3.1 Centrifugal Pendulum Vibration Absorbers:

Centrifugal pendulum vibration absorbers are a type of tuned dynamic absorber used for the reduction of torsional vibrations in rotating and reciprocating machines. They consist of masses that are constrained to move along specific paths relative to the rotational axis of the machine. Previous analytical studies have considered the performance of single absorber systems with general paths and of multi-absorber systems with a specific path type. The path along which each absorber mass moves is designed to achieve the desired circular paths for the absorbers. These circular paths work fine at small amplitudes, but the non-linear, amplitude-dependent frequency of these simple pendulums limit their effectiveness, and must be accounted for by intentionally mistuning the linearized frequency of the absorber and epicycloidal path absorbers are used in automotive engines. This epicycloidal path is very special, since it is the path that maintains a constant frequency for the absorber over all amplitudes, thereby keeping the absorbers as linear as possible over a large operating range. This path separates two basic types of paths that are considered in the present study. Paths such as circles exhibit softening non-linear behavior, that is, their frequency of oscillation decreases as the amplitude increases, which leads to many shortcomings in absorber performance. Paths such as cycloids exhibit hardening non-linear behavior in which the frequency of oscillation increases as the amplitude increases and avoid many of the difficulties encountered with circular paths. Due to spatial and balancing considerations the practical implementation of CPVA|s usually requires a number of individual absorbers stationed about the center of rotation. We consider a system of n CPVAs mounted on a rotor of inertia J, as shown in below.

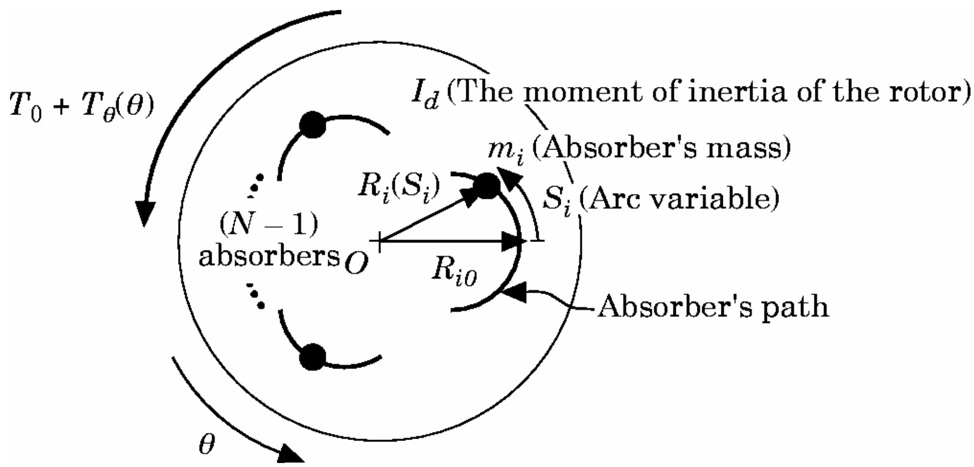

Fig. Schematic view of CPVAs mounted on a rotor.

the equation of motion for the *i*th absorber is

$$m_i\left[\ddot{S}_i+\tilde{G}_i(S_i)\ddot{\theta}-\frac{1}{2}\frac{dX_i}{dS_i}(S_i)\dot{\theta}\right]=-c_{ai}\dot{S}_i \qquad i=1,2,\ldots.N$$

and the equation of motion for the rotor is

$$J\ddot{\theta}+\sum_{i=1}^{N}m_i\left[\frac{dX_i}{dS_i}(S_i)\dot{S}_i\dot{\theta}+X_i(S_i)\ddot{\theta}+\tilde{G}_i(S_i)\ddot{S}_i+\frac{d\tilde{G}_i}{dS_i}(S_i)\dot{S}_i^2\right]=\sum_{i=1}^{N}c_{ai}\tilde{G}_i(S_i)\dot{S}_i-c_0\dot{\theta}+T_0+T(\theta)$$

where m_i = mass of *i*th absorber

R_i = distance of a point on the *i*th absorber path to the center of the rotor

S_i = arc length variable along the path of *i*th absorber

R_{i0} = the value of R_i at the verex of the path

θ = angular orientation of the rotor

c_0 = equivalent coefficient of viscous damping of the rotor

c_{ai} = equivalent coefficient of viscous damping of *i*th absorber

T_0 = mean component of the applied torque

$T(\theta)$ = fluctuating component of the applied torque the path functions $X_i(S_i)$ and $\tilde{G}(X_i)$ are given by

$$X_i(S_i) = R_i^2(\mathrm{S}_\mathrm{i}) \quad \text{and} \quad \tilde{G}(X_i) = \sqrt{X_i(S_i) - \frac{1}{4}\left(\frac{dX_i}{dS_i}(S_i)\right)^2}$$

each absorber is indirectly coupled to all the other absorbers through the dynamics of the rotor. From the equation of motion of the rotor it is clear that it is affected by each absorber through inertial and damping terms.

