2.1 Frahm's Dynamic Vibration Absorber:

In an undamped or lightly damped system when the excitation frequency nears the natural frequency the amplitude of the vibration can get extremely high. This phenomenon is called ***resonance***. If resonance occurs in a mechanical system it can be very harmful-- leading to eventual failure of the system. Consequently one of the major reasons for vibration analysis is to predict when resonance may occur and to determine what steps to take to prevent it from occurring. The magnitude of response can be reduced if the natural frequency can be shifted away from the forcing frequency by changing the stiffness or mass of the system. An additional degree of freedom can be added by attaching an auxiliary mass to the system through spring and/or damper called ***vibration absorber*** such that the natural frequency of the system are away from the excitation frequency. The auxiliary mass when attached to the system through elastic element (spring), called ***undamped vibration absorber*** reduces he system response by keeping the natural frequency of the system away from the excitation frequency. Undamped vibration absorber is also termed as Frahm's vibration absorber. It is the simplest type of vibration absorber.

The auxiliary mass when attached to the system through elastic element (spring), called ***undamped vibration absorber*** reduces he system response by keeping the natural frequency of the system away from the excitation frequency.

For the system as shown below

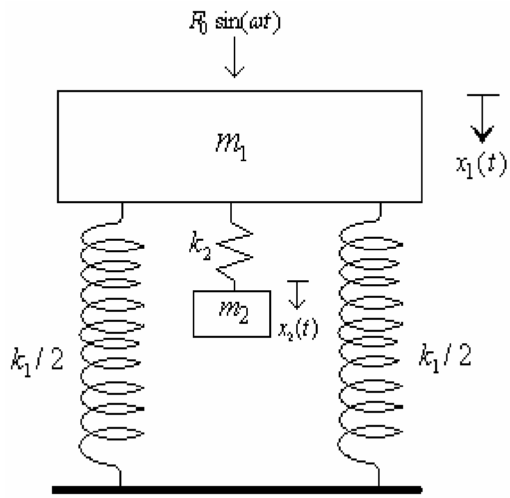

Fig. undamped dynamic vibration absorber.

the equation of motion is

$$m_1\ddot{x}_1 + k_1x_1 + k_2\left(x_1 - x_2\right) = F_0 \sin(\omega t)$$
$$m_2\ddot{x}_2 + k_2\left(x_2 - x_1\right) = 0$$

m_1 = mass of the system called primary mass

m_2 = mass of the absorber called secondary mass

k_1 = spring stiffness of the islator

k_2 = spring stiffness of the absorber

assuming harmonic solution

$$x_j(t) = X_j \sin(\omega t) \qquad j = 1, 2$$

the steady state amplitudes of the masses are

$$X_1 = \frac{\left(k_2 - m_2\omega^2\right)F_0}{\left(k_1 + k_2 - m_1\omega^2\right)\left(k_2 - m_2\omega^2\right) - k_2^2}$$

$$X_2 = \frac{k_2 F_0}{\left(k_1 + k_2 - m_1\omega^2\right)\left(k_2 - m_2\omega^2\right) - k_2^2}$$

for $m_1 = m_2$ and $k_1 = k_2$ response of the system becomes

$$\frac{k_1 X_1}{F_0} = \frac{1}{(2-r^2)(1-r^2)-1}$$

where $r = \dfrac{\omega}{\omega_n}$ and natural frequency of the system $\omega_n = \sqrt{\dfrac{k_1}{m_1}}$

