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AIRBORNE GRAVITY FIELD MEASUREMENTS BY USING INERTIAL NAVIGATION SYSTEMS AND DIFFERENTIAL GLOBAL POSITIONING SYSTEM

by Anastas Madjarov

Introduction

Aim of topic: Investigating the readings of accelerometers installed on a vehicle flying within the indicated range of altitudes and velocities by separating the inertial components and leaving only gravitational field measurements.

The aim presupposes a solution to an inverse problem in relation to the navigation one. The normal gravity field in the inertial navigation systems is separated from accelerometer readings by its mathematical model and then the flying vehicle velocities and location are determined by integrating the inertial components. Conversely, if we have accurate data about the flying vehicle velocities and location (e.g. when using GPS under a differential operating mode), then by applying a mathematical model we can separate the inertial components from the accelerometer readings thus leaving only gravity field measurements.

The technical system designed for solving the problem will be called Gravitation Measurement System (GMS).

Gravitation Measurement System

We shall assume that the system has been constructed according to the scheme, shown in Figure 1. On board the plane are installed:

- A receiver of signals from the ground correction station;
- Two receivers of signals from GPS satellites capable of operating under a differential mode [3];
- A control display unit (a computer connected with the receivers by RS-232 so as to send them control commands and receive, process and visualize their data);
- Inertial units consisting of three accelerometers and three gyroscopic angular rate sensors (RFOG for strapdown systems) with mutually perpendicular sensitive axes [2]. One of this is installed in the plane mass center along its construction lines. The control display unit uses this unit for solving the navigation problem and others for gravity field measurements using its readings.

- A GMS controller for solving the gravitation measurements problem. Additional inertial units are installed along the same sensitive axes (the plane construction lines) but at a certain distance.

The control display unit using the data from DGPS and the inertial unit installed in center of mass solves the navigation problem. Thus, on board the plane, besides velocities and location of the mass center, we also have a navigation apparatus base $\xi\eta\zeta$. The presence of data from the ground station about the true heading requires calculation of its azimuth and, therefore, calculation of its orientation according to the geodetic frame of reference $J_1J_2J_3$ (East, North, geodetic vertical). By using the navigation frame of reference $\xi\eta\zeta$ and measurements coming from the inertial measuring units, it is possible to calculate the angles of roll, pitch, true heading and their derivatives. This is required so that the inertial interference caused by the plane angular rates and accelerations can be separated from the accelerometer readings.

The readings from the accelerometers in additional units are collected, processed and compared in the GMS controller with those from the navigation controller. The readings, cleared from dynamic corrections, are reduced to $J_1J_2J_3$ and together with the coordinates and measuring time, are recorded in an archive device.

A ground correction station operates in the area of geophysical measurements, which has known geodetic coordinates: latitude B_e, longitude L_e, altitude above ellipsoid h_e, altitude above geoid H_e.

With respect to the data received by the station and by a satellite receiver of signals on board the plane, we shall assume that at one-second intervals the following parameters of motion of the plane center of mass are known:

- B_k, L_k, h_k (X_k, Y_k, Z_k) are geodetic polar or Cartesian coordinates;
- W_{Ek}, W_{Nk}, $W_{\zeta k}$ are eastern, northern and vertical ground velocities;

as well as ψ_k, t_k are true heading and time.

Formulation of task

The aim is to synthesize an algorithm for the operation of a gravimetric system which structural scheme is shown in Figure 1. This is an algorithm performed in the Gravitation Measurement Processing (Control Unit). At it output readings should be obtained about the projections g_{xk}, g_{yk}, g_{zk} of the gravity field in a measuring basis xyz of GMS. For a starting point of the measuring basis we assume the plane mass center. In order to achieve the aim it is necessary to extract the inertial components from accelerometer readings reduced to the measuring basis. These components can be divided into two types according to their origin.

The first type are inertial components of the relative and transfer movement of the center of xyz. They can be calculated in the presence of exact position and velocity data for the movement of the plane mass centre. Here only the measurements by the radio system are sufficient. Because of its discreet character as well as because of the possibility for the readings to fail for a certain period of time, it is necessary to conduct parallel calculation of velocities and location by the off-line inertial method.