These equations of motion represent an independent dynamical system, because the varying component of the applied torque, T(θ), which is expressed as a function of the rotor angle θ. For the purposes of analysis, it is convenient to choose the rotor angle as the independent variable, replacing time. To non-dimensionalize the problem, we first define a new variable ν as the ratio of the rotor angular velocity to the average rotor angular velocity Ω,

$$v = \frac{\dot{\theta}}{\Omega}$$

rotor angular velocity ν and arc length variables are variable of θ. Using the chain rule; we can obtain the following relationships between derivatives with respect to time and derivatives with respect to θ as

$$\ddot{\theta} = \frac{d^2\theta}{dt^2} = \Omega^2 v \frac{dv}{d\theta} = \Omega^2 v v'$$

$$\frac{d(d\theta/dt)}{dt} = \Omega v \frac{d(d\theta/dt)}{d\theta} = \Omega v (d\theta/dt)'$$

$$\frac{d^2(d\theta/dt)}{dt^2} = \Omega^2 v \frac{dv}{dt}\frac{d(d\theta/dt)}{d\theta} + \Omega^2 v^2 \frac{d^2(d\theta/dt)}{d\theta^2}$$

$$= \Omega^2 v v'(d\theta/dt)' + \Omega^2 v^2 (d\theta/dt)''$$

the equations of motion becomes

$$v s_i'' + \left[s_i' + \tilde{g}_i(s_i)\right]v' - \frac{1}{2}\frac{dx_i(s_i)}{ds_i} v = -\mu_{ai} s_i' \qquad i = 1, 2 \ldots N$$

$$\sum_{i=1}^{N} b_i \left[\frac{dx_i(s_i)}{ds_i} s_i' v^2 + x_i(s_i) v v' + \tilde{g}_i(s_i) s_i'' v^2 + \frac{d\tilde{g}_i(s_i)}{ds_i} s_i^2 v^2\right] + v v'$$

$$= \sum_{i=1}^{N} b_i \mu_{ai} \tilde{g}_i(s_i) s_i' v - \mu_0 v + \Gamma_0 + \Gamma(\theta)$$

where $\quad s_i = \dfrac{S_i}{R_{i0}}$, $\quad b_i = \dfrac{I_i}{J}$,

$$I_i = m_i R_{i0}^2 \ , \quad \mu_{ai} = \frac{c_{ai}}{m_i \Omega} \ , \quad \mu_0 = \frac{c_0}{J\Omega}$$

$$\Gamma_0 = \frac{T_0}{J\Omega^2} \ , \quad \Gamma(\theta) = \frac{T(\theta)}{J\Omega^2}$$

$$x_i(s_i) = \frac{X_i(s_i R_{i0})}{R_{i0}^2} \ \text{and} \ \ \tilde{g}_i(s_i) = \sqrt{x_i(s_i) - \frac{1}{4}\left(\frac{dx_i}{ds_i}(s_i)\right)^2}$$

now the system is dependent dynamic system, but its degree has been reduced, since only first derivatives in ν appear. Assuming that all the absorbers have the same mass, and all the paths have the same value of R_i at each vertex, i.e.,

$$m_i = m \text{ , and } R_{i0} = R_0 \qquad \forall_i \in [1, N]$$

Equation of motion of the rotor becomes

$$\frac{b_0}{N}\sum_{i=1}^{N}\left[\frac{dx_i(s_i)}{ds_i}s_i'v^2 + x_i(s_i)vv' + \tilde{g}_i(s_i)s_i'vv' + \tilde{g}_i(s_i)s_i''v^2 + \frac{d\tilde{g}_i(s_i)}{ds_i}s_i'^2v^2\right] + vv'$$

$$= \frac{b_0}{N}\sum_{i=1}^{N}\mu_{ai}\tilde{g}_i(s_i)s_i'v - \mu_0 v + \Gamma_0 + \Gamma(\theta)$$

where $\quad b_0 = \dfrac{I_0}{J} \;,\; I_0 = m_0 R_0^2 \;$ and $\; m_0 = Nm$

the fluctuating torque generally contains several harmonics. In most situations, only one or two harmonics have significant amplitude, and therefore we approximate the fluctuating torque by its dominant harmonic, taken to be of the order n, as

$$\Gamma(\theta) = \Gamma_0 \sin(n\theta)$$

the path for the ***i***th absorber by the local radius of curvature at any point on the path, given by

$$\rho_i = \sqrt{\rho_{i0}^2 - \lambda^2 S_i^2}$$

$$\rho_{i0} = \text{path's radius of curvature at the vertex}$$

value of λ varies from 0 to 1.