We have following plot for response of the system against frequency ratio

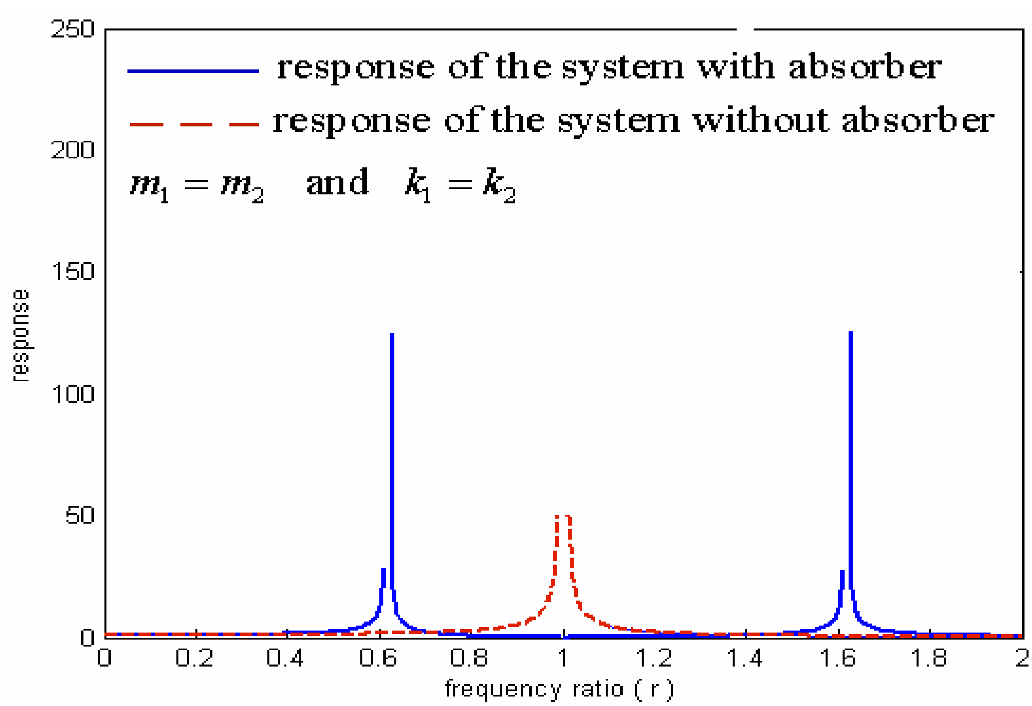

Fig. Response of the system with absorber for m_1=m_2 and k_1=k_2

2.2 Damped Vibration Absorber:

The auxiliary mass when attached to the system through elastic element (spring) along with an energy dissipating member (damper), called ***damped vibration absorber***. The amplitude of the system can be reduced by adding a damped vibration absorber as shown below.

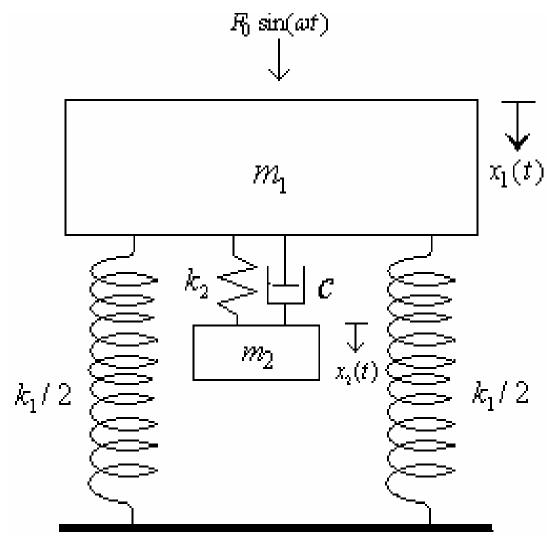

Fig. damped vibration absorber.

the equation of motion of the two masses is

$$m_1\ddot{x}_1 + k_1x_1 + k_2(x_1 - x_2) + c(\dot{x}_1 - \dot{x}_2) = F_0 \sin(\omega t)$$

$$m_2\ddot{x}_2 + k_2(x_2 - x_1) + c(\dot{x}_2 - \dot{x}_1) = 0$$

m_1 = mass of the system called primary mass

m_2 = mass of the absorber called secondary mass

k_1 = spring stiffness of the islator

k_2 = spring stiffness of the absorber

c = damping coefficient of the damper

assuming the solution

$$x_j(t) = X_j e^{i\omega t} \qquad j = 1, 2$$

the steady-state solution is

$$X_1 = \frac{F_0(k_2 - m_2\omega^2 + ic\omega)}{\left[(k_1 - m_1\omega^2)(k_2 - m_2\omega^2) - m_2 k_2 \omega^2\right] + ic\omega(k_1 - m_1\omega^2 - m_2\omega^2)}$$

