Determination of Orbital Parameters for Visual Binary Stars Using a Fourier-Series Approach

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Abstract

We expand on the Fourier transform method of Monet (ApJ **234**, 275, 1979) to infer the orbital parameters of visual binary stars, and we present results for several systems, both simulated and real. Although originally developed to address binary systems observed through at least one complete period, we have extended the method to deal explicitly with cases where the orbital data is less complete. This is especially useful in cases where the period is so long that only a fragment of the orbit has been recorded. We utilize Fourier-series fitting methods appropriate to data sets covering less than one period and containing random measurement errors. In so doing, we address issues of over-determination in fitting the data and the reduction of other deleterious Fourier-series artifacts. We developed our algorithm using the MAPLE mathematical software code, and tested it on numerous "synthetic" systems, and several real binaries, including Xi Boo, 24 Aqr, and Bu 738.

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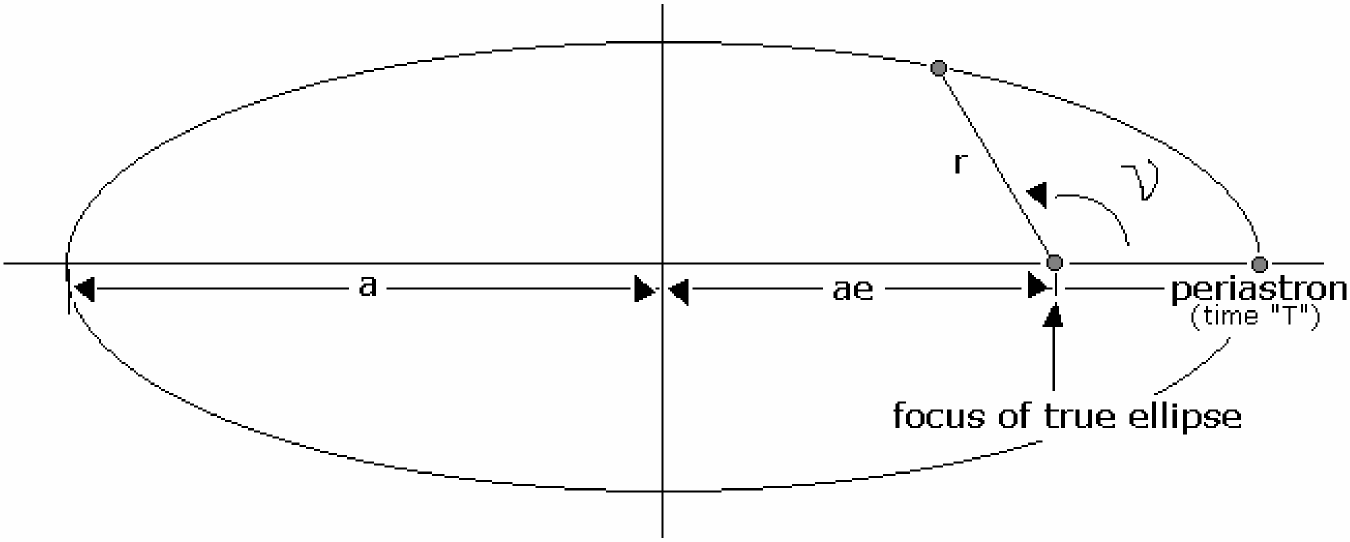
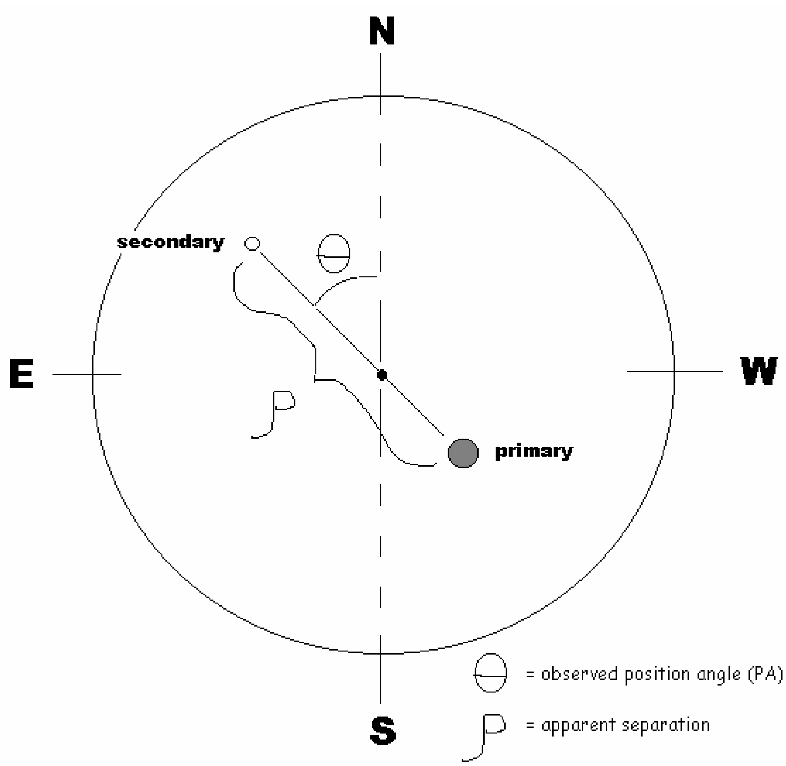
True Orbit