The second type are inertial components of the relative movement of the plane around xyz or around its mass centre. They could exist when measuring sensors of the gravity field (accelerometers) are positioned outside the mass centre and fixed to the plane body. If they were positioned on an ideal gyrostabilized horizontal platform, the effect of the plane angular orientation would not exist. In this case though, the accuracy and sensitivity requirements of the accelerometers would be extremely high for real technical devices operating in flight. The high price of the gyroplatforms and, moreover, the possibility for only one platform to be positioned sufficiently close to the plane mass centre, determine the need for measuring of the absolute angular rates of the plane. The use of sensors for these measurements allows to implement a non-platform variant for calculating the spatial (angular) plane orientation. Therefore, let us project the measurements of a great number of accelerometers positioned outside the plane mass centre onto a common basis. An inertial basis is closest to the nature of measuring the absolute angular rate. However, it is not suitable for use by GPS. A compromise is the use of a navigation frame of reference. The use of numerous accelerometers presupposes lower requirements for their accuracy when applying a suitable method for separating the useful signal from the gravity field. At the same time the use of sensors is required for determining the angular orientation of the plane.The need also arises for separating the second type of inertial components.

We shall assume that from the solution to the navigation problem only by the radioengineering method, in the User Control/Display Unit we know:

– B_k, L_k, h_k (X_k, Y_k, Z_k) - geodetic polar or Cartesian coordinates of the plane mass centre;

– $W_{Ek}, W_{Nk}, W_{\zeta k}$ - eastern, northern and vertical ground speeds;

as well as ψ_k, t_k - true heading and time entered into GMS at discrete time intervals at moments t_k. In the periods $T = t_{k+1} - t_k = const$, GMS uses the last readings of the differential GPS stored by the moment t_k. During the time *T*, the navigation problem is solved off-line, only by readings of accelerometers and gyroscopic sensors according to a strapdown algorithm (SINS) described in [1].

The presence of accurate position and velocity data at moments t_k allows to calculate the absolute angular rates of the navigation frame of reference, following the sequence of calculations [3]:

$$U_k^P = \begin{pmatrix} -\cos A\sin L_k + \sin A\sin B_k \cos L_k & \cos L_k \cos A + \sin A\sin B_k \sin L_k \\ -\sin A\sin L_k - \cos A\sin B_k \cos L_k & \sin A\cos L_k - \cos A\sin B_k \sin L_k \\ \cos B_k \cos L_k & \cos B_k \sin L_k \end{pmatrix}$$

$$\left.\begin{matrix} -\sin A\cos B_k \\ \cos A\cos B_k \\ \sin B_k \end{matrix}\right\}, \qquad A=-\psi_k(t_k),$$

$$\xi_k=(1-e^2 sin^2 B_k)^{-1/2},\; G_k=a\xi_k+h_k \;\; Q_k=a\xi_k^3(1-e^2)+h_k, \quad (1)$$

$$\Omega_\xi=\Omega u_{13}^p, \quad \Omega_\eta=\Omega u_{23}^p,\; \Omega_\zeta=\Omega u_{33}^p,$$

$$\omega_\xi^p=-\frac{W_{Nk}}{Q_k}\cos A-\frac{W_{Ek}}{G_k}\sin A,\; \omega_\eta^p=-\frac{W_{Nk}}{Q_k}\sin A+\frac{W_{Ek}}{G_k}\cos A, \quad (2)$$

$$\omega_\xi^{ap}=\omega_\xi^p+\Omega_\xi, \quad \omega_\eta^{ap}=\omega_2^p+\Omega_\eta, \quad \omega_\zeta^{ap}=\Omega_\zeta . \quad (3)$$

The calculated (3) allows to obtain calculated readings (references) for the accelerometers:

$$\begin{aligned} a_\xi(t_k)&=[W_{Ek}-W_{E(k-1)}]/T+(\omega_\eta^p+2\Omega_\eta)W_{\zeta k}-2\Omega_\zeta W_{Nk}-g_{\xi k}^T, \\ a_\eta(t_k)&=[W_{Nk}-W_{N(k-1)}]/T-(\omega_\xi^p+2\Omega_\xi)W_{\zeta k}+2\Omega_\zeta W_{Ek}-g_{\eta k}^T, \\ a_\zeta(t_k)&=[W_{\zeta k}-W_{\zeta(k-1)}]/T+(\omega_\xi^p+2\Omega_\xi)W_{Nk}-(\omega_\eta^p+2\Omega_\eta)W_{Ek}-g_{nk}^T, \end{aligned} \quad (4)$$

where the reference component of the specific gravity will also have a computational value [5]:

$$\vec{g}^T=\begin{pmatrix} g_{\xi k}^T \\ g_{\eta k}^T \\ g_{\zeta k}^T \end{pmatrix}=\begin{pmatrix} \Omega^2(a-a\xi_k-h_k)u_{13}^p u_{33}^p \\ \Omega^2(a-a\xi_k-h_k)u_{23}^p u_{33}^p \\ -g_e^T(1-2\frac{h_k}{a}+\beta u_{33})-\Omega^2(a-a\xi_k-h_k)(1-u_{33}^{p2}) \end{pmatrix}, \quad (5)$$

$$g_{nk}^T=g_{nk}+\Omega^2(a\xi_k+h_k)\cos^2 B_k .$$

Following this course of thinking, the structure of GMS from Figure 1 is proposed as well as the different purposes of the two computational control units.

Navigation algorithm

For an algorithm of the User Control/Display Unit we shall consider a standard algorithm of a GPS differential operation, supplemented with the algorithm described in Appendix 3 for a non-platform inertial system.

Its purpose is, under normal DGPS operation mode, to use its position and velocity data in GMS for calculating the inertial components of the first type.

At the same time, a SINS algorithm is performed which is corrected at each instant of valid readings from DGPS. The main purpose of SINS is to be used for determining the angular plane orientation: angles of heading, roll and pitch. These angles are necessary for projecting the readings of the many accelerometers onto a unified GMS measuring basis. The geodetic frame of reference is taken as such a basis which is oriented along the geodetic vertical axis determined by the standard value of the normal gravity field. When the DGPS readings fail, it is possible for GMS to continue operation on the output data from SINS.

We shall assume that the integration of the differential equations in SINS is performed at intervals $\tau = t_{i+1} - t_i = const$ which are sufficiently small for the change of the absolute angular rate of the plane $\omega^a_{x1}, \omega^a_{y1}, \omega^a_{z1}$ to be treated as constant. Generally, the proposed algorithm consists of a continuous non-stationary system of differential equations and its numerical integration requires that step τ be sufficiently small. We shall assume for convenience that $\tau < T;\quad j\tau = T$ is fulfilled, where j is an integer. Time τ will also be necessary for collecting and processing the data from several inertial units. If it turns out to be insufficient, then it must be saved for a while only in unit 2. The other equations can also be integrated by a larger step. The choice of an integration step is an independent problem with sufficient complexity for performing a numerical integration of a non-stationary system.

At the time instants t_k the initial values of the integrators are established according to the DGPS data: $D(t_k) = D[\psi_k, \gamma, \vartheta]$, $U(t_k) = U[Bt_k, L_k, -\psi_k]$, $W_\xi(t_k) = W_{Ek}$, $W_\eta(t_k) = W_{Nk}$, $W_\zeta(t_k) = W_{\zeta k}$. For this purpose, the equalities in unit 1 are substituted for (1-3), which is an efficient position (location) and velocity correction.

GMS algorithm

The purpose of the gravitation measurement system is to process the readings of a great number of accelerometers positioned outside the plane mass center by separating instrumental and methodological errors in the form of inertial components of the first and second type in order to obtain gravity field measurements according to instrumental basis xyz. DGPS is used for separating the first type of inertial components. The second type of inertial components are separated by using a suitable layout of an even number of sets of three accelerometers. The angular rate of the plane, obtained from SINS, is used for projecting the accelerometer readings onto xyz. The constant and slowly changing instrumental errors of the accelerometers, as well as of the gyroscopes, are separated by their stochastic model added to the SINS algorithm. Its parameters are specified at a stage of initial establishment for a stationary plane.

We shall divide the algorithm synthesis in two stages. **Firstly**, we shall use data only from the central unit of inertial sensors (unit 0) installed in the plane mass centre. At the second stage we shall include for use the other inertial units. Results from the GMS measurements will be obtained at moments t_k, as a result of the following computational process which is a continuation of (1)-(3):

$$\vec{g} = \begin{pmatrix} g_{\xi k} \\ g_{\eta k} \\ g_{nk} \end{pmatrix} = \begin{pmatrix} a\Omega^2 u^p_{13} u^p_{33} \\ a\Omega^2 u^p_{23} u^p_{33} \\ -g^T_e [1 + q - 2\dfrac{h}{a} + 0.5(3q - e^2) u^{p2}_{33}] \end{pmatrix}, \tag{6}$$