3.2 Untuned Torsional Vibration Absorber:

The distributing frequencies of torsional vibration oscillation are proportional to rotational speed. However there is generally more than one such frequencies and the centrifugal pendulum vibration absorber has the disadvantage that the several pendulums tuned to order number of the disturbance have been used. As compared to centrifugal pendulum vibration absorbers the untuned viscous torsional damper is effective over the wide operating range. It consists of a disk in a cylindrical cavity which is filled with viscous fluid as shown below. It is mounted on the rotating and reciprocating machine components to reduce the torsional vibration which may cause failure of the rotating and reciprocating machine component. The disk provides rotational inertia. The viscous fluid provides the damping effect for the absorber. This type of vibration absorbers are mostly used in automobile engines.

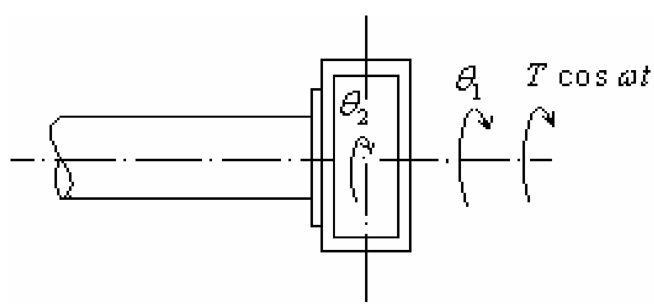

Fig. Untuned Torsional vibration absorber.

the equation of motion for above system is

$$I\ddot{\theta}_1 + k\theta_1 + c(\dot{\theta}_1 - \dot{\theta}_2) = T\cos\omega t$$

$$I_d\ddot{\theta}_2 - c(\dot{\theta}_1 - \dot{\theta}_2) = 0$$

$I =$ inertia of the shaft

$k =$ stiffness of the shaft

$c =$ damping factor of the viscous fluid inthe cavity

$I_d =$ mass moment of inertia of the absorber disk

$\theta_2 =$ angular displadement of the shaft

$\theta_2 =$ angular displadement of the absorber disk

$T =$ magnitude of applied oscillatory torque

Assuming harmonic solution

$$\theta_1 = (\theta_1)_0 \cos(\omega t + \phi)$$

$$\theta_2 = (\theta_2)_0 \cos(\omega t + \phi)$$

where $(\theta_1)_0$ and $(\theta_2)_0$ are amplitudes

Substituting these values in above differential equation we get,

$$\left[\left(\frac{k}{I} - \omega^2\right) + i\frac{c\omega}{I}\right](\theta_1)_0 - i\frac{c\omega}{I}(\theta_2)_0 = \frac{T}{I}$$

and $$\left(-\omega^2 + i\frac{c\omega}{I}\right)(\theta_2)_0 = i\frac{c\omega}{I_d}(\theta_1)_0$$

Solving these equations we have,

$$\frac{(\theta_1)_0}{T} = \frac{(\omega^2 I_d - ic\omega)}{\left[\omega^2 I_d (k - \omega^2 I) + ic\omega(\omega^2 I_d - (k - \omega^2 I))\right]}$$

the response of the system can also be written as

$$\frac{k(\theta_1)_0}{T} = \sqrt{\frac{\mu^2 r^2 + 4\zeta^2}{\mu^2 r^2 (1-r^2) + 4\zeta^2 (\mu r^2 + r^2 - 1)^2}}$$

where $r = \dfrac{\omega}{\omega_n}$, $\mu = \dfrac{m_2}{m_1}$ and $\zeta = \dfrac{c}{c_c}$

m_1 = mass of the shaft

m_2 = mass of the absorber disk

c_c = critical damping factor of the absorber

ω_n = natural freequecy of the system

from above equation it is clear that the response depends upon $\boldsymbol{r}$, μ and ζ.