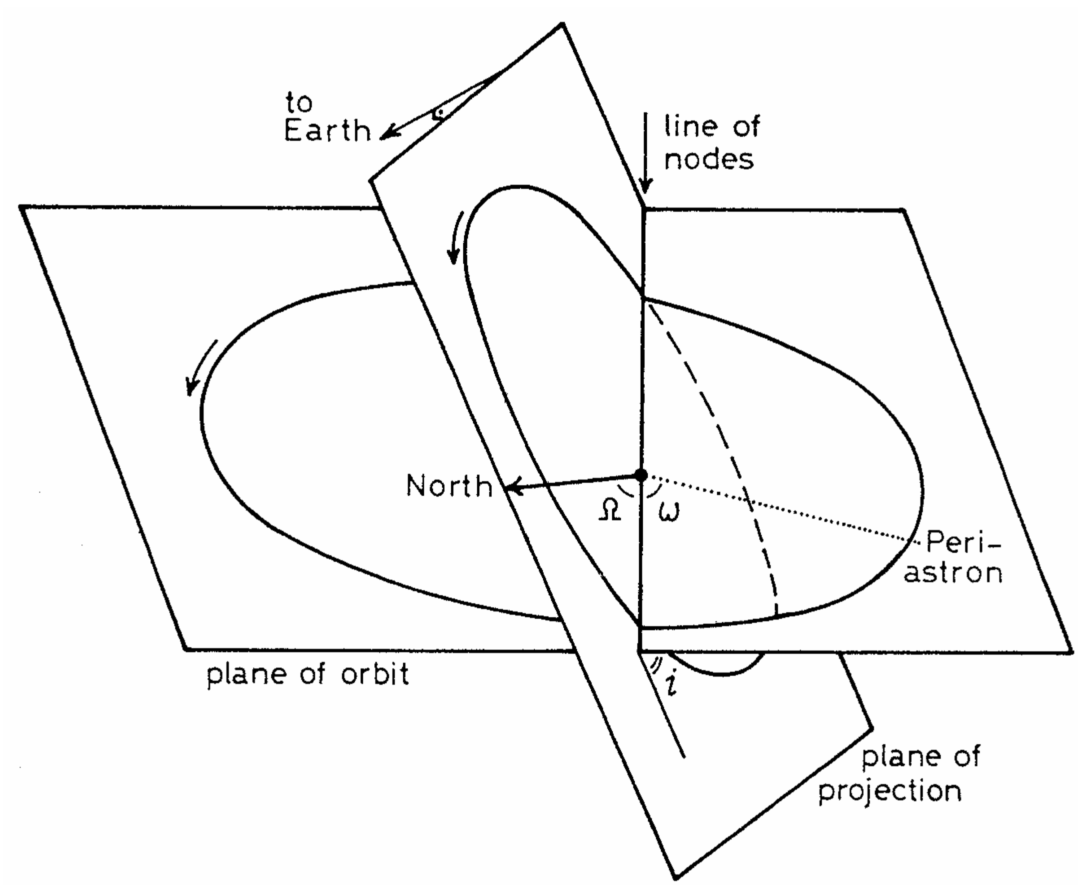


ν = true anomaly
r = true separation
a = semi-major axis
e = eccentricity
P = period
T = time of periastron passage

Observed Motion



Relationship Between True Orbit and Observed Motion



$$\rho \cos(\theta - \Omega) = r \cos(\nu + \omega)$$

$$\rho \sin(\theta - \Omega) = r \sin(\nu + \omega) \cos i$$

Operational Procedure

1. Begin with Observational Data Set

$$\{\rho_i, \theta_i, t_i\}$$

2. Select Trial Orbital Period

$$P$$

3. Fit Data to Fourier Series

$$x(t) = \rho\cos\theta = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n}{P}t\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi n}{P}t\right)$$

$$y(t) = \rho\sin\theta = c_0 + \sum_{n=1}^{\infty} c_n \cos\left(\frac{2\pi n}{P}t\right) + \sum_{n=1}^{\infty} d_n \sin\left(\frac{2\pi n}{P}t\right)$$

$$\{a_n, b_n, c_n, d_n\}$$

4. Monet's Method to Extract Orbital Parameters

$$\{a, e, T, \omega, \Omega, i\}$$

5. Iterate Period for Best Fit

$$\{a, e, P, T, \omega, \Omega, i\}$$

Method of Monet

Fourier Series Representation of Data

$$x(t) = \rho\cos\theta = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n}{P}t\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi n}{P}t\right)$$

$$y(t) = \rho\sin\theta = c_0 + \sum_{n=1}^{\infty} c_n \cos\left(\frac{2\pi n}{P}t\right) + \sum_{n=1}^{\infty} d_n \sin\left(\frac{2\pi n}{P}t\right)$$

Fourier Series Representation True Orbit

$$\frac{r}{a}\cos\nu = \sum_{n=0}^{\infty} F_n(e)\cos nM$$

$$\frac{r}{a}\sin\nu = \sum_{n=1}^{\infty} G_n(e)\sin nM$$

$$M \equiv \frac{2\pi}{P}t - \frac{2\pi}{P}T \equiv \frac{2\pi}{P}t - \Delta$$

Eccentricity-Dependent Functions

$$F_0(e) = -\tfrac{3}{2}e$$

$$F_n(e) = \frac{1}{\pi}\int_0^{2\pi} \cos E\cos\left[n\left(E - e\sin E\right)\right]\left(1 - e\cos E\right)dE$$

$$G_n(e) = \frac{1}{\pi}\sqrt{1-e^2}\int_0^{2\pi} \sin E\sin\left[n\left(E - e\sin E\right)\right]\left(1 - e\cos E\right)dE$$

Method of Monet

Fourier Series Representation Keplerian Orbit

$$x(t) = \rho\cos\theta = \left[\frac{r}{a}\cos\nu\right] af_3(\Gamma) + \left[\frac{r}{a}\sin\nu\right] af_4(\Gamma)$$

$$y(t) = \rho\sin\theta = \left[\frac{r}{a}\cos\nu\right] af_1(\Gamma) + \left[\frac{r}{a}\sin\nu\right] af_2(\Gamma)$$

$\Gamma \equiv \{\omega, \Omega, i\}$ $\qquad$ $f \equiv$ *trigonometric factors*

Extract Orbital Parameters from n=0, 1 Fourier Coefficients

Time of Periastron Passage…

$$\tan(n\Delta) = \frac{c_0 a_1 - a_0 c_1}{a_0 d_1 - c_0 b_1} \longrightarrow \Delta \longrightarrow T$$

Eccentricity (transcendental)…

$$\frac{F_1(e)}{F_0(e)} = \pm \frac{a_1\left(a_0 d_1 - c_0 b_1\right) + b_1\left(c_0 a_1 - a_0 c_1\right)}{a_0\left[\left(a_0 d_1 - c_0 b_1\right)^2 + \left(c_0 a_1 - a_0 c_1\right)^2\right]^{1/2}} \longrightarrow e$$

Angles and Semi-Major Axis…

$$\omega = \omega\left(F_1, G_1, \Delta, a_1, b_1, c_1, d_1\right)$$
$$i = i\left(F_1, G_1, \Delta, a_1, b_1, c_1, d_1\right)$$
$$\Omega = \Omega\left(F_1, G_1, \Delta, a_1, b_1, c_1, d_1\right)$$
$$a = a\left(F_1, G_1, \Delta, a_1, b_1, c_1, d_1\right)$$

$$\longrightarrow \omega, \Omega, i, a$$

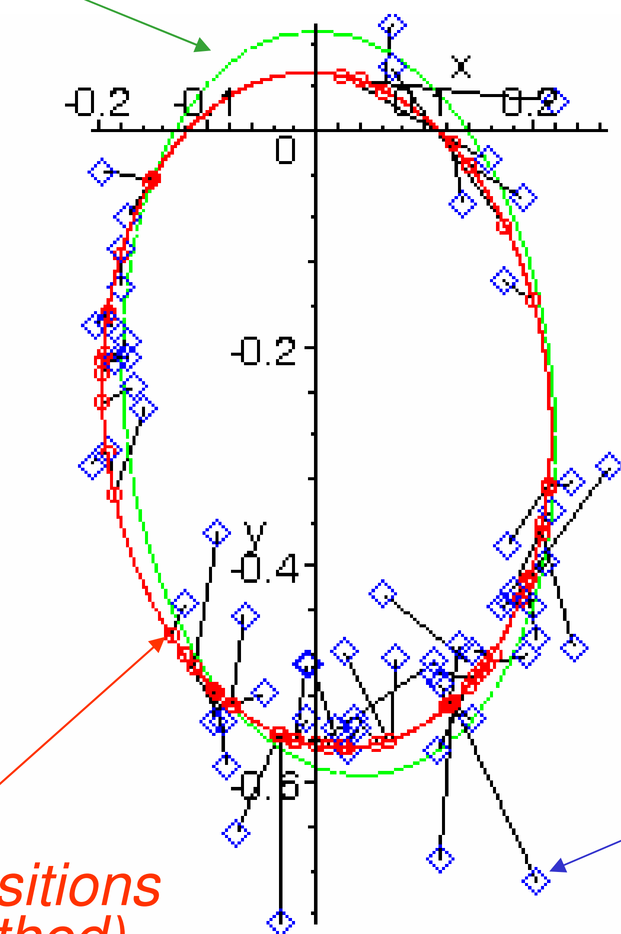
24 Aquarii (ADS 15176)

