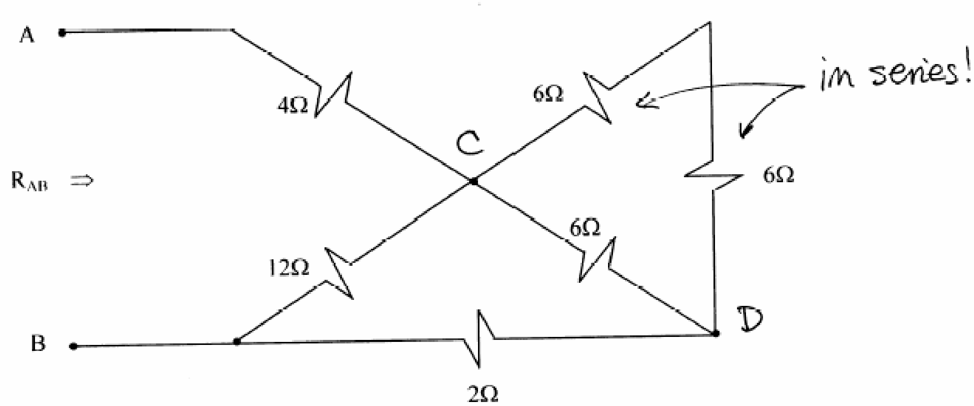
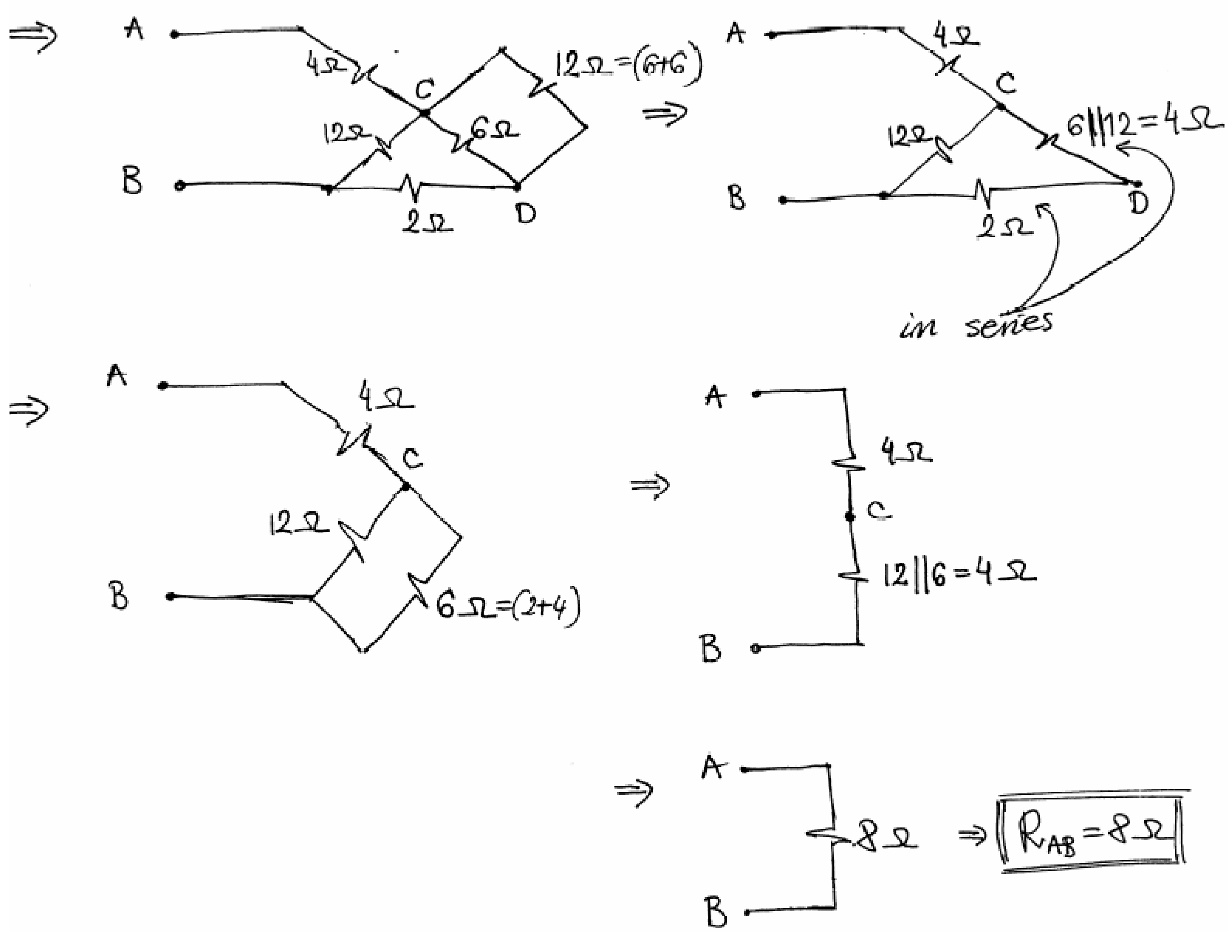
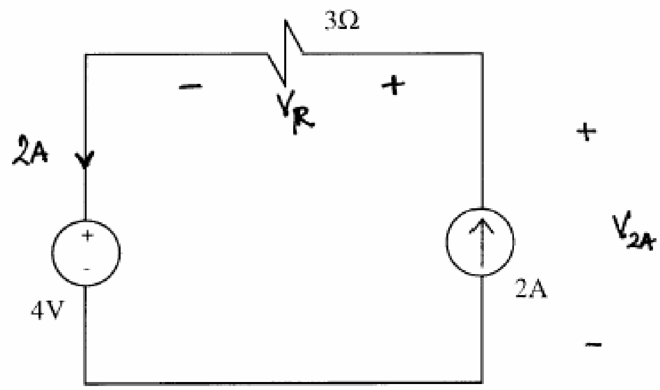
PRACTICE FINAL EXAM: PART I
SOLUTIONS