$$X_2 = \frac{X_1(k_2 + ic\omega)}{(k_2 - m_2\omega^2 + ic\omega)}$$

for $m_1 = m_2$ and $k_1 = k_2$ response of the system becomes

$$\frac{k_1 X_1}{F_0} = \left[\frac{(2\zeta r)^2 + (r^2-1)^2}{(2\zeta r)^2(2r^2-1)^2 + \left\{r^2 - (r^2-1)^2\right\}^2}\right]^{1/2}$$

where $r = \dfrac{\omega}{\omega_n}$, natural frequency of the system $\omega_n = \sqrt{\dfrac{k_1}{m_1}}$ and damping ratio $\zeta = \dfrac{c}{c_c}$

critical damping constant $c_c = 2m_2\omega_{11}$

We have following plot for response of the system against frequency ratio

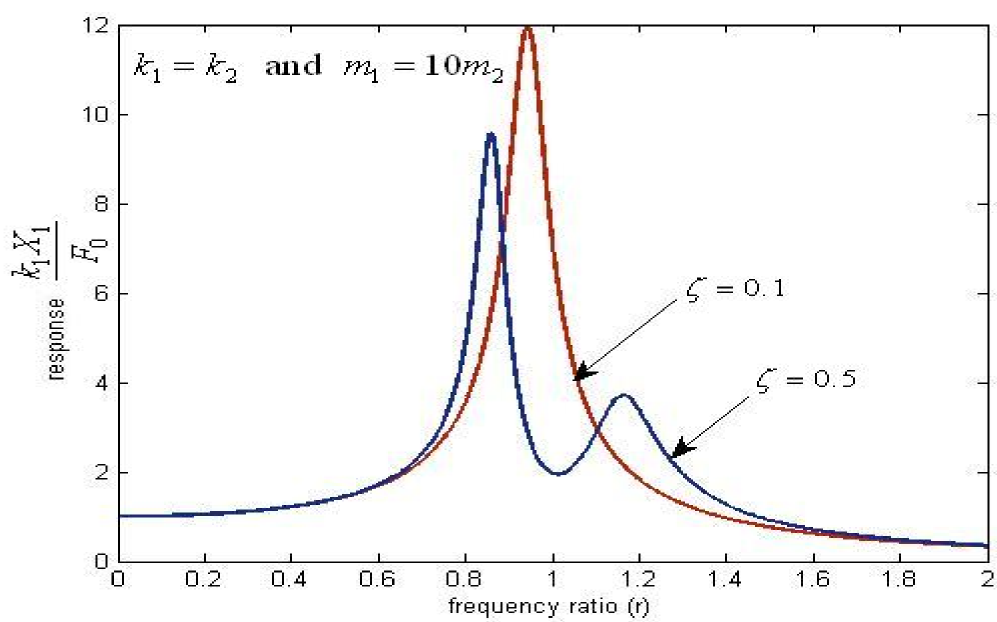

Fig. frequency response of the system with damped vibration absorber
for ζ = 0.1 and ζ = 0.5 at m_1=10m_2 and k_1=k_2