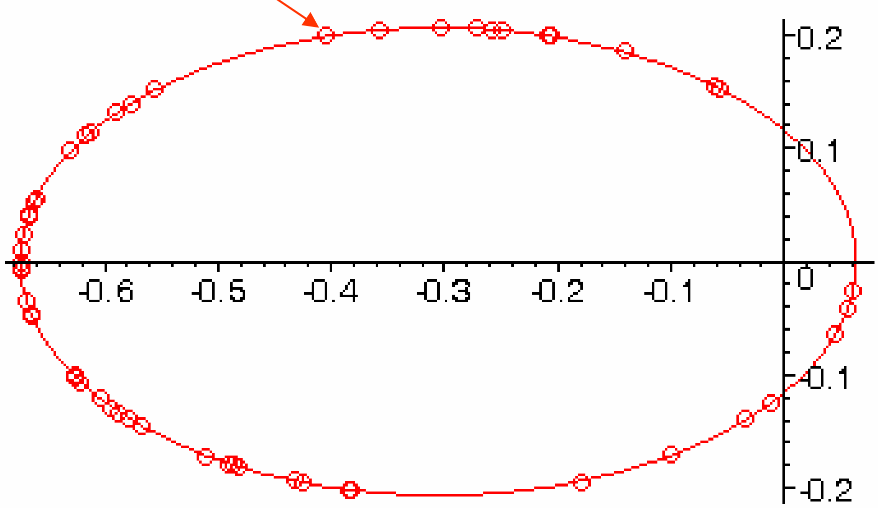
$n_{max} = 2$

$P = 47$

$\omega = 1.74$
$\Omega = -0.17$
$i = 0.58$
$a = 0.37$
$e = 0.83$
$\Delta = 4.58$

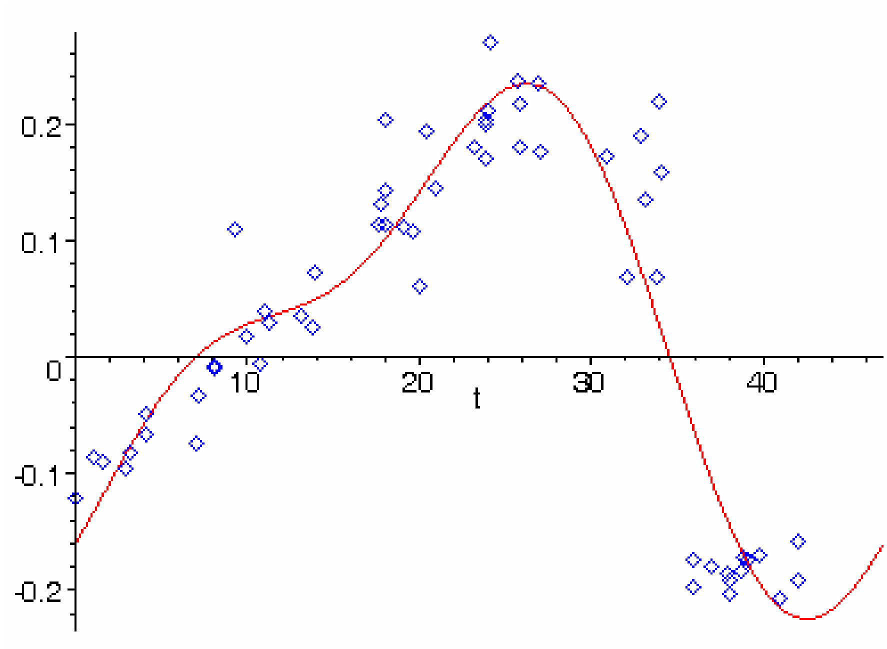
Error = 3.64



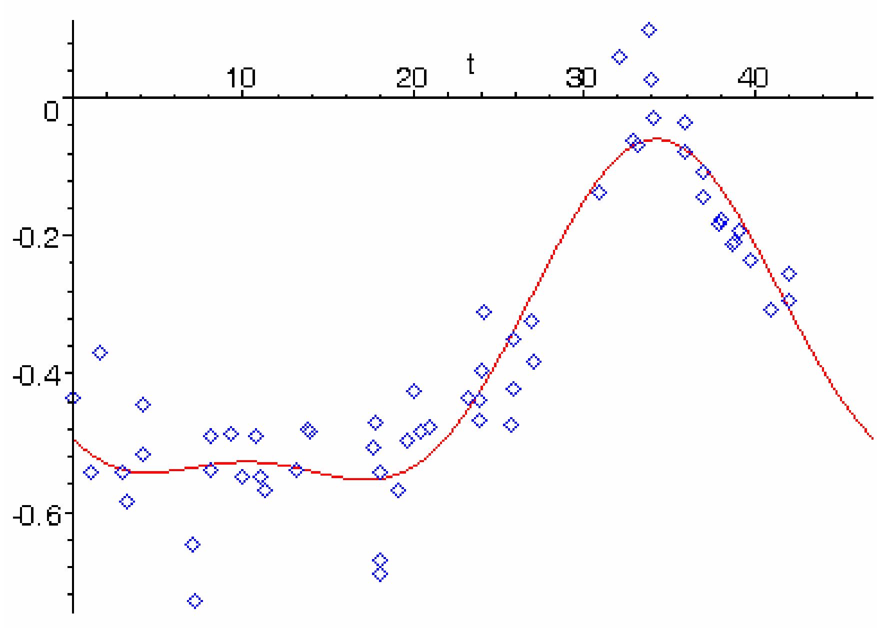


24 Aquarii (ADS 15176)

Fourier Fit to x(t)…



Fourier Fit to y(t)…



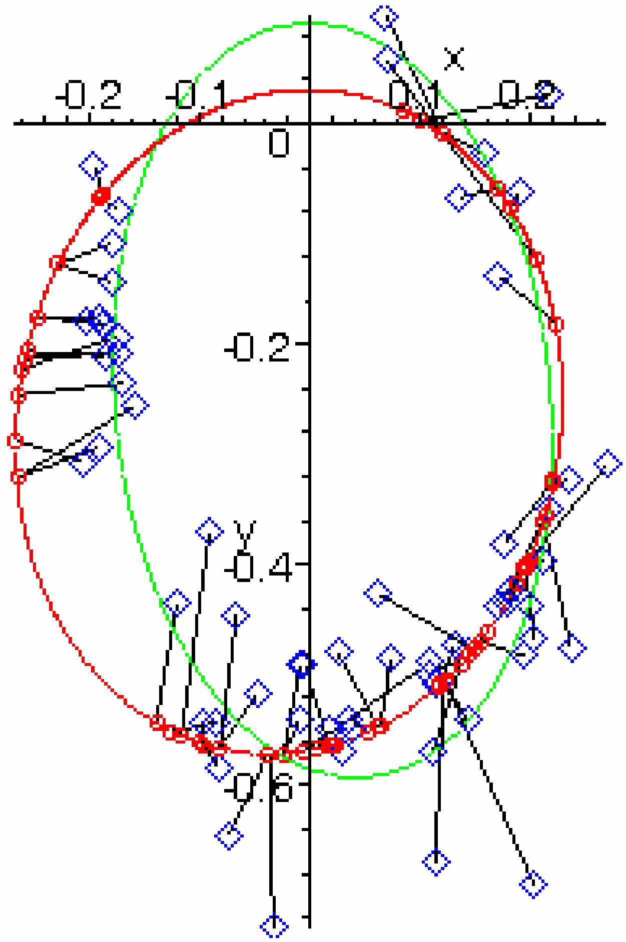
24 Aquarii (ADS 15176)