Find the equivalent resistance R_{AB}.





Find the power absorbed or supplied by each element.

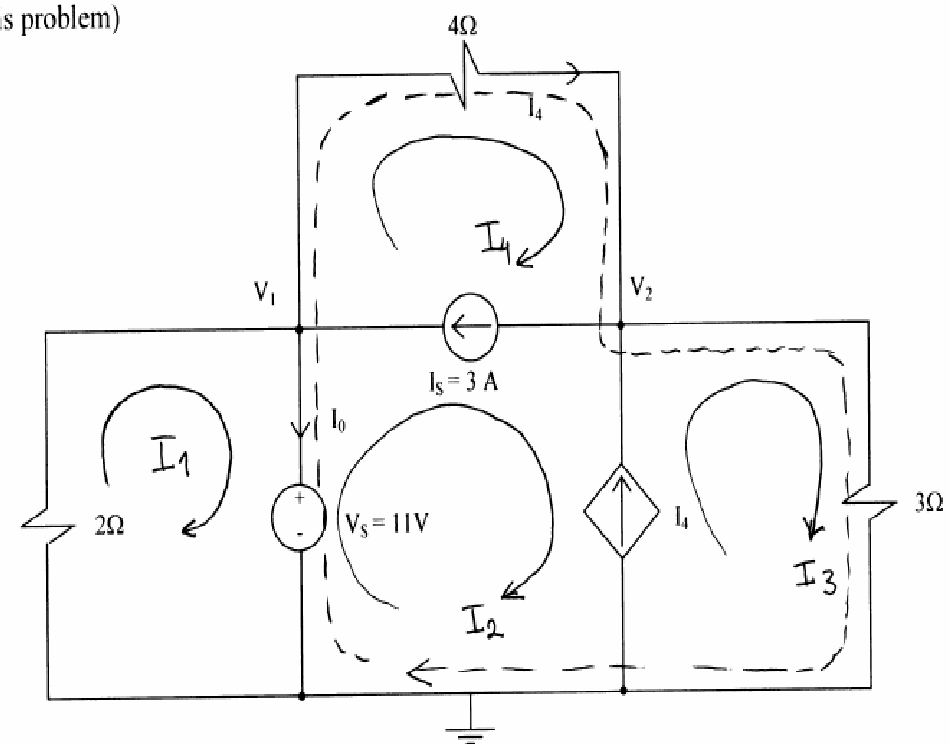


$P_R = V_R \cdot 2A = (3\Omega \cdot 2A) \cdot 2A = \underline{12\,W - \text{absorbed}}$

$P_{4V} = 4V \cdot 2A = \underline{8W - \text{absorbed}}$

$P_{2A} = V_{2A} \cdot 2A = (4V + V_R) \cdot 2A = (4V + 3\Omega \cdot 2A) \cdot 2A = 10V \cdot 2A = \underline{20\,W - \text{supplied}}$

Use mesh analysis to **find the current I_0** in the circuit below.
(Note: This is tge same circuit as in problem 4. You may use results from problem 4 to check you answers but you must write and solve mesh analysis equations to get credit for this problem)



mesh 1: $-V_S - 2I_1 = 0$ .... (1) $\Rightarrow I_1 = -\frac{11}{2} = -5.5A$

supermesh: $V_S - 4I_4 - 3I_3 = 0$ ... (2)

source I_S : $I_S = I_4 - I_2$ .... (3) $\Rightarrow I_2 = I_4 - 3$ ...... (5)

dep. sour. I_4 : $I_4 = I_3 - I_2$ .... (4)

(5) (4) : $I_4 = I_3 - I_4 + 3 \Rightarrow I_3 = 2I_4 - 3$ ... (6)

