Swap Curve Arbitrage Trade Strategy

Hedge Fund Strategies (B40.3321)

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Introduction[1]

A swap curve is a representation of the relationship between interest rates in the swaps market and their maturity. Just as the Treasury yield curve is used to value Treasury securities, the swaps market employs its own valuation vehicle, the swap curve. The swap curve is used for swap valuation as well as for pricing swap-related products such as forward rate agreements, Eurodollar futures contracts, caps, swaptions among others. A swap curve is usually constructed with benchmark swaps and related contracts.

An interest rate swap is a contract to exchange fixed and floating coupon payments between two counterparties for a specified period of time. Both the coupon payments are based on a notional amount, which is analogous to the principal amount of a bond and can be thought of as being exchanged at the swap maturity leading to a net zero exchange of principal at maturity. These two streams of cash flows are referred to as the fixed leg and the floating leg of the swap contract, respectively. The fixed-leg payments resemble the cash flows of a regular bond. However, unlike a bond that requires an up-front investment, a swap converts this initial amount into an equivalent set of floating coupon payments over the length of the contract, much like a floating coupon bond. In fact, a swap could be simply viewed as converting a fixed-rate bond into a floating bond. Receiving the fixed coupon on a swap, therefore, can be viewed as being long a fixed-rate bond and short a floating-rate one.

Constructing the Swap Curve

a. Data inputs used

We obtained daily rates for 2-year, 3-year, 5-year, 7-year and 10-year swaps from Bloomberg for approximately an ten-year timeframe, from June 21, 1996 to February 15, 2007. We then

[1] "The Swap Curve". Lehman Brothers Fixed-Income Research. August 26, 2002.

filtered the data such that only days with a complete set of data points were used. So, if on a given day, the 2-year rate was missing, we left out that set of data points.

Additionally, we discarded any data with obvious quality issues. For example, if an ask rate was lower than a bid rate, we discarded that record.

b. Fitting the swap curve

Based on the 2-year, 5-year, and 10-year data points, we fitted a quadratic function for each day's sample data. The quadratic function was constructed as follows to fit the three points – 2-year, 5-year and 10-year swap rates.

$$y = ax^2 + bx + c$$

a, b & c: constant coefficients to be solved for
x: swap maturity (in years)
y: swap rate (using the midpoint between the bid and ask prices)

Example: June 21, 1996
2-year rate: 6.55
5-year rate: 7.02
10-year rate: 7.31
The three equations in this instance for this particular day are:

$6.55 = a * 4 + b * 2 + c$ -------------------------(1)
$7.02 = a * 25 + b * 5 + c$ -------------------------(2)
$7.31 = a * 100 + b * 10 + c$ -------------------------(3)

With three equations and three unknowns, the coefficients a, b, and c can be determined for this particular day, hence giving us a fitted swap curve in the form of $y = ax^2 + bx + c$.

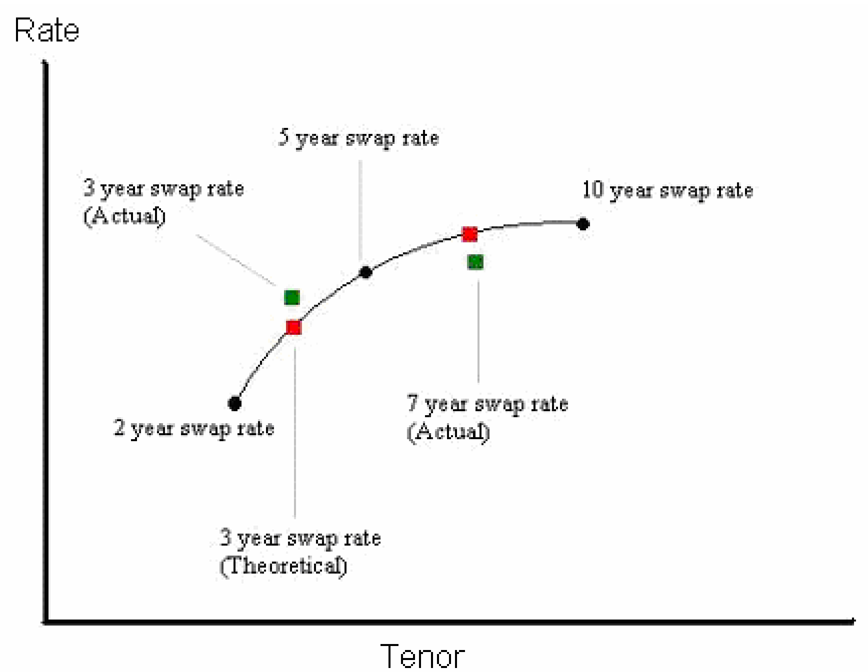
On solving the example above, we get the following equation that fits the data for June 21, 1996:

$$y = -0.01233\, x^2 + 0.243\, x + 6.1133$$

c. Obtaining the theoretical 3-year and 7-year swap rates

We assumed that the fitted curve obtained above is smooth under normal circumstances and all bumps (caused by deviations of the 3-year and 7-year rates from the fitted curve) revert to the theoretical swap rate on the fitted curve as a result of arbitrage.

Using the fitted curve, we solved for the theoretical 3-year and 7-year swap rates using the quadratic coefficients for that particular day. These theoretical values were compared to the actual 3- and 7-year rates (again, using the midpoint between the bid and ask prices) to determine if the actual rates were either too rich (i.e. overvalued) or too cheap (i.e. undervalued).



d. Quantifying the deviation of the 3 year and 7 year swap rates for each date

For each date (for both the 3- and 7-year data), we calculated the deviation of the rates (i.e. actual rate – theoretical rate). We then obtained the rolling average (trailing 90 days) of the deviation as well as its rolling 90-day standard deviation. If the current deviation was more than 2 standard deviations away from the rolling average, a trading opportunity existed.

Trading Strategy

As explained above, we obtained a rolling average for the deviations (i.e. actual swap rate – theoretical swap rate) for the trailing 90 days. We also calculated the standard deviation of this deviation for the trailing 90 days. If the deviation was more than 2 standard deviations away from the historical rolling average, we executed a butterfly trade.

<u>Example: June 25, 1996</u>

2-year rate	=	5.98%
5-year rate	=	6.17%
10-year rate	=	6.45%
3-year actual rate	=	6.27%
3-year theoretical rate	=	6.0402%
Deviation	=	0.2298%
Rolling average (trailing 90 days) deviation	=	0.02111%
2 standard deviations	=	0.02498%
Rolling average deviation + 2 standard deviations	=	0.07107%

Since Deviation > Rolling average + 2 standard deviations, we executed a trade
Therefore relevant butterfly trade ➔ Long (receive fixed) 3-year, Short 2-year, Short 5-year

Trading Rules for Entry and Exit

	Condition	**Trade 1**	**Trade 2**	**Trade 3**
E	3 year Deviation[1] > Rolling Average + 2 rolling σ	Long[2] 3 year	Short 2 year	Short 5 year
T	3 year Deviation < Rolling Average	Short 3 year	Long 2 year	Long 5 year
E	3 year Deviation < Rolling Average – 2 rolling σ	Short 3 year	Long 2 year	Long 5 year
T	3 year Deviation > Rolling Average	Long 3 year	Short 2 year	Short 5 year
E	7 year Deviation > Rolling Average + 2 rolling σ	Long 7 year	Short 5 year	Short 10 year
T	7 year Deviation < Rolling Average	Short 7 year	Long 5 year	Long 10 year
E	7 year Deviation < Rolling Average - 2 rolling σ	Short 7 year	Long 5 year	Long 10 year
T	7 year Deviation > Rolling Average	Long 7 year	Short 5 year	Short 10 year

[1] Deviation = Actual – Theoretical
[2] Firms have varying conventions when they say long or short swaps. In this paper, long swaps means receiving fixed and short swaps means paying fixed.
E = Execution of the Initial Trade
T = Termination of the Trade

Entry Rules:

If the current deviation is more than 2 (90-day rolling) standard deviations above the rolling 90-day average, receive fixed on the body, pay fixed on the wings. Else if, the current deviation is less than 2 (90-day rolling) standard deviations below the rolling 90-day average, pay fixed on the body, receive fixed on the wings.

Each of the trades are butterfly trades whereby the 3-year and 7-year swaps are the body and the corresponding pair of swaps above and below these form the wings. Each of the butterfly trades was weighted to be duration-neutral.