$n_{max} = 2$

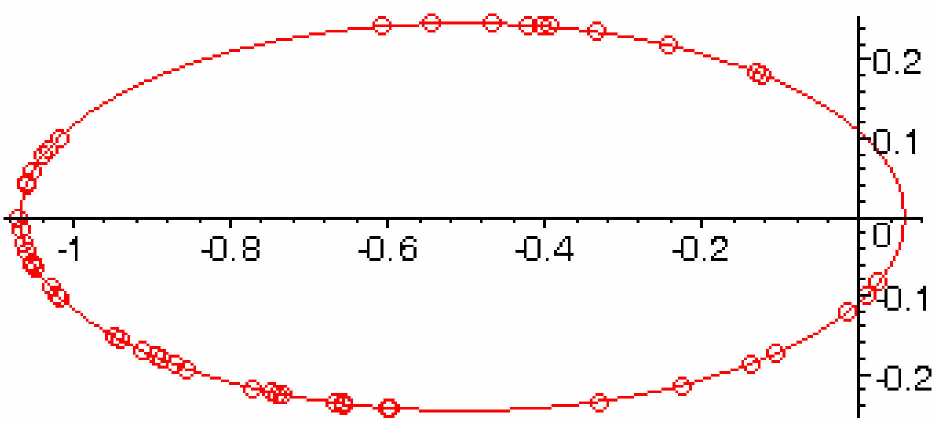
P = 55

$\omega = 1.53$
$\Omega = 0.01$
i = 1.01
a = 0.56
e = 0.89
$\Delta = 3.93$

Error = 4.27



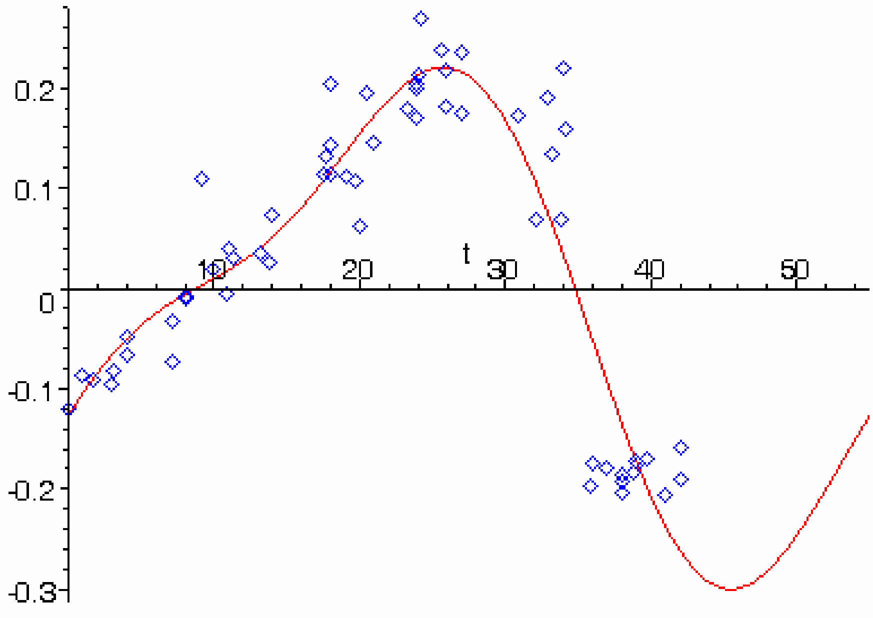
Computed Apparent Orbit



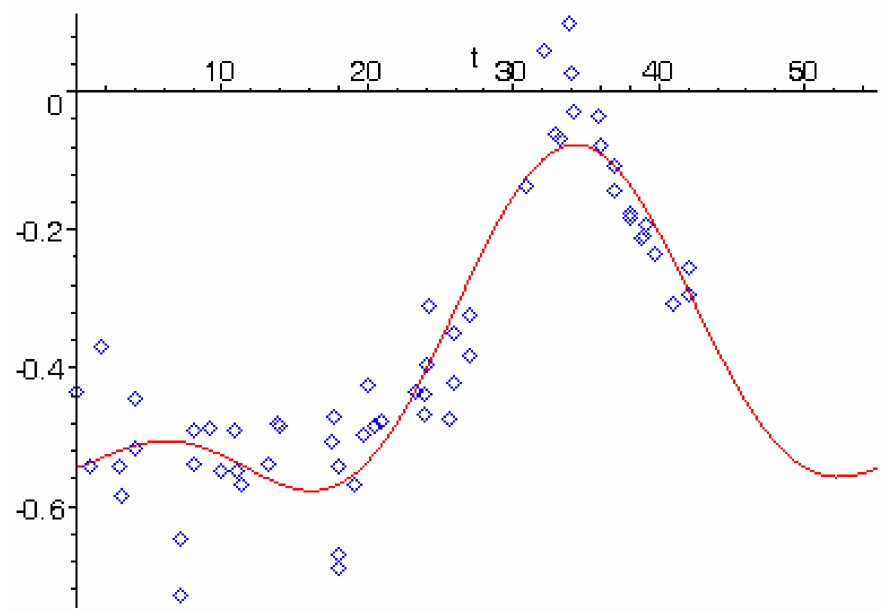
Computed True Orbit

24 Aquarii (ADS 15176)

Fourier Fit to x(t)…



Fourier Fit to y(t)…



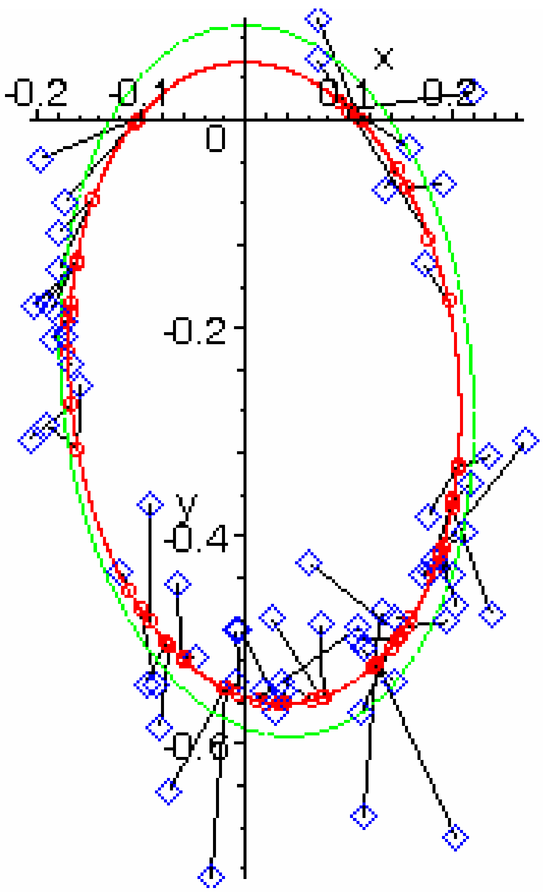
24 Aquarii (ADS 15176)