(6) (2) : $11 - 4I_4 - 3(2I_4 - 3) = 0 \Rightarrow 11 - 4I_4 - 6I_4 + 9 = 0$

$\Rightarrow 20 - 10I_4 = 0 \Rightarrow \underline{I_4 = 2A}$

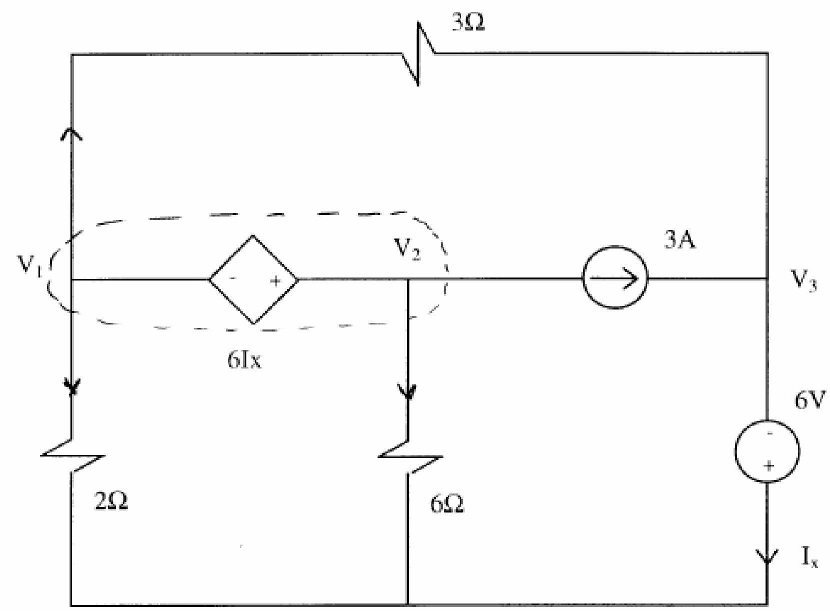
(5) $\Rightarrow I_2 = 2 - 3 \Rightarrow \underline{I_2 = -1A}$

$I_0 = I_1 - I_2 = -5.5 + 1 \Rightarrow \boxed{I_0 = -4.5A}$

Write nodal analysis equations for V_1, V_2 and V_3 in standard form:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{22} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

Do not solve the equations.



Supernode: $\left\{ \frac{V_1}{2} + \frac{V_1 - V_3}{3} + \frac{V_2}{6} + 3 = 0 \;\Big/ \cdot 6 \Rightarrow 3V_1 + 2V_1 - 2V_3 + V_2 = -18 \quad \ldots\ldots (1) \right.$

$V_2 - V_1 = 6 I_x \longrightarrow \Rightarrow V_2 - V_1 = 6\left(3 + \frac{V_1 - V_3}{3}\right) = 18 + 2(V_1 - V_3)$

@ V3 : $V_3 = -6 \quad \ldots (2)$

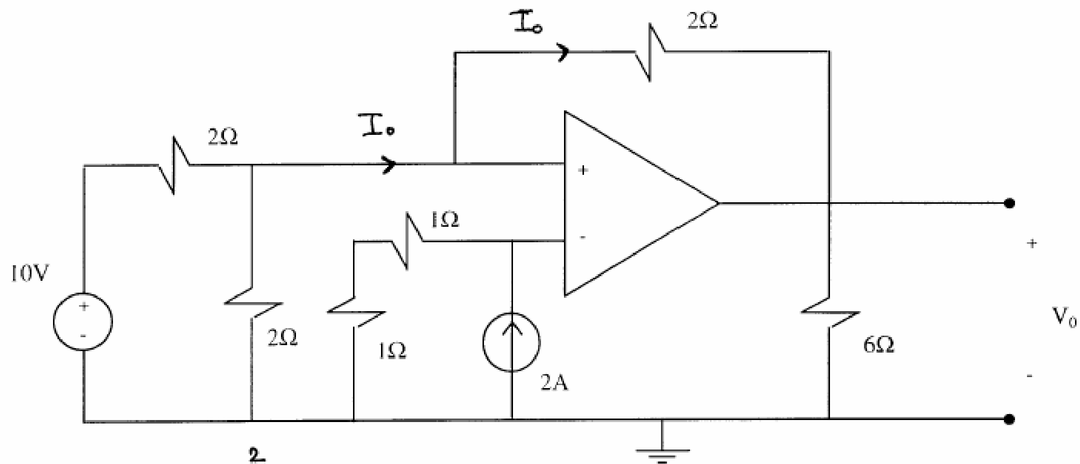
$I_x = 3 + \frac{V_1 - V_3}{3}$

$V_2 - V_1 = 18 + 2V_1 - 2V_3$

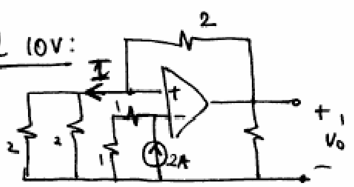
$3V_1 - V_2 - 2V_3 = -18 \quad \ldots\ldots (3)$

$$\begin{bmatrix} 5 & 1 & -2 \\ 0 & 0 & 1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -18 \\ -6 \\ -18 \end{bmatrix}$$

Use superposition to find V_0.



Kill 10V:



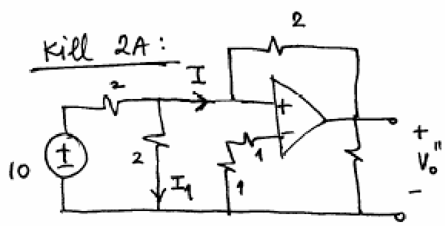
$V_- = 2A(1+1) = 4V$

$V_+ = V_- = 4V$

$I = \dfrac{V_+}{2 \| 2} = \dfrac{4V}{1\Omega} = 4A$

$V_o' = V_+ + 2 \cdot I = 4V + 2\Omega \cdot 4A = \underline{12V}$

Kill 2A:



$V_- = 0 \Rightarrow V^+ = 0 \Rightarrow I_1 = 0$

$\Rightarrow I = \dfrac{10 - V^+}{2} = 5A$

$V_o'' = V_+ - I \cdot 2\Omega = 0 - 5A \cdot 2\Omega = \underline{-10V}$

Total: $V_o = V_o' + V_o'' = 12V - 10V = \underline{\underline{2V}}$

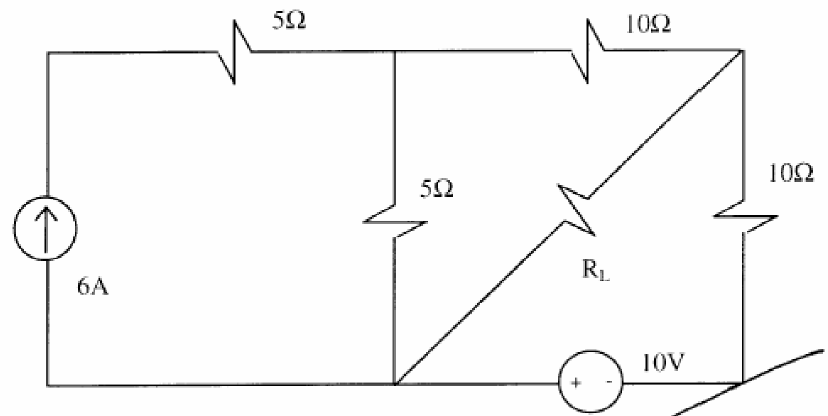
Check by analysing original circuit:

$V_- = 2A(1+1) = 4V \Rightarrow V_+ = 4V$

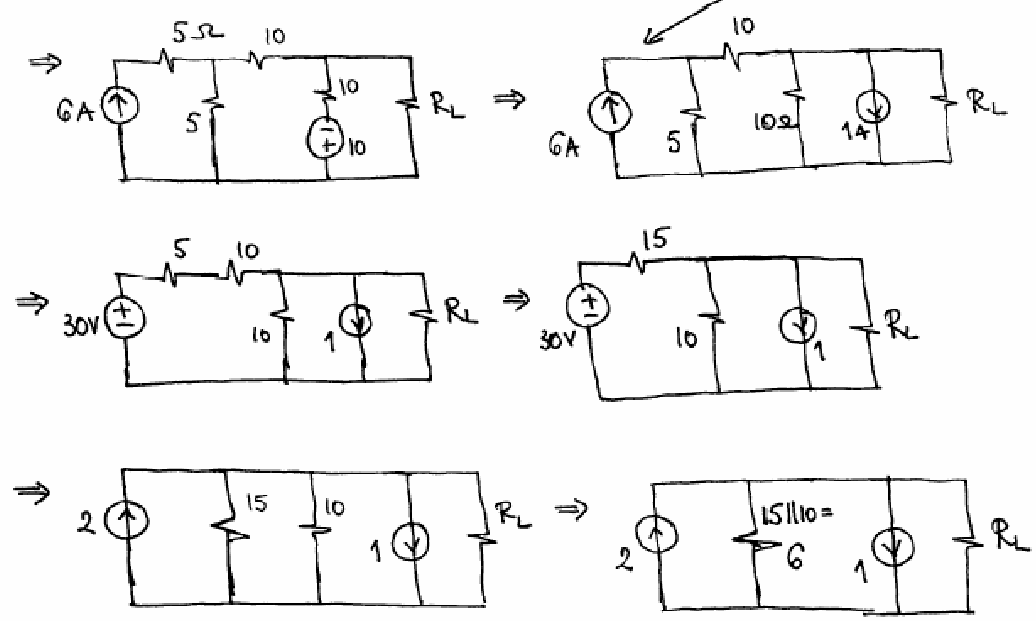
$I_o = \dfrac{10 - V_+}{2} - \dfrac{V_+}{2} = \dfrac{10-4}{2} - \dfrac{4}{2} = 3 - 2 = 1A$

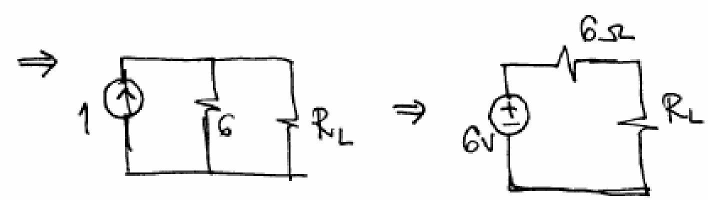
$V_o = V_+ - 2 \cdot I_o = 4 - 2 \Rightarrow \underline{\underline{V_o = 2V}}$ ✓

a) Use <u>source transformation</u> to find the value of R_L for maximum power transfer.
b) What is the maximum power that can be transferred to R_L?
NOTE: Repeat source transformation steps until R_L can be found without using any current or voltage division.



5Ω can be ignored since it is in series with a current source.