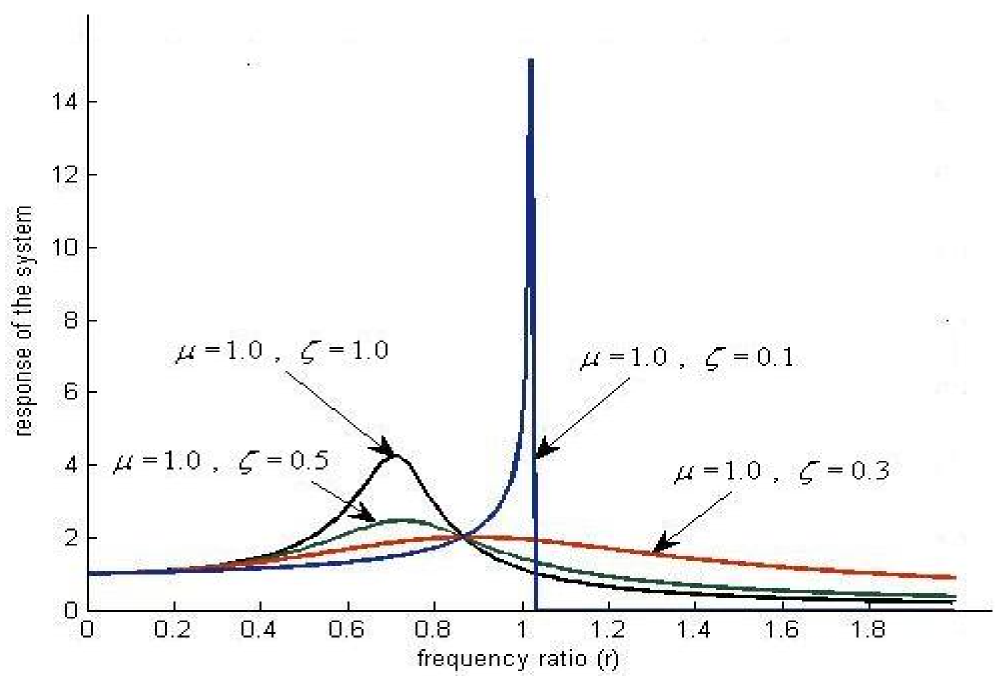

Fig. Response of the untuned torsional vibration absorber for μ=1.0 at ζ=0.1, ζ=0.3, ζ=0.5 and ζ=1.0

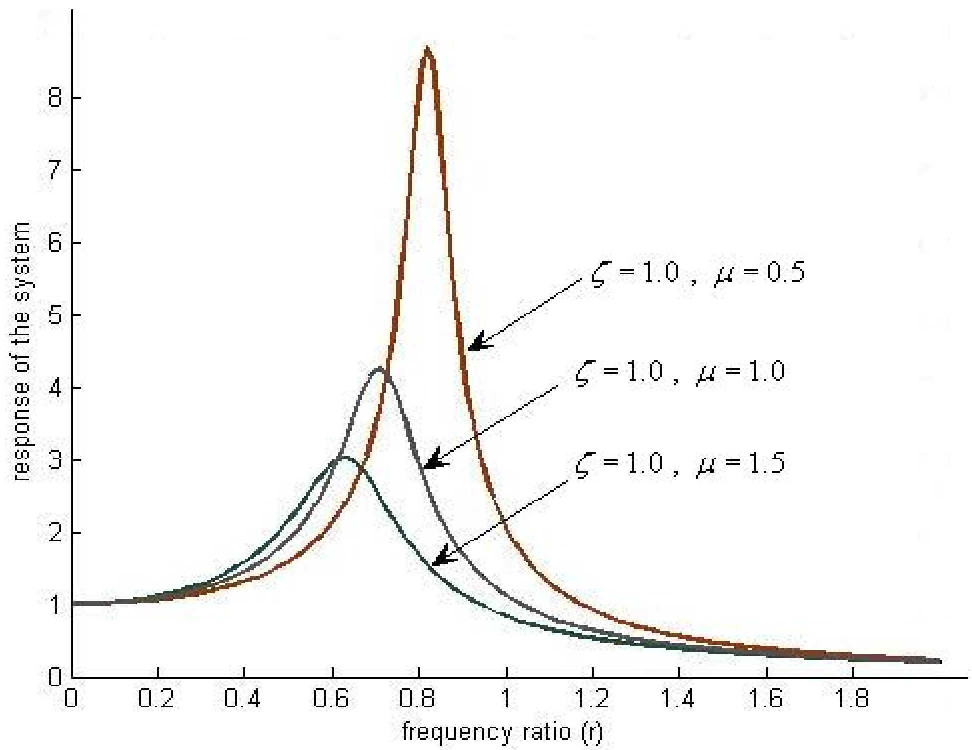

Fig. Response of the untuned torsional vibration absorber for ζ=1.0 at μ=0.5, μ=1.0 and μ=1.5