Homework Assignment #4 - SOLUTIONS

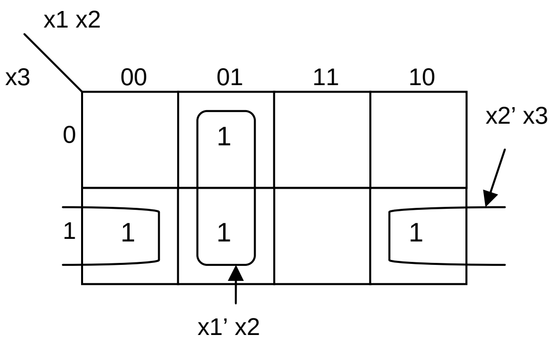
Assigned Thursday, February 10, 2005
Due Thursday, February 17, 2005

Do the following problems using Karnaugh maps.

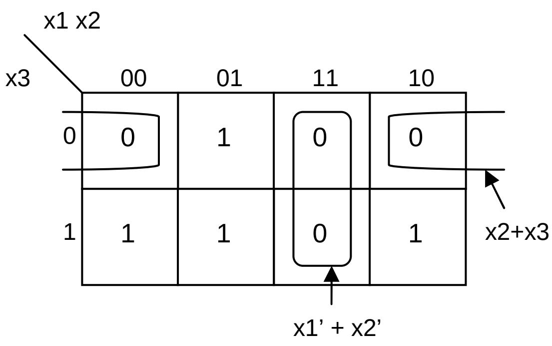
1. (10 pts) Do problem 4.1 in the Brown & Vranesic textbook.
- **(a) For the minimum cost SOP form, draw the corresponding circuit composed of NAND gates only. You may assume that the complements of the input signals are available as inputs, but use only NAND gates in your design.**
- **(b) For the minimum cost POS form, draw the corresponding circuit composed of NOR gates only. You may assume that the complements of the input signals are available as inputs, but use only NOR gates in your design.**

Solution:
f (x1, x2, x3) = Σ m(1, 2, 3, 5)

x1 x2 x3	**f**
0 0 0	0
0 0 1	1
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	0

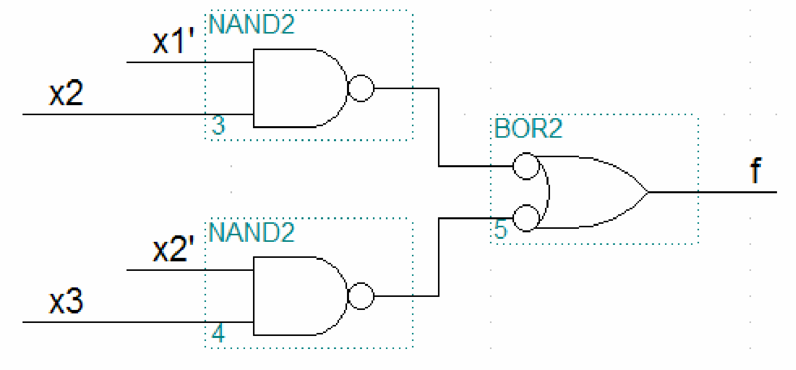


SOP: $f = \overline{x_1}x_2 + \overline{x_2}x_3$

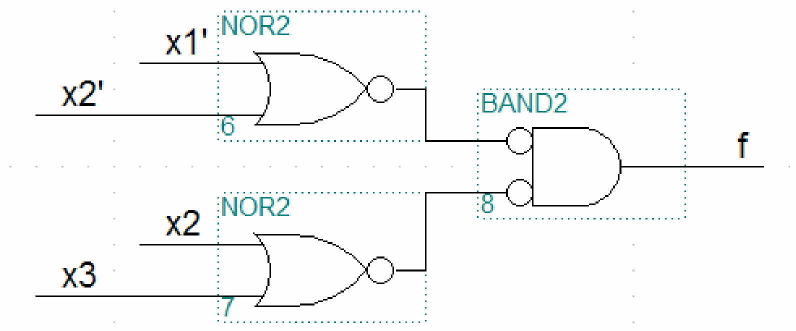


POS: $f = \left(\overline{x_1} + \overline{x_2}\right)\left(x_2 + x_3\right)$

(a) Note that the OR gate with inverted inputs is the same as a NAND gate, by DeMorgan's theorem.



(b) Note that the AND gate with inverted inputs is the same as a NOR gate, by DeMorgan's theorem.



2. (10 pts) Do problem 4.3 in the Brown & Vranesic textbook.