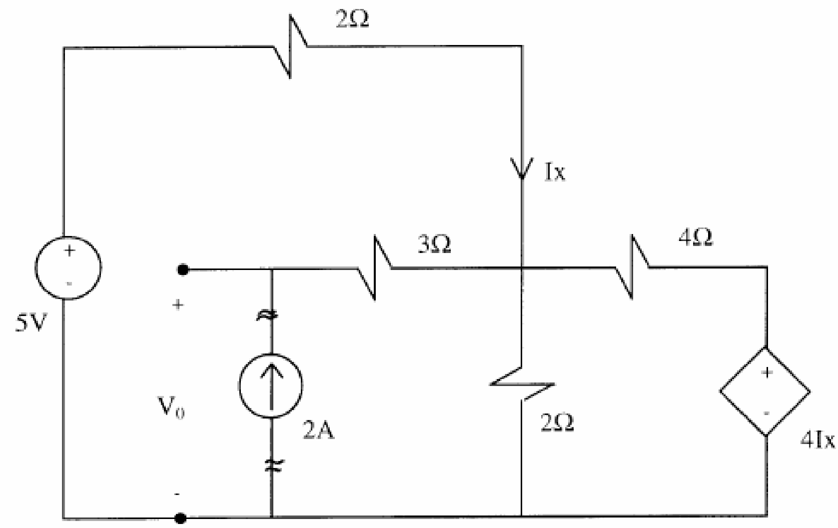


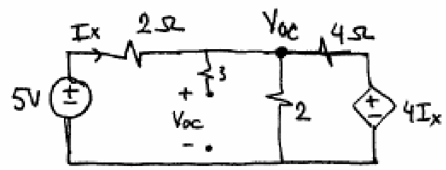


$\Rightarrow$ $\underline{R_L = 6\Omega}$ for maximum power transfer

$\Rightarrow$ $\underline{P_{max}} = \dfrac{6^2}{4 \cdot 6} = \underline{1.5\,W}$

Use Thevenen's theorem to find V_0.
HINT: Cut the circuit at the terminals of the 2A current source.





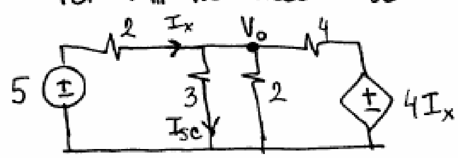
KCL @ V_{oc}: $\frac{5-V_{oc}}{2} = \frac{V_{oc}}{2} + \frac{V_{oc}-4I_x}{4}$

$I_x = \frac{5-V_{oc}}{2}$

$\Rightarrow \frac{5-V_{oc}}{2} = \frac{V_{oc}}{2} + \frac{V_{oc}}{4} - \left(\frac{5-V_{oc}}{2}\right) \Rightarrow 5-V_{oc} = V_{oc}\left(\frac{1}{2}+\frac{1}{4}\right) = V_{oc}\cdot\frac{3}{4}$

$\Rightarrow 5 = V_{oc} + \frac{3}{4}V_{oc} = \frac{7}{4}V_{oc} \Rightarrow \boxed{V_{oc} = \frac{20}{7}}$

For R_{TH} we need I_{sc}:



KCL@V_o: $\frac{5-V_o}{2} = \frac{V_o}{3} + \frac{V_o}{2} + \frac{V_o-4I_x}{4}$, $I_x = \frac{5-V_o}{2}$

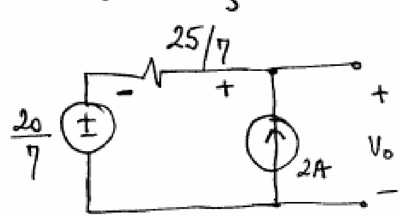
$\Rightarrow \frac{5-V_o}{2} = V_o\left(\frac{1}{3}+\frac{1}{2}+\frac{1}{4}\right) - \frac{5-V_o}{2}$

$\Rightarrow 5-V_o = V_o\cdot\frac{4+6+3}{12} \Rightarrow 5 = V_o\cdot\frac{13}{12} + V_o = \frac{25}{12}V_o \Rightarrow V_o = \frac{12\cdot5}{25} \Rightarrow V_o = \frac{12}{5}$

$I_{sc} = \frac{V_o}{3} = \frac{12}{3\cdot5} \Rightarrow I_{sc} = \frac{4}{5}$

$R_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{\frac{20}{7}}{\frac{4}{5}} = \frac{100}{28} = \frac{25}{7} = R_{TH}$

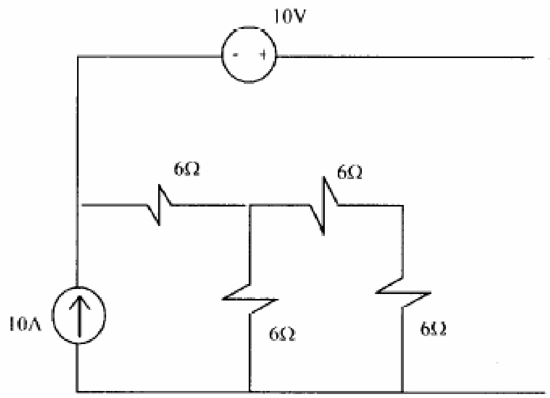
$\Rightarrow$



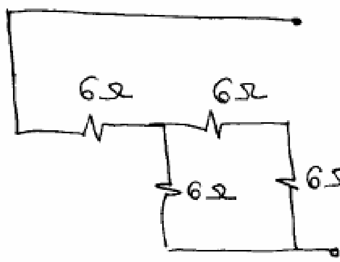
$V_o = V_{TH} + 2\cdot R_{TH} = \frac{20}{7} + 2\cdot\frac{25}{7} = \frac{20}{7} + \frac{50}{7} = \frac{70}{7}$

