ANALYSIS OF SERPENTINE COLLECTORS IN LOW FLOW SYSTEMS

Myrna Dayan
Sanford Klein
William Beckman
Solar Energy Laboratory
University of Wisconsin-Madison
1500 Engineering Drive
Madison, WI 53706

ABSTRACT

Existing analytical models to determine the collector heat removal factor for serpentine collectors have limitations both for low flow and a large number of tube bends. A finite-difference numerical model was compared to existing analytical models (Abdel-Khalik, 1976 and Zhang & Lavan, 1985). The finite difference model shows that if the serpentine collector has 10 or more bends (as expected in practical situations), the numerical results approach the analytical solution for one turn. As a consequence, the Hottel-Whillier equation (Duffie & Beckman, 1991) can be used to approximate the performance of a serpentine collector.

The major reason for the difference in performance between the conventional header-riser collector and the serpentine collector is the internal heat transfer coefficient. A serpentine collector and a header-riser collector with similar tube geometry and size were compared. An increase of about 3% in the simulated annual solar fraction collected was observed for flow rates of 0.004 kg/s-m^2 to 0.01 kg/s-m^2.

1. INTRODUCTION

Low flow solar domestic hot water systems offer the potential advantages of reduced piping costs, lower installation costs and lower parasitic power use compared to conventional systems which require 0.010-0.015 l/s-m^2. Low-flow systems can result in increased tank stratification. Increased tank stratification can result in lower collector inlet temperatures and consequently increased collector efficiency.

The disadvantage of low flow systems is the collector heat removal factor, F_R decreases with decreasing flow rate. A serpentine collector has the potential to perform better than a conventional header-riser collector in low-flow systems due to the earlier onset of turbulent flow which enhances the internal heat transfer coefficient. The onset of turbulent flow is a function of tube diameter and flow rate per tube.

Serpentine collectors consist of a flow duct that is bonded to the absorber plate in a serpentine or zigzag fashion. A serpentine collector is shown in Fig. 1.

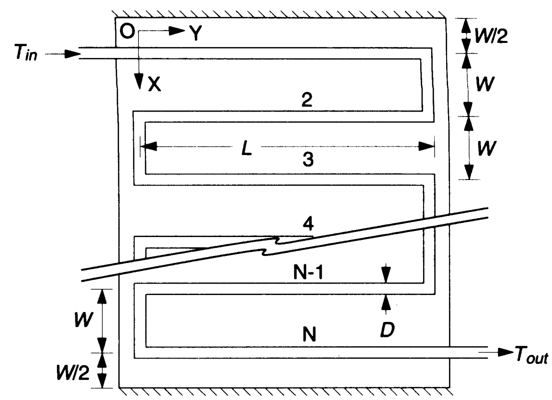

Fig. 1: Serpentine flat-plate collector

The heat removal factor for a serpentine collector is much more difficult to determine than for a conventional flat-plate collector. Unlike the analysis for the header-riser flat-plate where the fins between the tubes are assumed adiabatic at the center of the tube spacing, there is heat transfer between the tubes for a serpentine collector, resulting in a two-dimensional heat transfer problem.

Abdel-Khalik (1976) analyzed the heat removal for a flat-plate solar collector with a serpentine tube. This analysis produced graphical results that can be used to obtain the heat removal factor. Fig. 2 represents the generalized chart for estimating the heat removal factor, F_R, for flat-plate collectors with serpentines of arbitrary geometry and number of bends. The parameters F_1 and F_2, given in equations 1 and 2 respectively, are functions of physical design parameters, including plate thickness, conductivity and tube spacing.

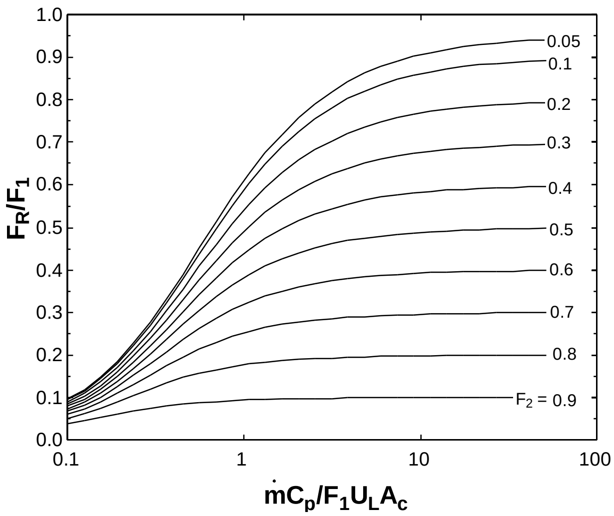

Fig. 2: Generalized chart for estimating the heat removal factor by Abdel-Khalik

$$F_1 = \frac{NkL}{U_L A_c} \frac{kR(1+g)^2 - 1 - g - kR}{[kR(1+g)-1]^2 - (kR)^2} \tag{1}$$