Duration on swaps was as follows[2]:

2-year	3-year	5-year	7-year	10-year
1.64	2.52	4.15	5.61	7.53

Given these durations, the 3-year butterfly was weighted as –65% / 100% / -35% and the 5-year butterfly was weighted as –57% / 100% / -43%.

Exit Rules:

If a trade is on, and the deviation reverts back to the 90-day rolling average, then unwind the trade. Else if the trade has been on for more than 90-days, unwind the trade.

Marking-to-Market the Swaps

The profit / loss an each trade was calculated by using the durations of generic at-the-money swaps and multiplying by the yield change. Given that most yield changes were small and the length of time that trades were on was short, duration is a good approximation for the profit / loss. The use of a single duration (as of 3/21/2007) to calculate the profit / loss of our trades is a limitation of our model. However, we believe that had we used the actual durations during the period we analyzed, our final conclusion would not be significantly different.

[2] Obtained with Blackrock's Anser system as of 3/21/2007

Example: Receive Fixed 11/25/1996, Exit Trade 11/26/1996

	B	C	D	E	F	H	I	J	K	L
5		3-Year Butterflies				**Duration**	1.64	2.52	4.15	
6						**Weightings**	-65%	100%	-35%	
7			2	3	5	**Tenor**	2	3	5	**Total P&L**
8	11/25/1996	Enter	5.98	6.27	6.17					
9	11/26/1996	Exit	5.80	5.94	6.14		(1,918.80)	8,190.00	(435.75)	**5,835.45**
10										
11										
12										
13										

=-(D9-D8)*I$5/100*I$6*B1

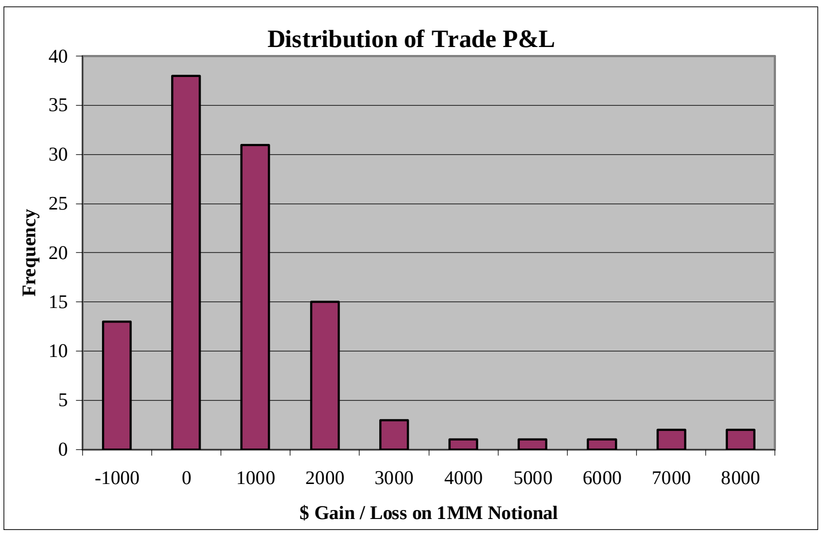
=-(E9-E8)*J$5/100*J$6*B1

=-(F9-F8)*K$5/100*K$6*B1

=SUM(I9:K9)

Results and Returns

The following graph illustrates the P&L distribution over the period analyzed:



The following table summarizes key statistics of the portfolio over the entire 10-year period:

Entire Portfolio	
Mean	$432.21
Standard Error	$170.71
Median	$24.72
Standard Deviation	$1,765.86
Sample Variance	$3,118,251.51
Kurtosis	6.46
Skewness	2.35
Range	$9,732.17
Minimum	-$1,796.87
Maximum	$7,935.30
Sum	$46,245.96
Count	107
Avg Days Per Trade	18.00
Periods in Trading Year	13.89
Mean Return	0.04%
Annualized Return	0.60%
Mean SD	0.18%
Annualized SD	0.66%
Sharpe Ratio	0.91470

The annualized return of the trades without leverage, based on the notional amount of $1 million is 0.60%. Given typical margin requirements of 25-100 basis points and reasonable risk levels, leverage of 20x is typical for such transactions, providing a return of 12.0%.

