Input-Output Analysis

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Session 3.2

Updating Methods

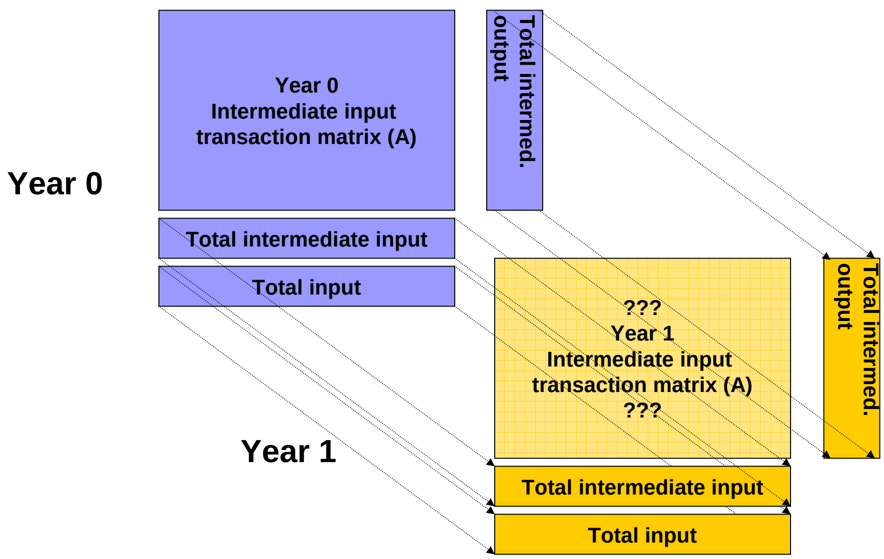
Updating IO tables

- IO coefficients need to be updated from time to time in order to reflect changes in the economy
  - What changes: prices, technology, preferences, etc
- The best way to update the coefficients is by conducting the survey again
- But survey is usually expensive and time consuming
  - Questions are too detail, coverage too wide (nationally)
- Need a more efficient way to update the table
- There is non-survey method, However, more relevant to update regional tables, and is too simple an adjustment

Partial-survey methods

- Partial-survey method is a compromise.
- Survey is still needed, but not as detailed as it would have been with the full-survey method.
- Still needs firms to elaborate the input, but not in detailed. Firms need to provide information about the total only: total intermediate inputs, total intermediate output (or total final demand), and total input or output

Basic principle of RAS method



The problem is…

- With

$$\mathbf{A(0)} = \begin{bmatrix} a_{11}(0) & a_{12}(0) & a_{13}(0) \\ a_{21}(0) & a_{22}(0) & a_{23}(0) \\ a_{31}(0) & a_{32}(0) & a_{33}(0) \end{bmatrix}$$

- And the help of

$$\mathbf{X(1)} = \begin{bmatrix} X_1(1) \\ X_2(1) \\ X_3(1) \end{bmatrix} \quad \mathbf{U(1)} = \begin{bmatrix} U_1(1) \\ U_2(1) \\ U_3(1) \end{bmatrix}$$

$$\mathbf{V(1)} = \begin{bmatrix} V_1(1) & V_2(1) & V_3(1) \end{bmatrix}$$

$$\mathbf{A(0)} = \begin{bmatrix} 0{,}1 & 0{,}2 \\ 0{,}3 & 0{,}3 \end{bmatrix}$$

$$\mathbf{X(1)} = \begin{bmatrix} 1500 \\ 2600 \end{bmatrix}$$

$$\mathbf{U(1)} = \begin{bmatrix} 700 \\ 1400 \end{bmatrix}$$

$$\mathbf{V(1)} = \begin{bmatrix} 600 & 1500 \end{bmatrix}$$

- We want to estimate

$$\mathbf{A(1)} = \begin{bmatrix} a_{11}(1) & a_{12}(1) & a_{13}(1) \\ a_{21}(1) & a_{22}(1) & a_{23}(1) \\ a_{31}(1) & a_{32}(1) & a_{33}(1) \end{bmatrix}$$

Adjustment process (1)

- Multiply

$$\mathbf{A(0)\hat{X}(1)} = \begin{bmatrix} 0,1 & 0,2 \\ 0,3 & 0,3 \end{bmatrix} \begin{bmatrix} 1500 & 0 \\ 0 & 2600 \end{bmatrix} = \begin{bmatrix} 150 & 520 \\ 450 & 780 \end{bmatrix}$$

Transaction matrix
if A(0) = A(1)

- Get row summation, compare with U(1)

$$\mathbf{U}^1 = \left[\mathbf{A(0)\hat{X}(1)}\right]\mathbf{i} = \begin{bmatrix} 150 & 520 \\ 450 & 780 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 670 \\ 1230 \end{bmatrix}$$

- As they are not the same, adjustment is needed. The adjustment is

$$\mathbf{R}^1 = \left[\mathbf{\hat{U}(1)}\right]\left(\mathbf{\hat{U}}^1\right)^{-1} = \begin{bmatrix} 700 & 0 \\ 0 & 1400 \end{bmatrix} \begin{bmatrix} 670 & 0 \\ 0 & 1230 \end{bmatrix}^{-1} = \begin{bmatrix} 1,04478 & 0 \\ 0 & 1,13821 \end{bmatrix}.$$