SOP: $f = \overline{x}_1 x_2 x_3 \overline{x}_4 + x_1 x_2 \overline{x}_3 x_4 + \overline{x}_2 x_3 x_4$

POS: $f = (\overline{x}_1 + x_4)(x_2 + x_3)(\overline{x}_2 + \overline{x}_3 + \overline{x}_4)(x_2 + x_4)(x_1 + x_3)$

3. (10 pts) Do problem 4.4 in the Brown & Vranesic textbook.

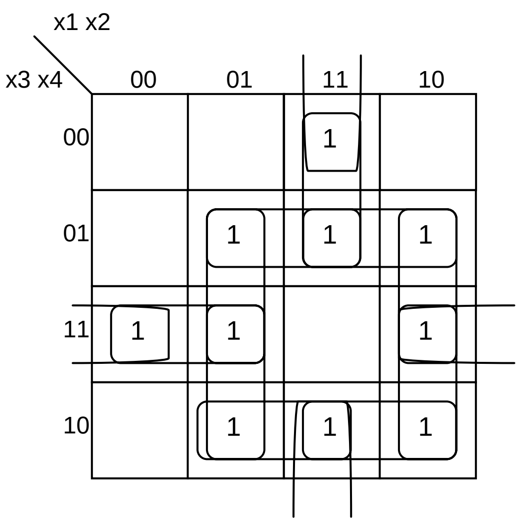
SOP: f = x2' x3' + x2' x4' + x2 x3 x4
POS: f = (x2' + x3)(x2 + x3' + x4')(x2' + x4)

4. (10 pts) Do problem 4.10 in the Brown & Vranesic textbook.

Solution:

x1 x2 x3 x4	**f**
0 0 0 0	0
0 0 0 1	0
0 0 1 0	0
0 0 1 1	1
0 1 0 0	0
0 1 0 1	1
0 1 1 0	1
0 1 1 1	1
1 0 0 0	0
1 0 0 1	1
1 0 1 0	1
1 0 1 1	1
1 1 0 0	1
1 1 0 1	1
1 1 1 0	1
1 1 1 1	0

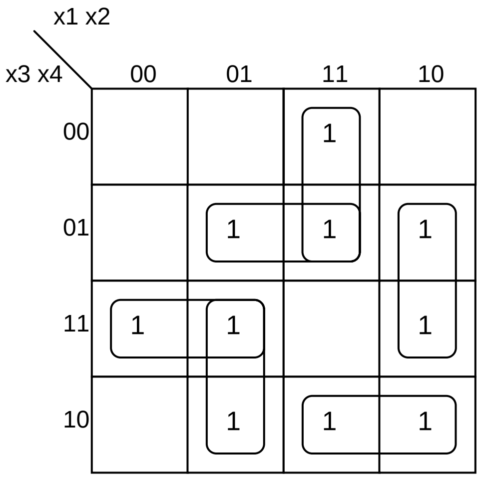
f (x1, x2, x3, x4) = Σ m(3, 5, 6, 7, 9, 10, 11, 12, 13, 14)



There are a lot of prime implicants, and none of them are essential. So you need to just pick a subset that covers the ones with as few prime implicants as possible.

Here is one solution for the SOP (there may be more with the same cost):

f = x1 x2 x3' + x1 x2' x4 + x1 x3 x4' + x1' x2 x3 + x1' x3 x4 + x2 x3' x4



Similarly, there may be more than one minimal POS solution. Here is one:

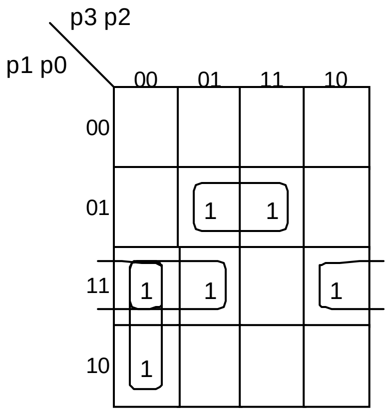
f = (x1 + x2 + x3) (x1 + x2 + x4) (x1 + x3 + x4) (x2 + x3 + x4) (x1' + x2' + x3' + x4')

The POS form has lower cost.

5. (15 pts) Consider a prime number detector that inputs a four bit number (i.e., values 0 to 15) and outputs a 1 if the number is prime. (The numbers 0 and 1 are not prime.) Give the truth table and minimum sum of products.

Truth table:

p3 p2 p1 p0	f
0 0 0 0	0
0 0 0 1	0
0 0 1 0	1
0 0 1 1	1
0 1 0 0	0
0 1 0 1	1
0 1 1 0	0
0 1 1 1	1
1 0 0 0	0
1 0 0 1	0
1 0 1 0	0
1 0 1 1	1
1 1 0 0	0
1 1 0 1	1
1 1 1 0	0
1 1 1 1	0



$$f = p_2 \overline{p_1} p_0 + \overline{p_3} p_1 p_0 + \overline{p_2} p_1 p_0 + \overline{p_3}\, \overline{p_2} p_1$$

6. (15 pts) The following is the minimal product-of-sums (POS) expression for a function F:

F(W,X,Y,Z) = (X' + Y)(X' + Z')(Y' + Z')

It has a cost of 13 (4 gates + 9 inputs to the gates).

