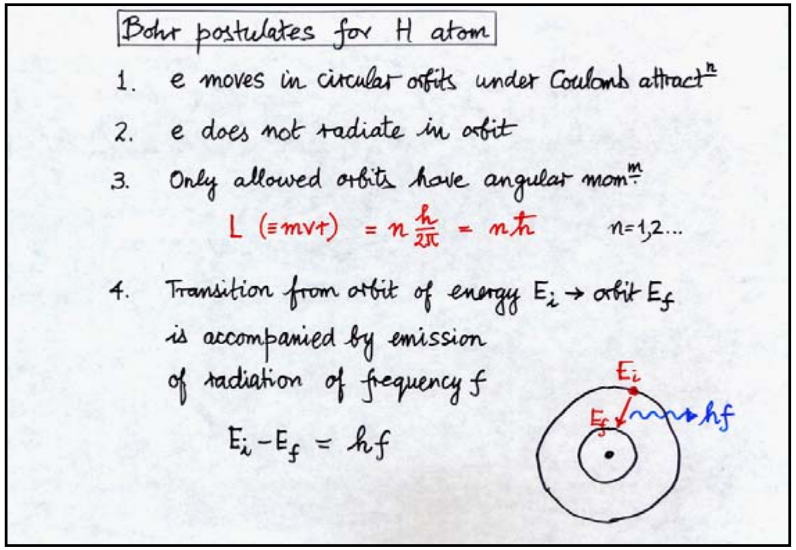
Scattering formula derived from repulsive force $\frac{1}{4\pi\varepsilon_0}\frac{2e\,Ze}{r^2}$ fitted the observed angular distribution.

Still the major flaw: e continuously accelerated (towards centre) radiates energy, spirals in in 10^{-12} secs!

ENTER BOHR

First, a bit of history....

- Gold has Z = 79, but original Ruth. expts. gave Z = 97 and 114?
- Rutherford first presented his model on March 7th 1911 to the Manchester Literary + Philosophical Society
- Exactly same society to which Dalton presented his hypothesis on Mass of Chemical Elements, a century before.



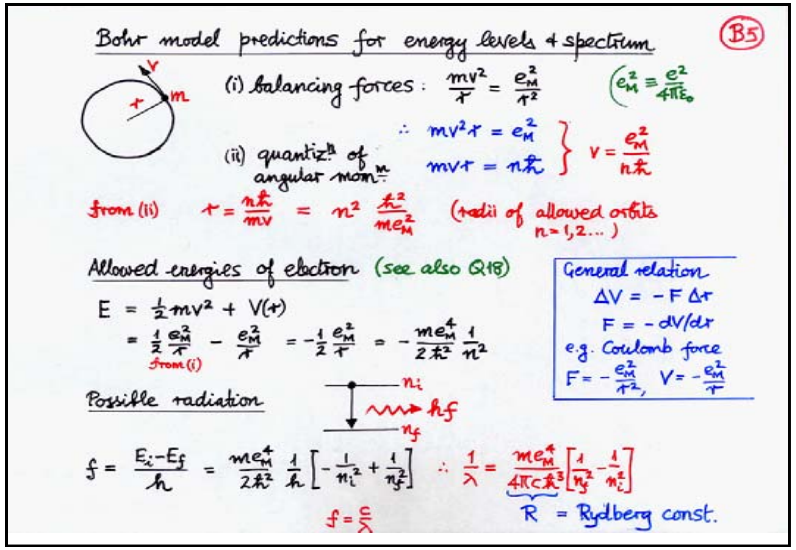
Bohr postulates for H atom

1. e moves in circular orbits under Coulomb attract^{n}
2. e does not radiate in orbit
3. Only allowed orbits have angular mom^{m}:

$$L \ (\equiv mvr) = n\frac{h}{2\pi} = n\hbar \qquad n=1,2...$$

4. Transition from orbit of energy $E_i \rightarrow$ orbit E_f is accompanied by emission of radiation of frequency f

$$E_i - E_f = hf$$



Bohr model predictions for energy levels + spectrum

Ⓑ5

(i) balancing forces : $\frac{mv^2}{r} = \frac{e_M^2}{r^2}$ $\left(e_M^2 \equiv \frac{e^2}{4\pi\varepsilon_0}\right.$

$\therefore \; mv^2 r = e_M^2$

(ii) quantiz^{n} of angular mom^{m} $mvr = n\hbar$

$\left.\begin{matrix} mv^2 r = e_M^2 \\ mvr = n\hbar \end{matrix}\right\} \; v = \frac{e_M^2}{n\hbar}$

from (ii) $r = \frac{n\hbar}{mv} = n^2 \frac{\hbar^2}{me_M^2}$ (radii of allowed orbits $n = 1, 2 \ldots$)

Allowed energies of electron (see also Q18)

$$E = \tfrac{1}{2}mv^2 + V(r)$$

$$= \underbrace{\frac{1}{2}\frac{e_M^2}{r}}_{\text{from (i)}} - \frac{e_M^2}{r} = -\frac{1}{2}\frac{e_M^2}{r} = -\frac{me_M^4}{2\hbar^2}\frac{1}{n^2}$$

General relation
$\Delta V = -F\,\Delta r$
$F = -dV/dr$
e.g. Coulomb force
$F = -\frac{e_M^2}{r^2}, \; V = -\frac{e_M^2}{r}$

Possible radiation

$$f = \frac{E_i - E_f}{h} = \frac{me_M^4}{2\hbar^2}\frac{1}{h}\left[-\frac{1}{n_i^2} + \frac{1}{n_f^2}\right] \quad \therefore \; \frac{1}{\lambda} = \underbrace{\frac{me_M^4}{4\pi c\hbar^3}}_{R}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$f = \frac{c}{\lambda}$

R = Rydberg const.