$\Rightarrow \boxed{V_o = 10\,V}$

1. Find both Thevenen's and Norton's equivalent of the following circuit.

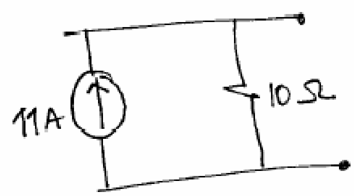


Find R_{TH}:

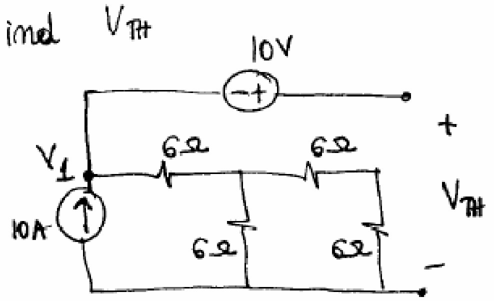


$$R_{TH} = 6 + 6 \| (6+6)$$
$$= 6 + 6 \| 12$$
$$= 6 + 4$$
$$\boxed{10\,\Omega = R_{TH}}$$

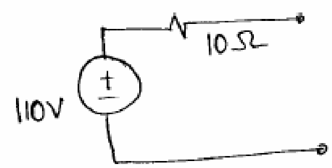
Norton's Equivalent



Find V_{TH}



Thevenen's Equivalent

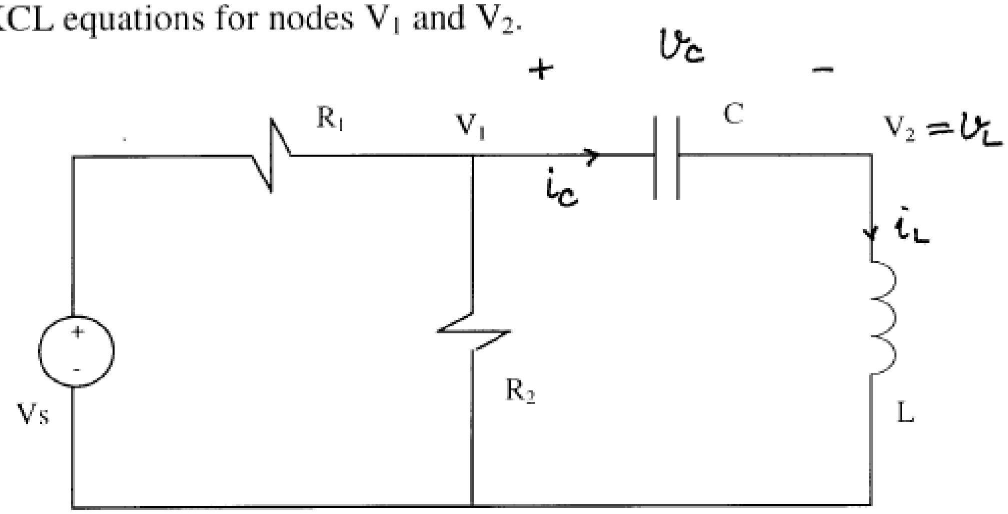


$$V_1 = 10A \cdot \left(6 + 6 \| (6+6)\right)$$
$$= 10\,A \cdot 10\,\Omega = 100\,V$$
$$V_{TH} = V_1 + 10V \Rightarrow \boxed{V_{TH} = 110\,V}$$
$$I_{NT} = \frac{V_{TH}}{R_{TH}} = \frac{110\,V}{10\,\Omega} \Rightarrow \boxed{I_{NT} = 11\,A}$$

2. If the load is $R_L = 1K\Omega$, does this circuit behaves more like a current or a voltage source ?

$R_g = 10\,\Omega \Rightarrow R_g \ll R_L \Rightarrow$ This circuit behaves more line a voltage source.

Write KCL equations for nodes V_1 and V_2.



$$@\,V_1:\quad \frac{V_s - V_1}{R} = \frac{V_1}{R} + i \quad ; \quad i = i_c = i_L \quad ; \quad i_L = \frac{1}{L}\int v_1\,dt$$

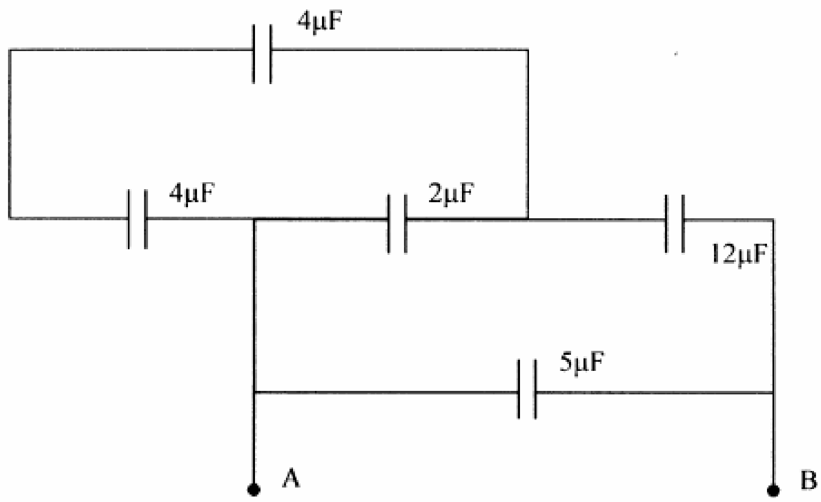
$$\Rightarrow \underline{\frac{v_s - v_1}{R} = \frac{v_1}{R} + \frac{1}{L}\int v_1\,dt} \quad \ldots\ (1)$$