(Correction to this problem: F as given is not minimal.)

(a) Find the minimal sum-of-products (SOP) expression for F. What is the cost?

Solution: The K-map is

YZ \ WX	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	0	0	0	0
10	1	1	1	1

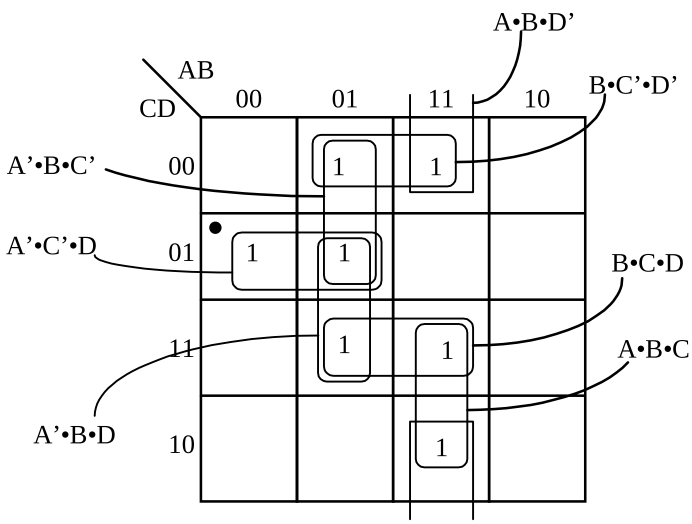
If we cover the 1's instead, we get

F(W,X,Y,Z) = Y Z' + X' Y'. The cost is 9 (3 gates + 6 inputs).

(b) Now assume that the input combination (W,X,Y,Z) = (1,0,1,1) will never occur. Why doesn't this simplify the SOP?

Solution: If we make the value for (1011) a "don't care", this won't change our result. It is simpler to ignore that term and not try to cover it.

7. (15 pts) The minimum cost realization of a function is not necessarily unique (ie, there can be more than one solution that is equally as good). Find all four minimum cost SOP expressions for the function
f(A,B,C,D) = =Σ m(1,4,5,7,12,14,15)

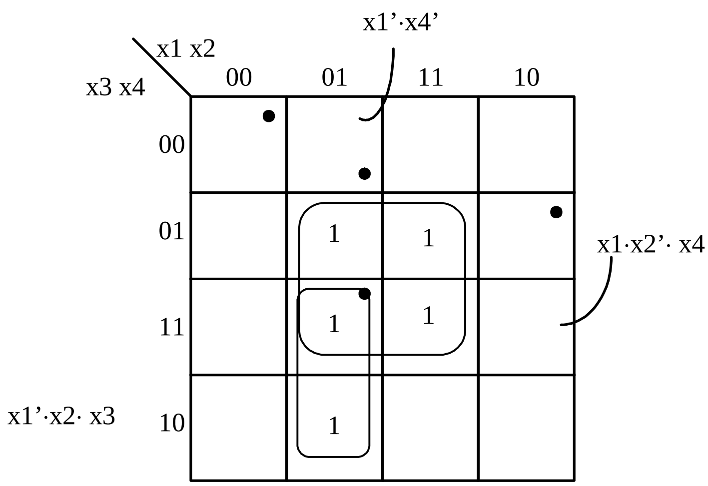


There are four minimal solutions, each equally good:
F = A’•C’•D + B•C’•D’ + A•B•C + B•C•D
F = A’•C’•D + B•C’•D’ + A•B•C + A’•B•D
F = A’•C’•D + B•C’•D’ + A•B•D’ + B•C•D
F = A’•C’•D + A’•B•C’ + A•B•D’ + B•C•D

8. (15 pts) A circuit with two outputs has to implement the following functions.
f(x1,x2,x3,x4) = Σ m(5, 6, 7, 13, 15)
g(x1,x2,x3,x4) = Σ m(2, 4, 5, 6, 7, 12, 13, 15)