$$F_2 = \frac{1}{kR(1+g)^2 - 1 - g - kR} \tag{2}$$

where

$$k = \frac{kdm}{(W-D)\,\boldsymbol{sinh}\,m} \tag{3}$$

$$g = -2\,\boldsymbol{cosh}\,m - \frac{DU_L}{k} \tag{4}$$

$$m = (W-D)\sqrt{\frac{U_L}{kd}} \tag{5}$$

In the above equations, D (m) is the outer tube diameter, k (W/m-K) represents the plate conductivity, d (m) is the plate thickness, R (m-K/W) is the resistance between the tube and the plate and N is the number of turns in the serpentine collector plus one.

Abdel-Khalik states that the differences in the values of F_R/F_1 for one turn, (N=2) and those obtained numerically for higher values of N are less than 5 %. These differences vanish completely for values of $\dot{m}c_p / F_1 U_L A_c$ greater than unity. In other words, the graphical results are valid within 5% for all practical situations.

Zhang and Lavan (1985) argue that this is not the and present an analytical solution to Abdel-Khalik's analysis for N=2 or for the case where the parameter, $\dot{m}c_p / F_1 U_L A_c$ is greater than unity. They also provide analytical solutions for N=3 and N=4, however these are in matrix form and difficult to implement.

Zhang and Lavan state that the heat removal factor, F_R, is generally a maximum at N=1 and is generally a minimum at N=2. As N increases, F_R increases, but at a decreasing rate. For $N \rightarrow \infty$, F_R seems to approach the value for F_R at N=1. As the number of turns increases, the tube length increases for a given area. The surface area exposed to solar radiation increases and F_R increases. When N=1 the serpentine collector acts as a header-riser flat-plate and F_R is the largest since there is no heat transfer between tubes.

For practical applications, serpentine collectors have many turns, and therefore it is necessary to calculate F_R with a simple method. The matrix solutions are cumbersome to implement.

2. FINITE DIFFERENCE TECHNIQUE

A finite difference technique was developed. Abdel-Khalik presents analytical equations for heat flow per unit length entering the base of the tube, given in equations 6. In these equations, m is given by equation 5 and T_{bi} (K) is the temperature at the base of the plate for the segment i.

$$\begin{aligned} q_i^+ &= k\left[q_{i-1} - q_i \,\mathbf{cosh}\, m\right] \quad (2 \le i \le N) \\ q_i^- &= k\left[q_{i+1} - q_i \,\boldsymbol{cosh}\, m\right] \quad (1 \le i \le N-1) \\ q_1^+ &= kq_1(1 - \mathbf{cosh}\, m) \\ q_N^- &= kq_N(1 - \mathbf{cosh}\, m) \end{aligned} \tag{6}$$

where

$$q_i = T_{bi} - T_a - \frac{G_T}{U_L} \tag{7}$$

The useful energy gain to the tubes is given by equations 8a and 8b.

$$q_{useful} = q_i - DU_L q_i \tag{8a}$$

$$q_{useful} = \frac{(T_{bi} - T_{fi})}{R} \tag{8b}$$

where the quantity $[-DU_L q_i]$ is the energy collected per unit time and per unit length above the tube.

Figure 3 is a representation of the finite difference technique. The values of q represent useful energy, given (equations 8), transferred to the tube from the upper and lower parts of the absorber plate and g is an intermediate temperature.