$n_{max} = 5$

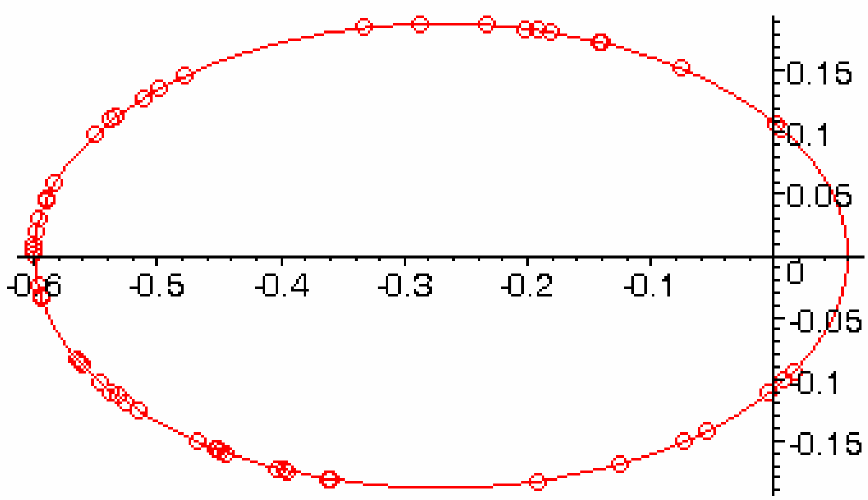
$P = 47$

$\omega = 1.89$
$\Omega = -0.27$
$i = 0.38$
$a = 0.33$
$e = 0.82$
$\Delta = 4.67$