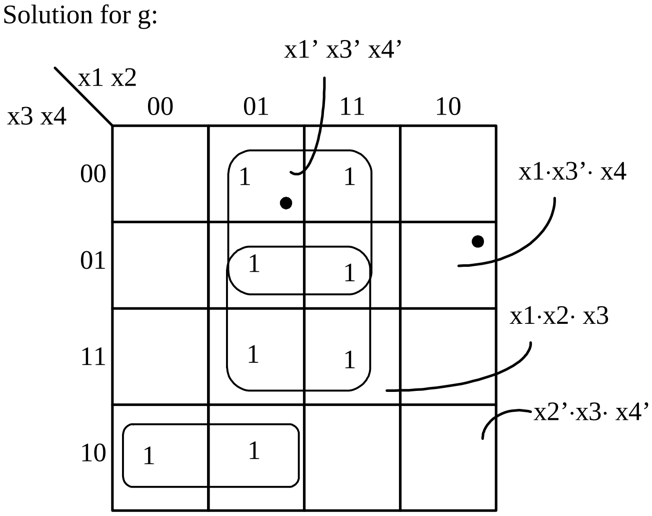
Design the minimum cost SOP circuit, assuming that you can share product terms between the two outputs. Compare the cost of the combined circuit with the costs of two circuits that implement f and g separately.

Solution for f:



The minimum SOP is f(x1,x2,x3,x4) = x1' x4' + x1' x2 x3 + x1 x2' x4
Cost is 4 gates plus 11 inputs = 15.

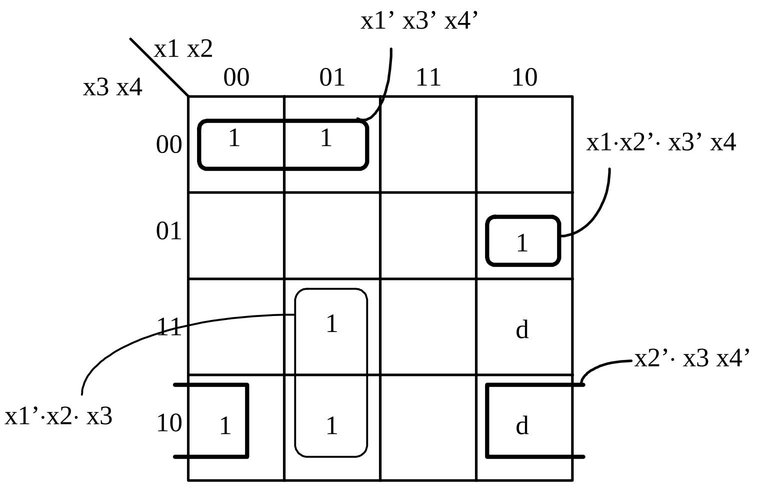
Solution for g:



The minimum SOP is g(x1,x2,x3,x4) = x1' x3' x4' + x1 x3' x4 + x1 x2 x3 + x2' x3 x4'
Cost is 5 gates plus 16 inputs = 21.

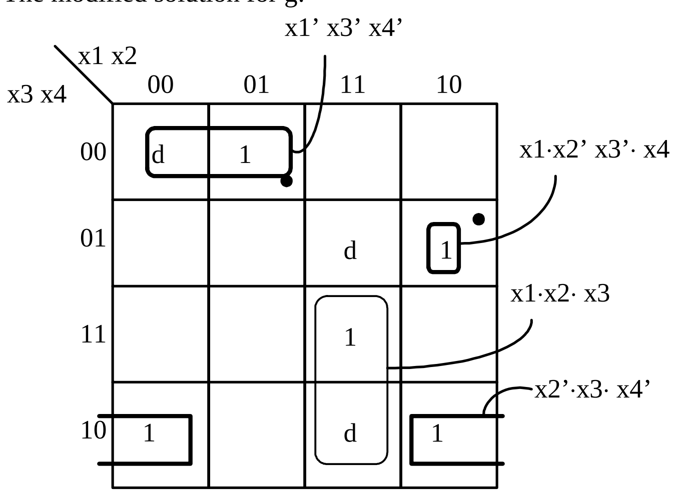
So the total cost if we implemented f and g separately, would be 15 + 21 = 36.

Instead, we shall look for common implicants between the two K-maps.
The modified solution for f:



f(x1,x2,x3,x4) = (x1’ x3’ x4’ + x2’ x3 x4’ + x1 x2’ x3’ x4) + x1’ x2 x3

The modified solution for g:



g(x1,x2,x3,x4) = (x1’ x3’ x4’ + x2’ x3 x4’ + x1 x2’ x3’ x4) + x1 x2 x3

The 3 product terms in parentheses (shown in heavy lines in the figure) can be shared.
It takes 3 gates with 10 inputs to generate the 3 shared product terms.

To implement f takes an additional 2 gates with 7 inputs.
To implement g takes an additional 2 gates with 7 inputs.

The new total cost is:
(3+10) + (2+7) + (2+7) = 31.