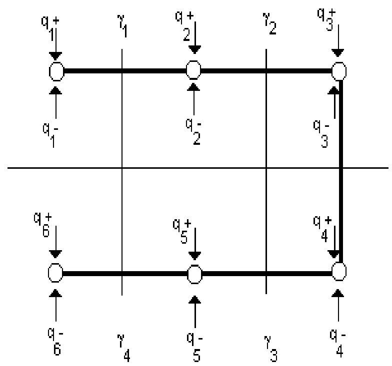

Fig. 3: Representation of the finite difference technique

The heat transferred to each node is represented in equations 9.

$$
\begin{aligned}
&\dot{m}C_p\,(g_1 - T_1) = q_1 \frac{\Delta Y}{2}\\
&\dot{m}C_p\,(g_2 - g_1) = q_2 \Delta Y\\
&\dot{m}C_p\,(g_3 - g_2) = (q_3 + q_4)\Delta Y \qquad\qquad (9)\\
&\dot{m}C_p\,(g_4 - g_3) = q_5 \Delta Y\\
&\dot{m}C_p\,(T_6 - g_4) = q_6 \frac{\Delta Y}{2}
\end{aligned}
$$

where

$$
\begin{aligned}
&\frac{g_1 + g_2}{2} = T_2\\
&\frac{g_2 + g_3}{2} = T_3 = T_4\\
&\frac{g_3 + g_4}{2} = T_5
\end{aligned}
$$

Special care was taken in the algorithm to ensure the boundary conditions at each turn were met. The boundary condition in the above example is T_3=T_4. That is, it is assumed that there is no energy conducted to the tube near the edge of the collector.

The main advantage of this finite difference technique is that assumptions need not be made that certain flow or geometry conditions are met, such as $\dot{m}c_p \,/\, F_1 U_L A_c$ being greater than unity.

The finite difference technique was compared to the solution given by Abdel-Khalik. Figure 4 represents the comparison between the methods. The collector was chosen to have a constant area of one square meter, thus varying the number of turns changed the tube spacing. Constant values for the fluid heat-transfer-coefficient, h_{fi}, of 1500 W/m^{2}K and heat loss coefficient, U_L of 5 W/m^{2}K were used.

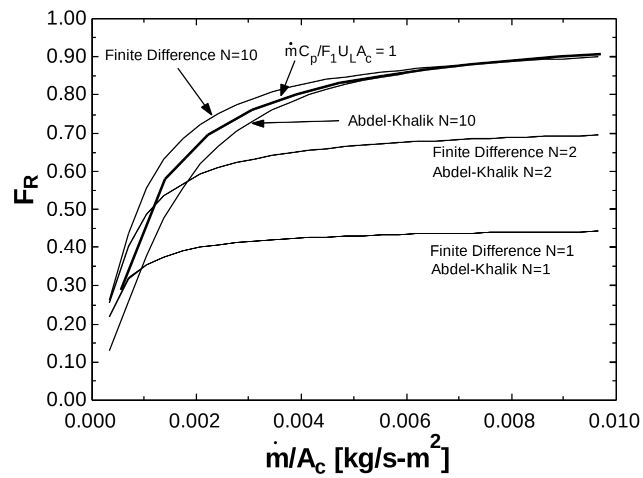

Fig. 4: Comparison of the finite difference and Abdel-Khalik model

The locus of $\dot{m}c_p \,/\, F_1 U_L A_c$ equal to unity was also plotted. For values of $\dot{m}c_p \,/\, F_1 U_L A_c$ greater than unity, the Finite Difference and Abdel-Khalik model compare favorably. At all flow rates, the two methods for one and two turns yield identical results. For N=4 the values for the collector heat removal factor compare reasonably within 4% with the largest discrepancies occurring when $\dot{m}c_p \,/\, F_1 U_L A_c$ is less than unity. The parameter $\dot{m}c_p \,/\, F_1 U_L A_c$ is equal to unity at a flow rate per unit area of about 0.06 kg/sm^2 with a serpentine model of N=10. Unfortunately, for the region of interest, at low flow rates and high values of the collector heat removal factor, the parameter $\dot{m}c_p \,/\, F_1 U_L A_c$ is less than unity and Abdel-Khalik's analysis does not hold for $N > 2$. At a flow rate of 0.002 kg/s.m^2 the percentage difference in the values of F_R for the finite difference and Abdel-Khalik's analysis is about 15 %.

The effect of number of tubes on F_R for a tube spacing of 10 cm is presented in Fig. 5. A minimum value for F_R occurs when N=2 and the maximum value occurs when N=1. As the number of turns increases, the values of F_R approach the values of F_R for N=1, therefore it can be postulated for N=∞ the values of F_R equal the values of F_R at N=1. This agrees with the results obtained by Zhang and Lavan.