Error = 3.79



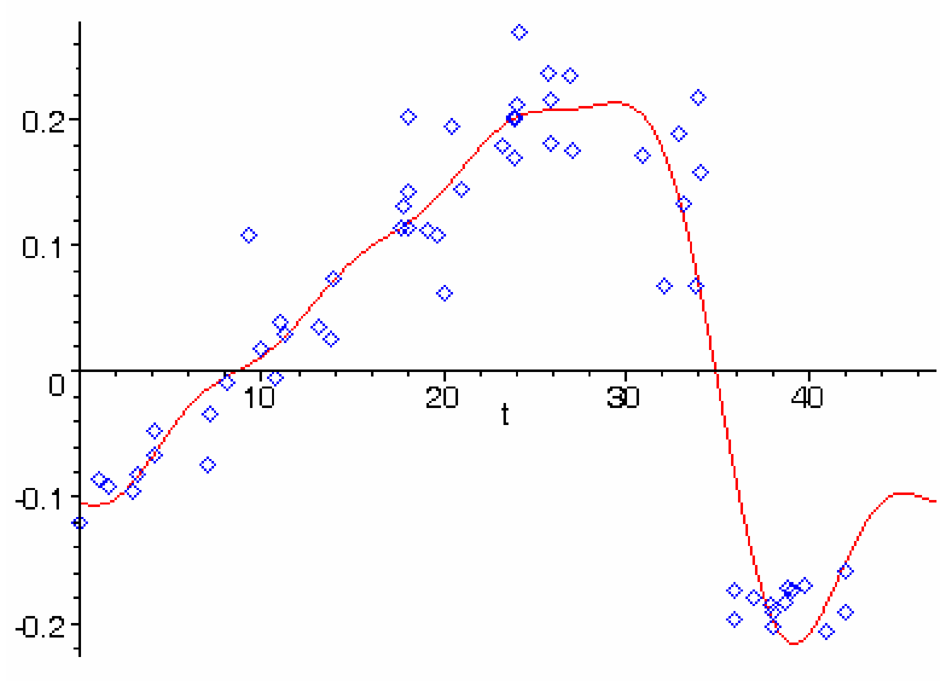
Computed Apparent Orbit



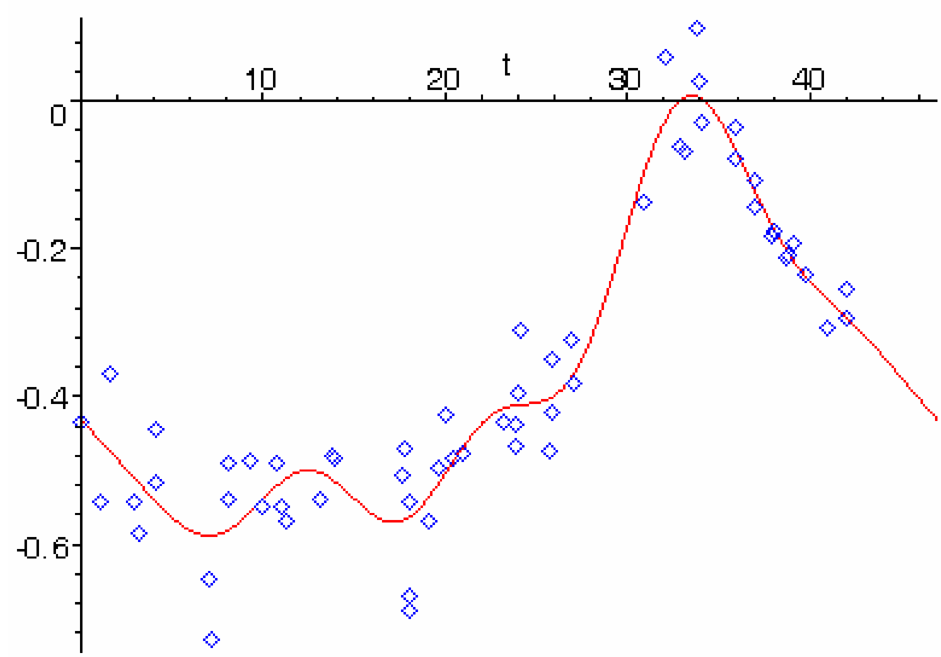
Computed True Orbit

24 Aquarii (ADS 15176)

Fourier Fit to x(t)…



Fourier Fit to y(t)…



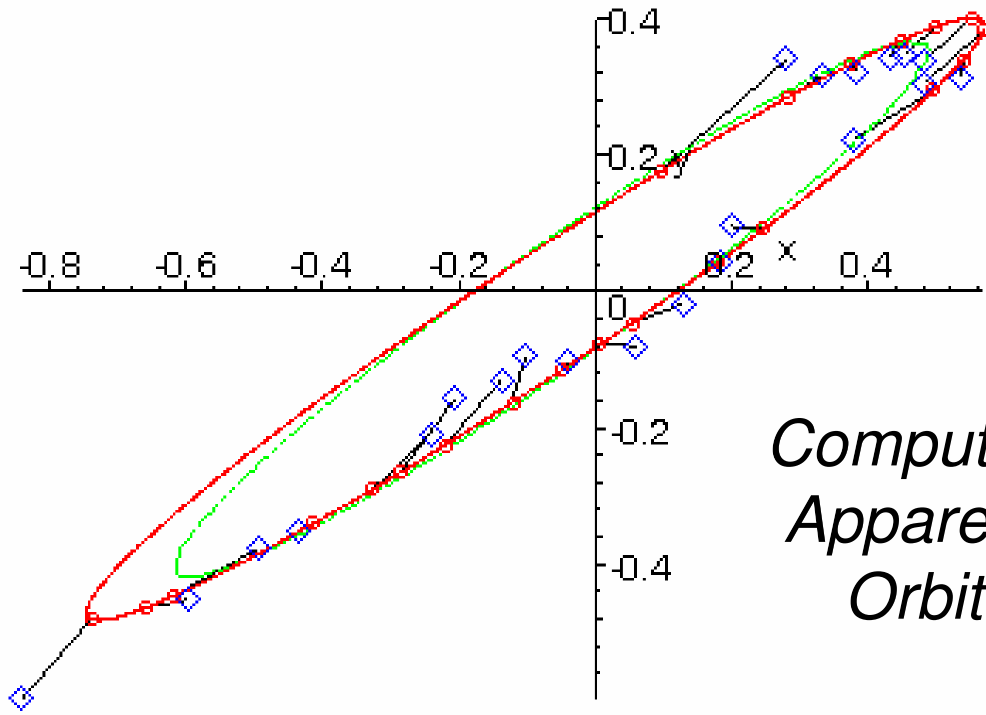
BU 738

$n_{max} = 1$

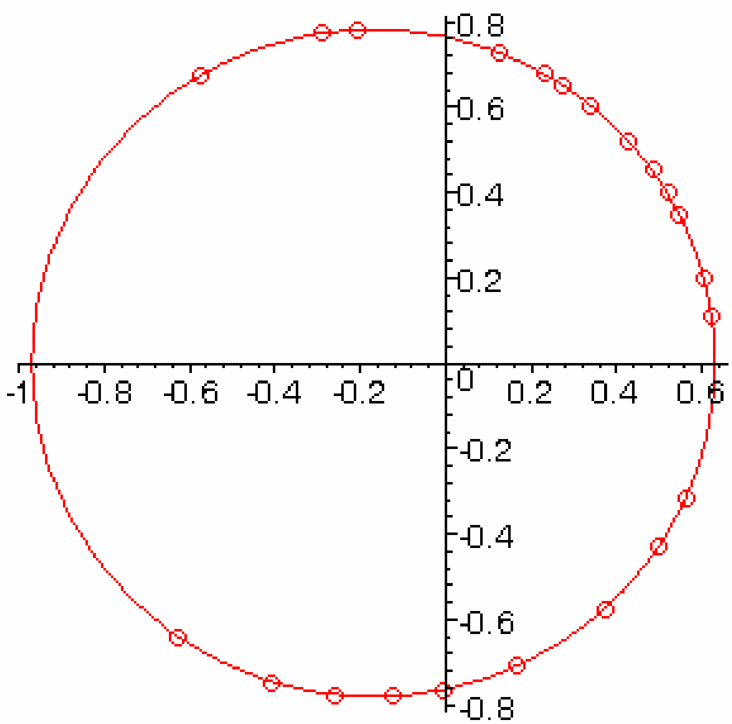
P = 100

$\omega = 0.97$
$\Omega = -0.59$
i = 1.47
a = 0.80
e = 0.21
$\Delta = 2.01$