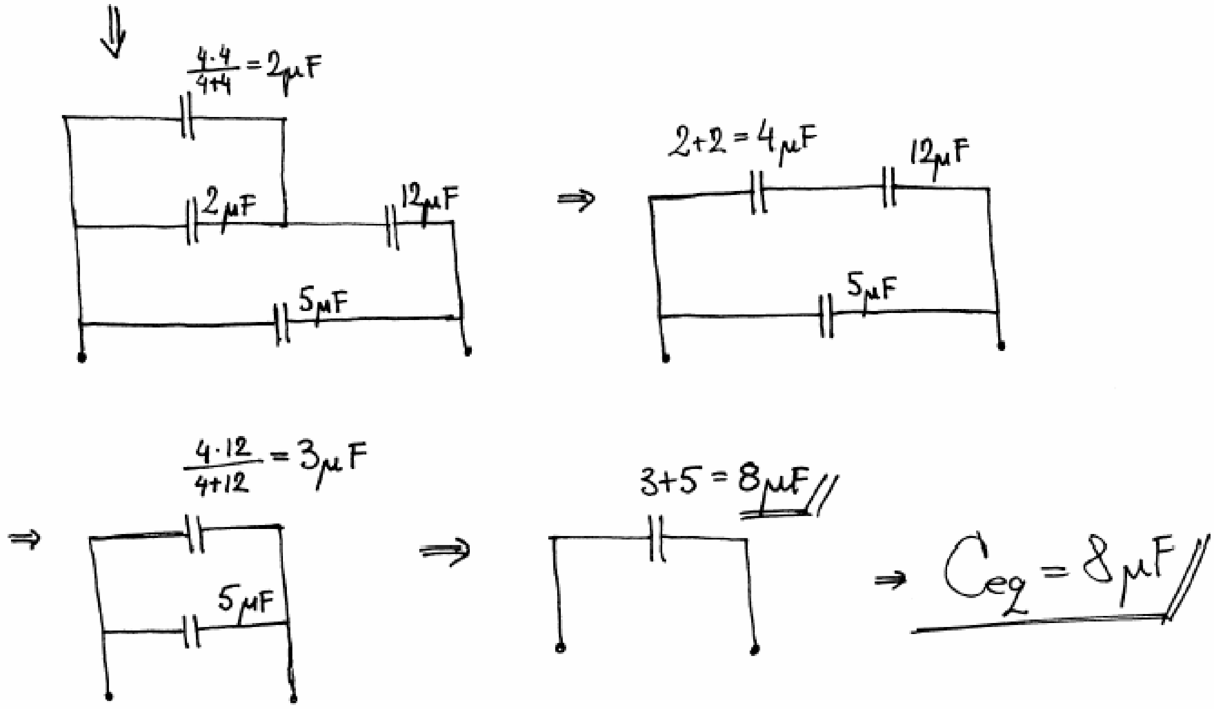
$$@\,V_2:\quad i_c = i_L$$

$$i_c = C \cdot \frac{d}{dt} \cdot v_c = C \cdot \frac{d}{dt}\left(v_1 - v_2\right) = C\frac{dv_1}{dt} - C\frac{dv_2}{dt}$$

$$\Rightarrow \underline{C\frac{dv_1}{dt} - C\frac{dv_2}{dt} = \frac{1}{L}\int v_1\,dt} \quad \ldots\ (2)$$

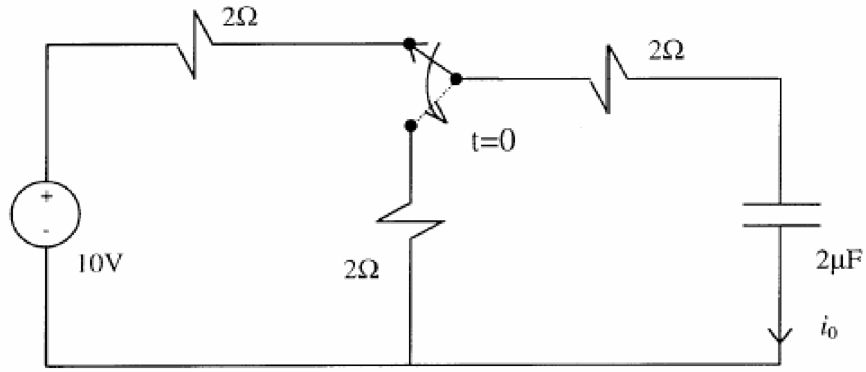
Find the equivalent capacitance between points A and B.