$$\varpi_{\xi k}=[W_{Ek}-W_{E(k-1)}]/T+(\omega_{\eta}^{p}+2\Omega_{\eta})W_{\zeta k}-2\Omega_{\zeta}W_{Nk}+\Omega^{2}G_{k}u_{13}^{p}u_{33}^{p},$$
$$\varpi_{\eta k}=[W_{Nk}-W_{N(k-1)}]/T-(\omega_{\xi}^{p}+2\Omega_{\xi})W_{\zeta k}+2\Omega_{\zeta}W_{Ek}+\Omega^{2}G_{k}u_{23}^{p}u_{33}^{p}, \tag{7}$$
$$\varpi_{\zeta k}=[W_{\zeta k}-W_{\zeta(k-1)}]/T+(\omega_{\xi}^{p}+2\Omega_{\xi})W_{Nk}-(\omega_{\eta}^{p}+2\Omega_{\eta})W_{Ek}-\Omega^{2}G_{k}(1-u_{33}^{p2}),$$

$$\begin{pmatrix} g_{x}(t_{k}) \\ g_{y}(t_{k}) \\ g_{z}(t_{k}) \end{pmatrix}=\begin{pmatrix} \varpi_{\xi k} \\ \varpi_{\eta k} \\ \varpi_{\zeta k} \end{pmatrix}-D^{*}\begin{pmatrix} a_{x1}^{*0} \\ a_{y1}^{*0} \\ a_{z1}^{*0} \end{pmatrix}. \tag{8}$$

The synthesized algorithm (1)-(8) uses B_k, L_k, h_k, $W_{Ek}, W_{Nk}, W_{\zeta k}$, ψ_k, t_k valued from DGPS and readings $a_{x1}^{*0}, a_{y1}^{*0}, a_{z1}^{*0}$ of accelerometers positioned in the plane mass centre. Matrix D^* is taken from unit 2 of the SINS algorithm and is used in (8) as a function of the heading, roll and pitch for projecting $a_{x1}^{*0}, a_{y1}^{*0}, a_{z1}^{*0}$ on the axes of the GMS measuring basis.

If from equations (8) we take out the model (6) of standard gravity field [4], we shall calculate the gravitation deviations:

$$\begin{pmatrix} \Delta g_{\xi}(t_{k}) \\ \Delta g_{\eta}(t_{k}) \\ \Delta g_{\zeta}(t_{k}) \end{pmatrix}=\begin{pmatrix} g_{\xi k} \\ g_{\eta k} \\ g_{nk} \end{pmatrix}-\begin{pmatrix} g_{x}(t_{k}) \\ g_{y}(t_{k}) \\ g_{z}(t_{k}) \end{pmatrix}=\begin{pmatrix} g_{\xi k} \\ g_{\eta k} \\ g_{nk} \end{pmatrix}-\begin{pmatrix} \varpi_{\xi k} \\ \varpi_{\eta k} \\ \varpi_{\zeta k} \end{pmatrix}+D^{*}\begin{pmatrix} a_{x1}^{*0} \\ a_{y1}^{*0} \\ a_{z1}^{*0} \end{pmatrix}. \tag{9}$$

The difference between the computational standards (4) and those measured by the accelerometers of unit 0 is:

$$\begin{pmatrix} \Delta a_{\xi} \\ \Delta a_{\eta} \\ \Delta a_{\zeta} \end{pmatrix}=D^{*}\begin{pmatrix} a_{x1}^{*0} \\ a_{y1}^{*0} \\ a_{z1}^{*0} \end{pmatrix}-\begin{pmatrix} a_{\xi}(t_{k}) \\ a_{\eta}(t_{k}) \\ a_{\zeta}(t_{k}) \end{pmatrix}. \tag{10}$$

The differences between the calculated (3) and those measured by gyroscopes of unit 0 absolute angular rates of the navigation frame of reference are, respectively:

$$\begin{pmatrix} \Delta\omega_{\xi}^{a} \\ \Delta\omega_{\eta}^{a} \\ \Delta\omega_{\zeta}^{a} \end{pmatrix}=D^{*}\begin{pmatrix} \omega_{x1}^{a*0} \\ \omega_{y1}^{a*0} \\ \omega_{z1}^{a*0} \end{pmatrix}-\begin{pmatrix} \omega_{\xi}^{ap} \\ \omega_{\eta}^{ap} \\ \omega_{\zeta}^{ap} \end{pmatrix}, \tag{11}$$

Equalities (10) and (11) are the difference between the GMS and SINS measurements and can serve for equation of the relation. They would be equal to zero in the absence of instrumental errors in the sensors of unit 0 and when there is an accurately prescribed model of the gravity field in SINS (in the absence of or completely known and read anomalies [5]). For a known field and a stationary plane at a stage of initial establishment, they can be used for calculating the constant and slowly changing instrumental errors in unit 0. In this case, of course, for quite a long time we can rely on the exceptional accuracy of GPS.