Error = 1.82



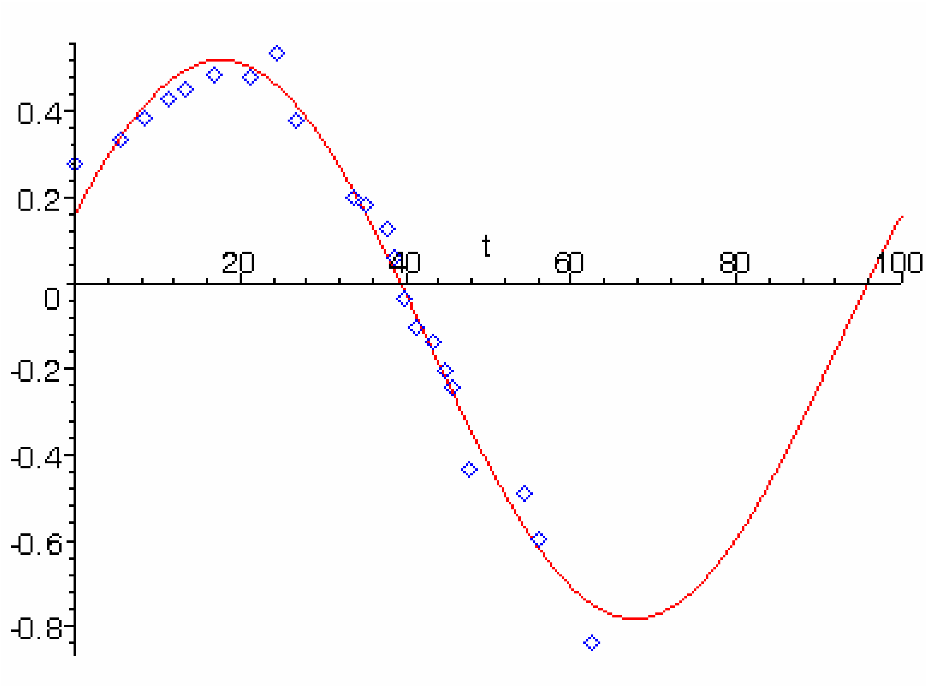
Computed Apparent Orbit



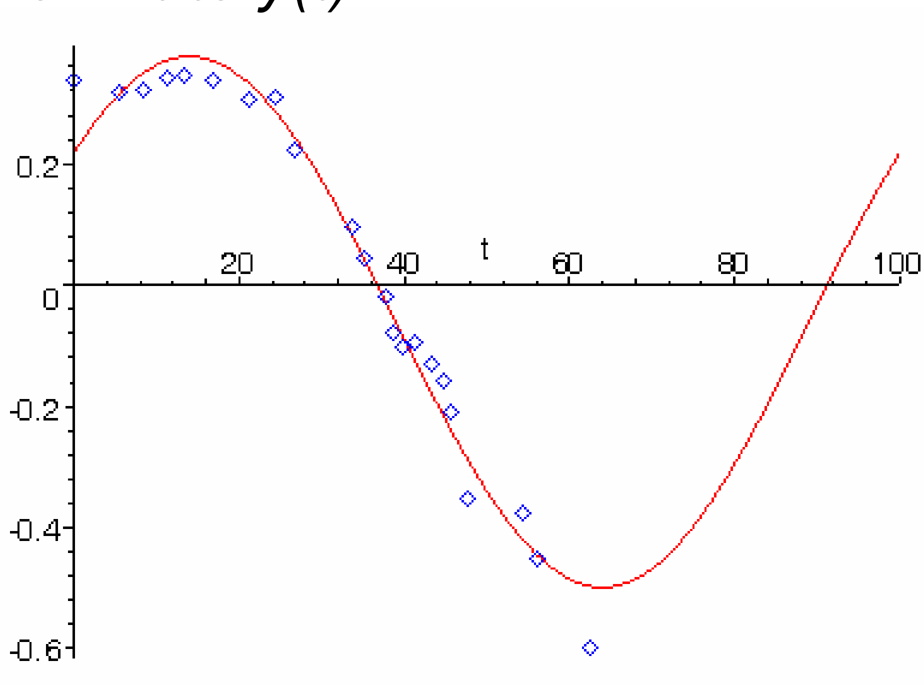
Computed True Orbit

BU 738

Fourier Fit to x(t)…



Fourier Fit to y(t)…



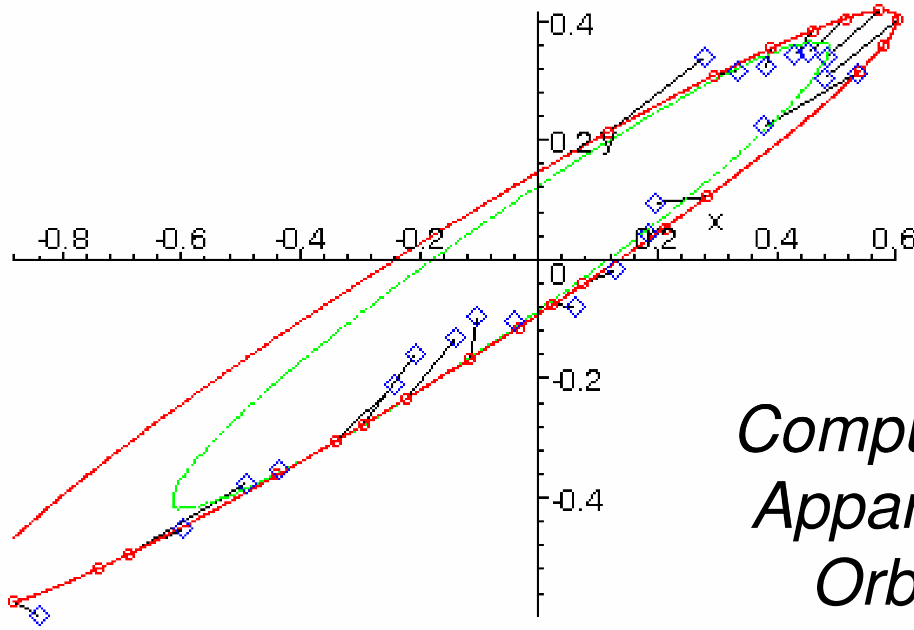
BU 738

$n_{max} = 1$

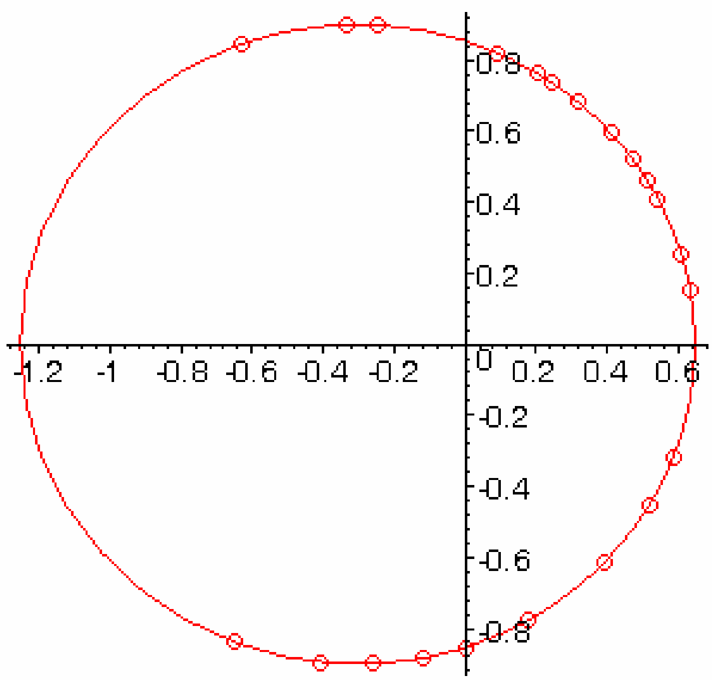
P = 120

ω = 0.86
Ω = -0.57
i = 1.46
a = 0.95
e = 0.32
Δ = 1.65

Error = 2.08



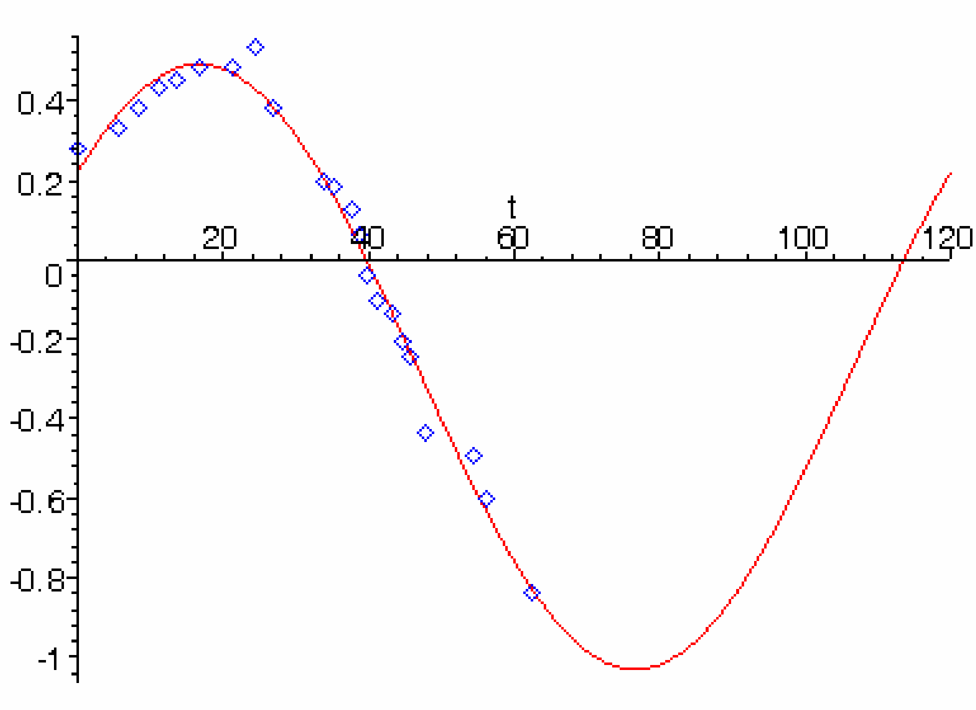