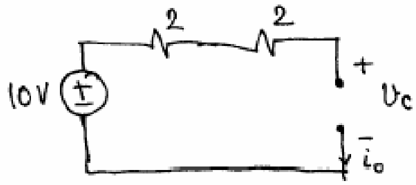


$\Rightarrow$ $C_{eq} = 8\mu F$

Find $i_0(0^+)$.



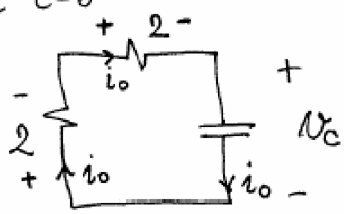
@ $t=0^-$: STEADY STATE:



$i_o(0^-)=0$

$v_c(0^-)=10V$

@ $t=0^+$

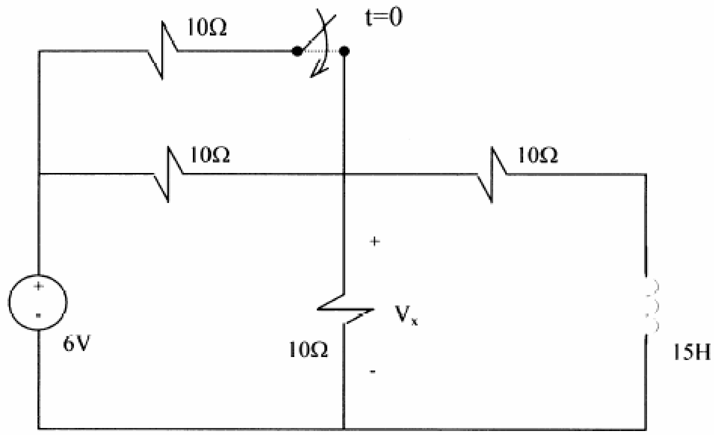


KVL: $2i_o(0^+)+2i_o(0^+)+v_c(0^+)=0$

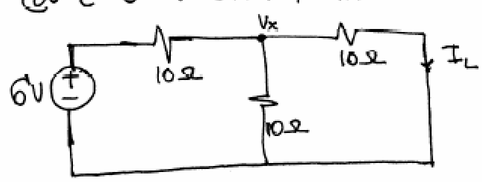
$$\Rightarrow i_o(0^+) = -\frac{v_c(0^+)}{4} = -\frac{v_c(0^-)}{4} = -\frac{10}{4}$$

$$\Rightarrow \underline{i_o(0^+)=-2.5A}$$

Find voltage $V_x(t)$, for $t > 0$s. Sketch $V_x(t)$, for $t > -1$s.



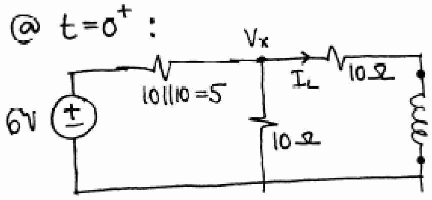
@ $t=0^-$: STEADY STATE:



$$\underline{V_x(0^-)} = \frac{10 \| 10}{10 + 10\|10} \cdot 6V = \frac{5}{15} \cdot 6 = \underline{2V}$$

$$I_L(0^-) = \frac{V_x}{10} = 0.2A$$

@ $t=0^+$:



a) non-discontinuity condition for inductor:

$$I_L(0^+) = I_L(0^-) = 0.2A$$

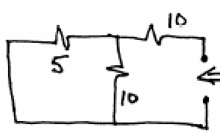
b) KCL @ V_x :

$$\frac{6 - V_x}{5} = \frac{V_x}{10} + I_L \Rightarrow 12 - 2V_x = V_x + 10I_L$$

$$\Rightarrow V_x = \frac{1}{3}(12 - 10I_L)$$

$$\Rightarrow \underline{V_x(0^+) = \frac{10}{3}V}$$

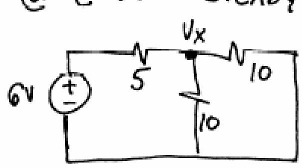
time constant: $\tau = \frac{L}{R_{TH}}$



$$\Leftarrow R_{TH} = 10 + 5\|10 = 10 + \frac{50}{15} = 10 + \frac{10}{3} = \frac{40}{3}$$

$$\Rightarrow \tau = \frac{15 \cdot 3}{40} = \frac{45}{40} \Rightarrow \underline{\tau = \frac{9}{8}}$$

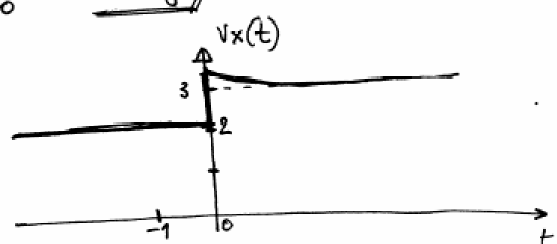
@ $t \to \infty$: STEADY STATE



$$V_x = \frac{10\|10}{5 + 10\|10} \cdot 6$$

$$= \frac{5}{10} \cdot 6$$

$$\underline{V_x = 3V}$$



$$\Rightarrow V_x(t) = V_x(\infty) + [V_x(0^+) - V_x(\infty)]e^{-t/\tau} \Rightarrow \boxed{V_x(t) = 3 + \frac{1}{3}e^{-8t/9}, \quad t > 0}$$