Adjustment process (2)

- Thus, the new A matrix which guarantees correct U(1) is

$$\mathbf{A}^1 = \mathbf{R}^1\mathbf{A(0)} = \begin{bmatrix} 1,04478 & 0 \\ 0 & 1,13821 \end{bmatrix} \begin{bmatrix} 0,1 & 0,2 \\ 0,3 & 0,3 \end{bmatrix} = \begin{bmatrix} 0,10448 & 0,20896 \\ 0,34146 & 0,34146 \end{bmatrix}.$$

Transaction matrix
if A¹ = A(1)

- Next, must check the V(1) or the column
- Multiply

$$\mathbf{A}^1\mathbf{\hat{X}(1)} = \begin{bmatrix} 0,10448 & 0,20896 \\ 0,34146 & 0,34146 \end{bmatrix} \begin{bmatrix} 1500 & 0 \\ 0 & 2600 \end{bmatrix} = \begin{bmatrix} 156,71642 & 543,28358 \\ 512,19512 & 887,80488 \end{bmatrix}.$$

- Check column sum

$$\mathbf{V}^1 = \mathbf{i}'\left[\mathbf{A}^1\mathbf{\hat{X}(1)}\right] = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 156,71642 & 543,28358 \\ 512,19512 & 887,80488 \end{bmatrix} = \begin{bmatrix} 668,91154 & 1431,08848 \end{bmatrix}.$$

Adjustment process (3)

- As the column sum is different from V(1), adjustment is needed. The adjustment is

$$\mathbf{S}^1 = \left[\hat{\mathbf{V}}(\mathbf{1})\right]\left(\hat{\mathbf{V}}^1\right)^{-1} = \begin{bmatrix} 600 & 0 \\ 0 & 1500 \end{bmatrix}\begin{bmatrix} 668,91154 & 0 \\ 0 & 1431,80488 \end{bmatrix}^{-1} = \begin{bmatrix} 0,89698 & 0 \\ 0 & 1,04815 \end{bmatrix}.$$

- Thus, the new A matrix which guarantees correct V(1) is

$$\mathbf{A}^2 = \mathbf{A}^1\mathbf{S}^1$$
$$= \begin{bmatrix} 0,10448 & 0,20896 \\ 0,34146 & 0,34146 \end{bmatrix}\begin{bmatrix} 0,89698 & 0 \\ 0 & 1,04815 \end{bmatrix} = \begin{bmatrix} 0,09371 & 0,21902 \\ 0,30629 & 0,35791 \end{bmatrix}.$$

- But as we fix the column, the row sum is disturbed. Need to fix back the column. This process should go indifinitely until convergence is reached

Adjustment process (4)

- Series of column and row adjustments may be made
- Recall that

$$\mathbf{A}^2 = \mathbf{A}^1\mathbf{S}^1$$
$$\mathbf{A}^2 = \mathbf{R}^1\mathbf{A(0)}\mathbf{S}^1$$
$$\mathbf{A}^3 = \mathbf{R}^2\mathbf{R}^1\mathbf{A(0)}\mathbf{S}^1$$
$$\mathbf{A}^4 = (\mathbf{R}^2\mathbf{R}^1)\mathbf{A(0)}(\mathbf{S}^1\mathbf{S}^2)$$
$$\mathbf{A}^5 = (\mathbf{R}^3\mathbf{R}^2\mathbf{R}^1)\mathbf{A(0)}(\mathbf{S}^1\mathbf{S}^2)$$

and if we continue ....

$$\mathbf{A}^{2n} = (\mathbf{R}^n \ldots \mathbf{R}^3\mathbf{R}^2\mathbf{R}^1)\mathbf{A(0)}(\mathbf{S}^1\mathbf{S}^2\mathbf{S}^3 \ldots \mathbf{S}^n)$$

- Hence is the name of RAS method

Rounds of adjustment

Value of elements of A^k, in each round of adjustment

k	a_{11}	a_{12}	a_{21}	a_{22}
0	0,1	0,2	0,3	0,3
1	0,10448	0,20896	0,34146	0,34146
2	0,09317	0,21902	0,30629	0,35791
3	0,09239	0,21593	0,30849	0,36048
4	0,09219	0,21612	0,30781	0,36080
5	0,09216	0,21606	0,30785	0,36085
6	0,09216	0,21606	0,30784	0,36086
7	0,09216	0,21606	0,30784	0,36086

Final matrix is

$$\mathbf{A(1)} = \begin{bmatrix} 0.09216 & 0.21606 \\ 0.30784 & 0.36086 \end{bmatrix} \Longleftrightarrow \mathbf{A(0)} = \begin{bmatrix} 0{,}1 & 0{,}2 \\ 0{,}3 & 0{,}3 \end{bmatrix}$$

Other adjustment methods

- There are other adjustment methods which usually is used in preparing regional input-output tables
- Will elaborate this in the regional input-output section