The Sharpe Ratio of the portfolio is 0.91 (conditional on trading). The Sharpe Ratio, which is a reward to risk ratio, is independent of the leverage used so long as the standard deviation is small, which in our case is 0.66%. We recommend that the fund take full advantage of the maximum leverage permissible of 20x, as explained above, to extract the a return of 12.0% given that each of the trades are notional amount and duration neutral.

Risks

Trades were constructed to be duration neutral which hedges against the risk of parallel shifts in the yield curve. However, there is still exposure to curve re-shapings which could result in losses. A more sophisticated approach might use principal component analysis to determine the major factors that affect yield curve movements and hedge out the majority of the principal factor risks.

There was also an assumption that deviations (actual – theoretical) of points on the swap curve are mean reverting. To the extent that this is not true and deviations continue to diverge from expected values over a long period of time, this strategy would lose money.

Transaction Costs

For the purpose of the trading strategy, we have assumed a bid/ask spread of 1 basis point, similar to Duarte, Longstaff, and Yu (2005). So, for example, when we receive a fixed rate, we actually receive the fixed rate – 0.5 basis points. When we pay a fixed rate, we actually pay the fixed rate + 0.5 basis points. This results in the 1 basis point spread we mentioned above.

If the same trades were executed with transactions costs incurred based on the reported bid-ask spreads from Bloomberg, the strategy loses a significant amount of money as any profits from this trading strategy are typically more than offset by the actual transaction costs, resulting in the fund losing money. Trading costs overwhelm any fundamental value opportunities that arise for hedged trades on the swap curve.

To get some perspective, we calculated our profits and losses based on the 1 basis point spread and compared that to our profits and losses based on the actual bid-ask spreads as reported by Bloomberg, based on a notional value of $1 million.

Reported Bid/Ask

	Received Fixed	Pay Fixed
3-Year Butterflies	(13,683)	(25,662)
7-Year Butterflies	(82,427)	(52,667)
Total		(174,439)

Assumed Bid/Ask of 1 BP

	Received Fixed	Pay Fixed
3-Year Butterflies	14,771	8,183
7-Year Butterflies	6,926	16,366
Total		46,246

In addition, out of the ten trades that we first looked at which generated a profit under the 1 basis point scenario, eight of those turned out to be losses using the reported bid-ask spread. Realistically in speaking with traders who actively trade in these markets, they claim 0.5 basis point is the typical bid-ask spread for large institutional traders. Therefore, we are comfortable with their assumption of 1 basis point and believe it errs on the conservative side.

Conclusion

Based on our analysis above, the swap curve arbitrage is a profitable trading strategy earning a cumulative amount of $46,246 on a notional capital of $1,000,000 over a 10-year period and an annualized return of 0.60%, conditional on trading. As demonstrated by the Share Ratio of 0.91, returns on a risk adjusted basis are respectable and can be further enhanced through the use of leverage.

Alternate strategies for the swap curve arbitrage that may be built on our basic model include:

- Reduce the trigger level for entering into trades from the current 2 standard deviations to 1 standard deviation. However, we tested this strategy and found it to be relatively less profitable on both an absolute and risk adjusted basis even though it entailed a larger number of trades (approximately 2.5 times more). The annualized return is lower at 0.22% and the Sharpe Ratio is also lower at 0.36.

Mean	$131.08
Standard Error	$89.77
Median	-$145.55
Standard Deviation	$1,491.32
Sample Variance	2,224,033.08
Kurtosis	18.18
Skewness	3.68
Range	$13,090.36
Minimum	-$1,974.40
Maximum	$11,115.96
Sum	$36,177.71
Count	276
Avg Days Per Trade	14.90
Periods in Trading Year	16.78
Mean Return	0.01%
Annualized Return	0.22%
Mean SD	0.15%
Annualized SD	0.61%
Sharpe Ratio	0.3604

	Received Fixed	**Pay Fixed**
3-Year Butterflies	20,578	12,588
7-Year Butterflies	(1,722)	4,732
Total		36,178

- Use principal component analysis to analyze the major factors that affect yield curve movements and hedge out the dominant factors such as PC1 (level) and PC2 (steepness) and capitalize on perceived dislocations of PC3 (curvature).

While this trading strategy in and of itself does not constitute a profitable hedge fund, it is a valuable strategy for a multi-strategy hedge fund to have in its arsenal.