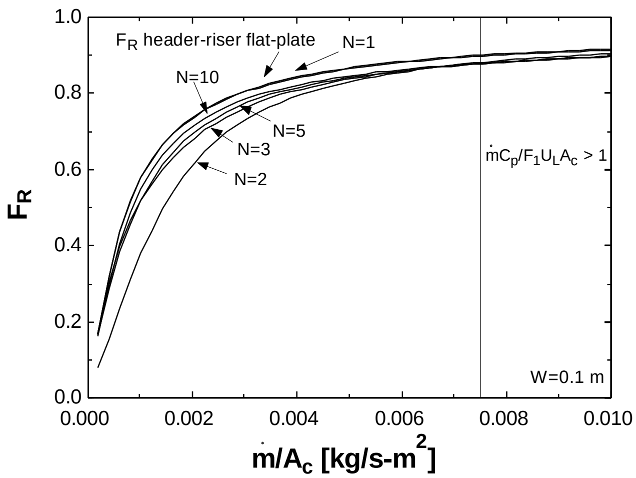

Fig. 5: Effect of the number of turns on collector performance

The results are also presented in Fig. 6 as the ratio of F_R to $F_{R,flat}$ (for various numbers of turns).

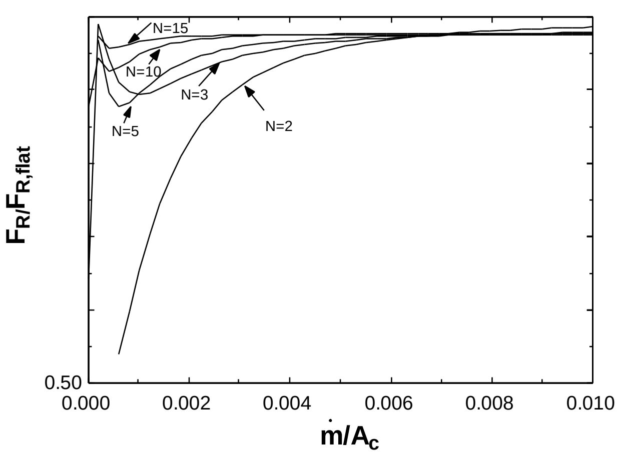

Fig. 6: Comparing the number of turns of the serpentine collector to the one turn collector

The difference for F_R between the 15 turn serpentine collector and the N=1 collector is at worst less than five percent for a flow rate of 0.004 kg/s.m^2. This flow rate is well below the expected operating range. For a flow rate of 0.002 kg/s.m^2 the difference between the models is less than 3 percent.

A serpentine collector may have more than 15 turns. In this case, the analysis for a long straight collector with no turns will hold. Therefore, the model is very close to the model for the flat-plate collector, with the exception being that the internal heat transfer coefficient will be different. A collector of N=1 could also be made by using a conventional collector with many turns and creating long cuts between the tubes, effectively decoupling the collector tubes.

The internal heat transfer coefficient is dependent on the flow rate through the tubes, the diameter of the tubes, the length of the tubes and the flow regime, that is, whether it is laminar or turbulent.

Fig. 7 represents the effect of tube diameter for a given flow rate of 0.002 kg/s.m^2. It can be seen that the tube diameter plays little importance in the collector heat removal factor for the header-riser flat-plate collector. However, the tube diameter is very important in serpentine collectors. In order to promote turbulent flow, the tube diameter should be small.

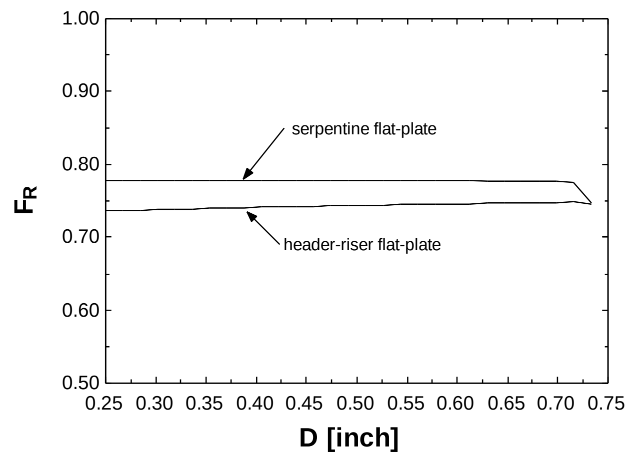

Fig. 7: Effect of tube diameter on collector performance

The serpentine collector of 19 tubes in parallel with 18 turns was compared to the conventional header-riser flat-plate, Fig. 8. The serpentine collector has better performance due to the higher heat transfer coefficient at collector flow rates greater than approximately 0.001 kg/s.m^2. The flow through the serpentine collector is 19 times greater than the flow through each riser of the conventional collector.