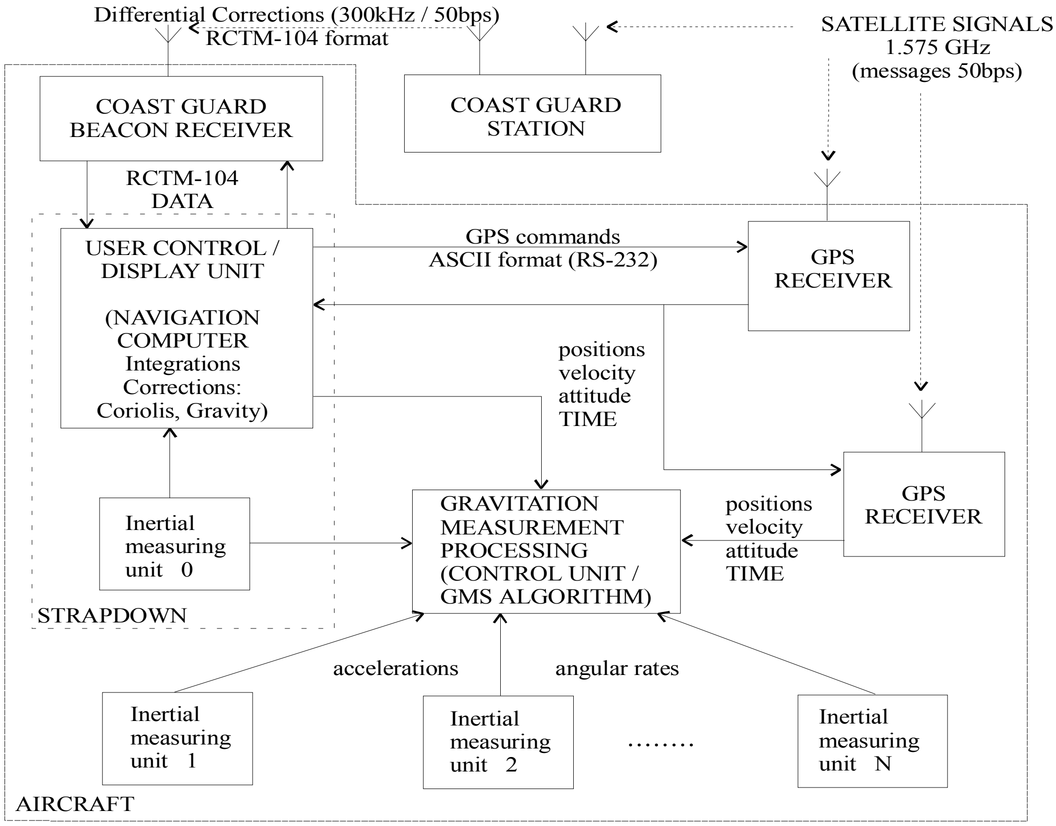

Figure 1 Gravitation measurements in flight

Secondly, we shall synthesize an algorithm by using data from an even number of inertial sensors (unit n), n=2N+1, where N is the number of pairs of inertial units. The aims in using numerous sensors are firstly, eliminating the dynamic error in the accelerometer readings of the second type, and secondly, achieving error decrease by correcting the sensor readings to a measuring basis xyz. The first aim can be achieved by positioning of additional measuring units in pairs, symmetrically in relation to the plane mass center, and with measuring axes being parallel to the plane construction lines $x_1 y_1 z_1$ and mutually parallel.

Problems related to gravitation measurements in flight

Gravity field board sensors are understood to be sensors measuring the first, second and other derivatives of the Earth's gravity potential. Gravimeters and variometers for ground measurements, as well as other instruments, designed especially for marine and aircraft gravity field surveys, are used as such sensors.