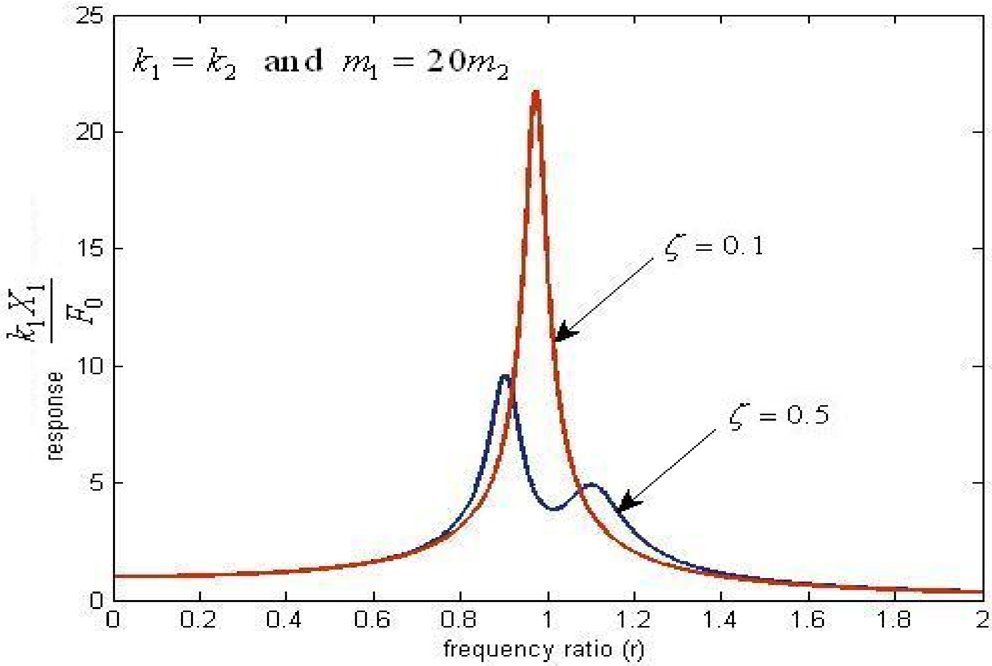

Fig. frequency response of the system with damped vibration absorber for $\zeta = 0.1$ and $\zeta = 0.5$ at m_1=20m_2 and k_1=k_2

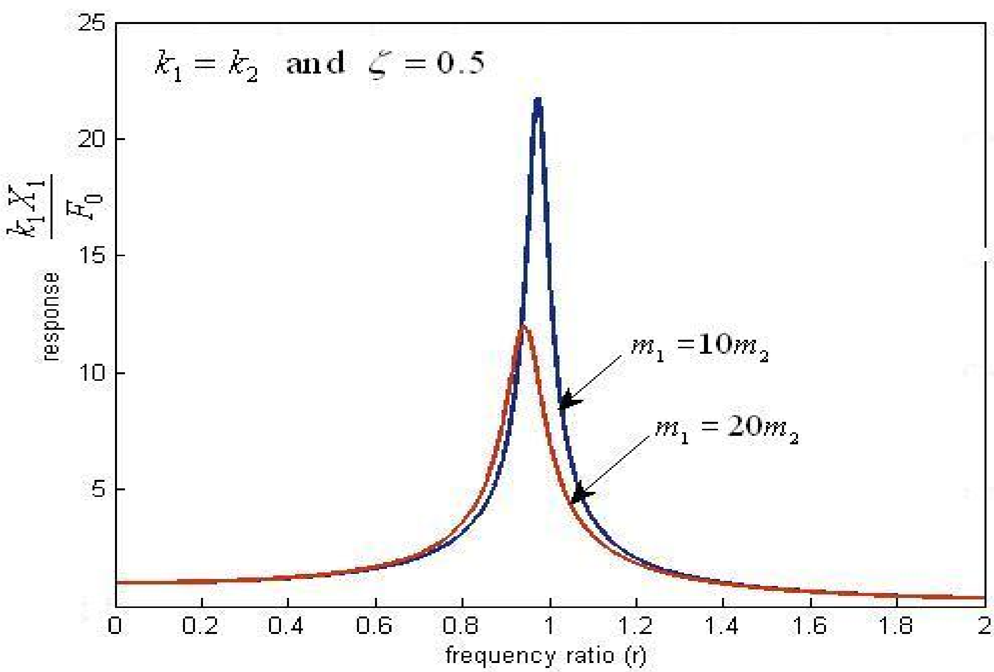

Fig. frequency response of the system with damped vibration absorber for m_1=10m_2 and m_1=20m_2 at $\zeta = 0.5$ and k_1=k_2