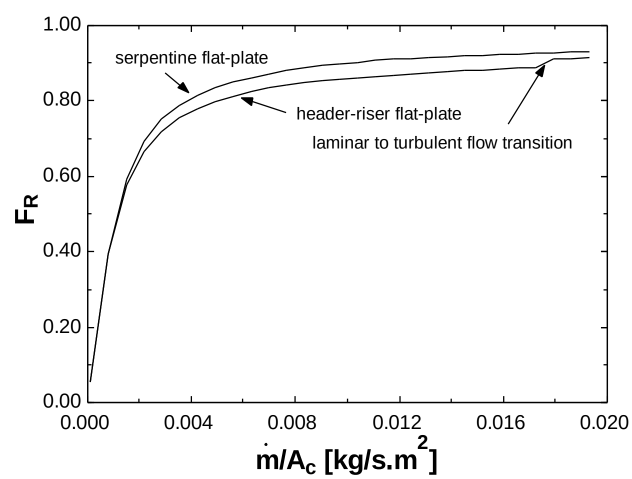

Fig. 8: Comparison of the heat removal factor for the header-riser and serpentine flat-plate collectors

Serpentine collectors have been disregarded in the past because of the large pumping requirements at higher flow rates, however at low flow rates the pumping requirement is much smaller. The theoretical pumping power requirement for a flow rate of 0.002 kg/s.m^2 is approximately 0.1 W. The fluid through the collector could be driven by a PV powered pump.

3. TRNSYS MODEL

In order to calculate the system performance, a TRNSYS (1998) model of the serpentine collector was developed. The model uses the assumption that it can be modeled as a long collector, with no bends. This is essentially the model for the conventional header-riser flat-plate collector, however the internal heat coefficient has been modified to account for the higher flow rate and longer tube.

Using a simple TRNSYS program the header-riser collector was compared to the performance of the serpentine collector. Fig. 9 represents the performance of the serpentine collector and the header-riser collector with similar tube geometry and size for various flow rates.

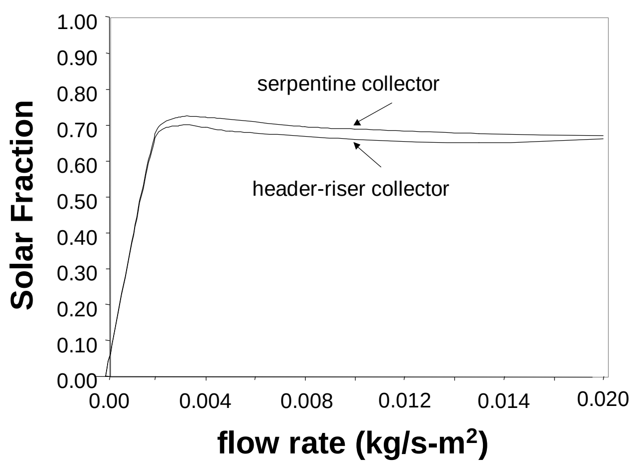

Fig. 9: Comparison of yearly performance of serpentine and header-riser flat-plate collectors for various flow rates.

An increase of about 3% in the annual solar fraction collected was observed for flow rates of 0.004 kg/s-m^2.

4. SERPENTINE COLLECTORS IN PARALLEL

In order to reduce the pressure drop across a serpentine collector it may be necessary to add collectors in parallel. It can be shown that the addition of collectors in parallel will have little effect on the collector heat removal factor given that the flow rate per total area remains the same.

5. CONCLUSIONS

Serpentine collectors perform slightly better than a header-riser collector with the same area, tube spacing and tube diameter at a low flow rate. The serpentine collectors perform better due to the earlier onset of turbulent flow, which enhances the internal heat transfer coefficient. The onset of turbulent flow is a function of the tube diameter and flow rate. However, as soon as the flow becomes turbulent, the pumping power increases substantially.

6. REFERENCES

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