A major difficulty in measuring the gravitation force on board a moving object under vibrations and overload of different amplitude range is the complexity in separating the useful gravitation signal from the background of large inertial interference. The directions for solving the problem can be summarized in the following:

- Developing new methods and samples of measuring sensors having higher sensitivity;
- Using precise gyrostabilized platforms for the main sensors and separate autonomous sensors for the inertial interference;
- Using numerous sensors designed by different methods of measuring and, hence, having a different frequency range of measurement. Using frequency methods for processing the readings and error filtering.
- In order to diminish the Eötvös effect, it is necessary to have accurate data about spatial coordinates and direction of movement of the board carrier;
- The difficulties in separating the gravitation and inertial accelerations when measuring first derivatives of the gravity potential show that it is a promising enterprise to develop board sensor for the second potential derivatives. In this case, the main interference will come from angular rates and accelerations;
- Designing mathematical models of sensor errors and their use in the algorithm of the gravitation measurement system. Thus, for the period of measuring there will be preliminary data about instrumental and methodological errors of the system. The preliminary planning of the route and performance time could also be used for decreasing or taking into account the inertial movements of the carrier.

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ГРАВИТАЦИОННИ ИЗМЕРВАНИЯ В ПОЛЕТ ЧРЕЗ ИНЕРЦИАЛНИ НАВИГАЦИОННИ СИСТЕМИ И ГЛОБАЛНА РАДИОНАВИГАЦИОННА СИСТЕМА, РАБОТЕЩА В ДИФЕРЕНЦИАЛЕН РЕЖИМ

А. Н. МАДЖАРОВ

В днешно време, промишлеността за гражданска авиация предлага евтини Инерциални навигационни системи (ИНС) с много на брой миниатюрни оптични жироскопи, поместени в един интегрален чип. Това е неоценимо за точно управление на боеприпаси и за навигационни устройства, използващи GPS. Вместо това, тази статия изследва възможността за използване на нов алгоритъм за геофизически наблюдения по време на полет. Ако разполагаме с точна информация за скорости и местоположение на летателния апарат (каквато дава използването на GPS в диференциален режим), то чрез математически модел могат да се отделят от показанията на акселерометри инерционните съставки, за да останат само гравитационни измервания.

Техническата система, предназначена да решава поставената задача ще се нарича система за гравитационни измервания (GMS - Gravitation Measurement System).

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БОРТОВЫЕ ИЗМЕРЕНИЯ ГРАВИТАЦИОННОГО ПОЛЯ, ИСПОЛЬЗУЯ ИНЕРЦИАЛЬНЫЕ НАВИГАЦИОННЫЕ СИСТЕМ И ГЛОБАЛЬНОЙ РАДИОНАВИГАЦИОННОЙ СИСТЕМЫ, РАБОТАЮЩЕЙ В ДИФФЕРЕНЦИАЛЬНОМ РЕЖИМЕ

А. Н. МАДЖАРОВ

В настоящее время, коммерческая промышленность авиакомпаний обеспечивает очень дешевые Инерциальные навигационные системы (ИНС) с многими микро-оптическими гироскопами в одном объединенном кристалле. Это могло быть неоценимо для точных управляемых боеприпасов и для штурманов, использующие GPS. В этой статье, вместо этого исследуются возможности использования нового алгоритма для бортовых геофизических наблюдений. Анализ показует, что это является реалистической целью. Если мы имеем точные данные для скоростей и местоположения летального аппарата (например при использовании GPS в дифференциалном режиме), то

применяя математическую модель мы можем отделить инерционные компоненты от показания акселерометров, что бы остались только гравитационные составляющие.

Техническая система, разработанная для решения проблемы будет называться Измерительной системой Гравитации (GMS).

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AIRBORNE GRAVITY FIELD MEASUREMENTS BY USING INERTIAL NAVIGATION SYSTEMS AND DIFFERENTIAL GLOBAL POSITIONING SYSTEM

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Currently, the commercial airline industry provides very low-cost Inertial Navigation Systems (INS) with many micro-optic gyros in one integrated chip. It could be invaluable for precision-guided munitions and GPS-aided navigators. This paper instead explores the possibilities of using a new algorithm for airborne geophysical observations. The analyze shows that it is realistic aim. If we have accurate data about the flying vehicle velocities and location (e.g. when using GPS under a differential operating mode), then by applying a mathematical model we can separate the inertial components from the accelerometer readings thus leaving only gravity field measurements.

The technical system designed for solving the problem will be called Gravitation Measurement System (GMS).