Computed Apparent Orbit



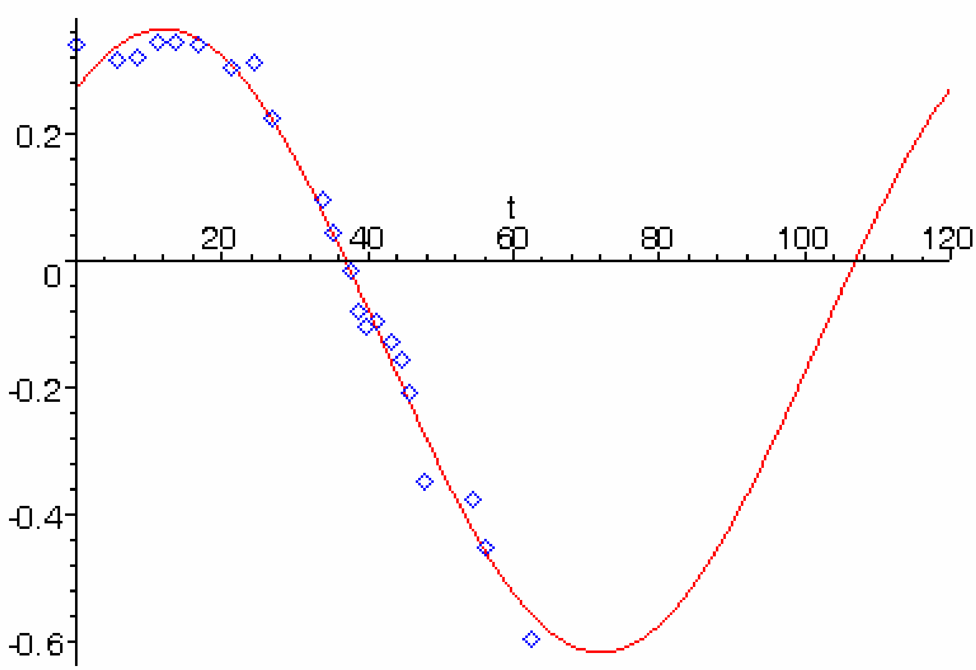
Computed True Orbit

BU 738

Fourier Fit to x(t)…



Fourier Fit to y(t)…



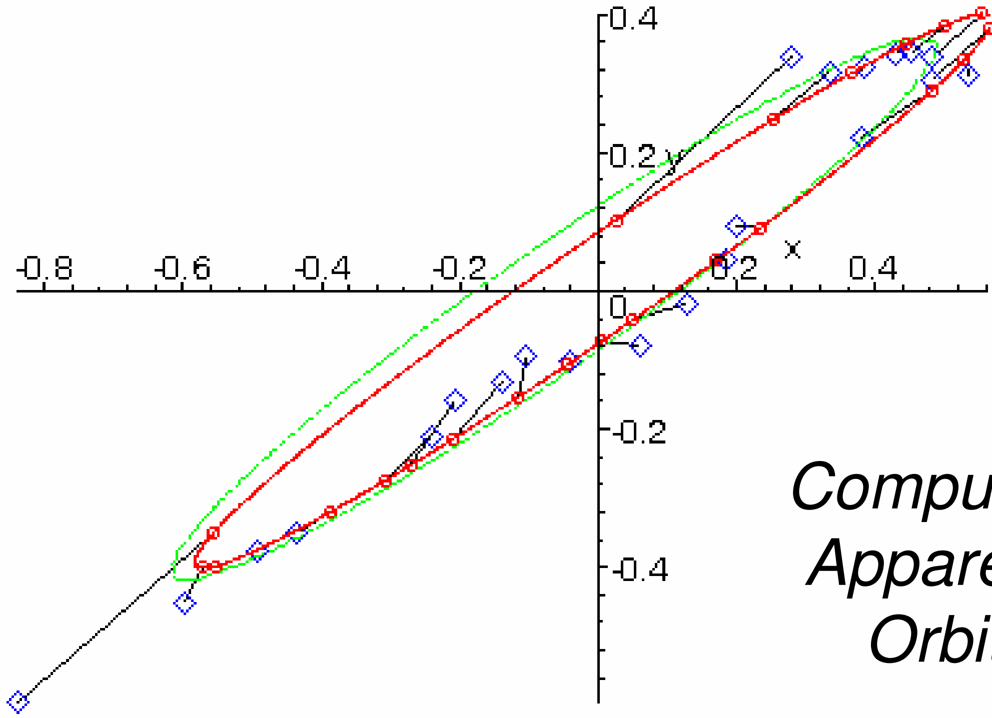
BU 738

n_{max} = 1

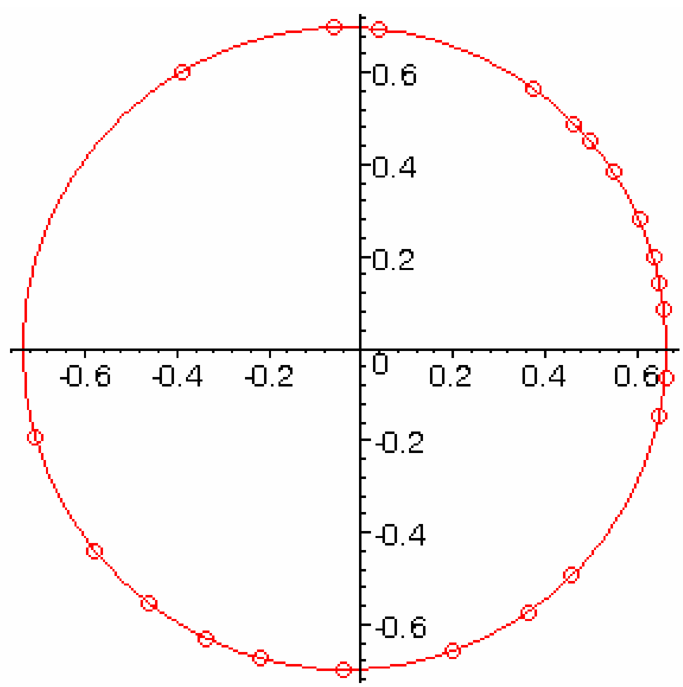
P = 80

ω = 1.41
Ω = -0.60
i = 1.48
a = 0.70
e = 0.05
Δ = 2.85

Error = 2.04



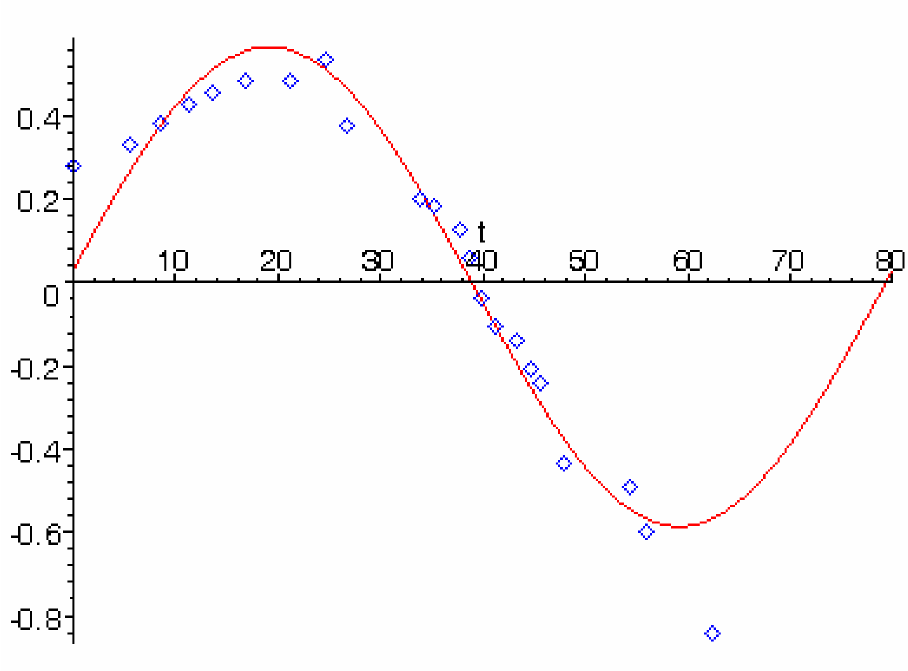
Computed Apparent Orbit



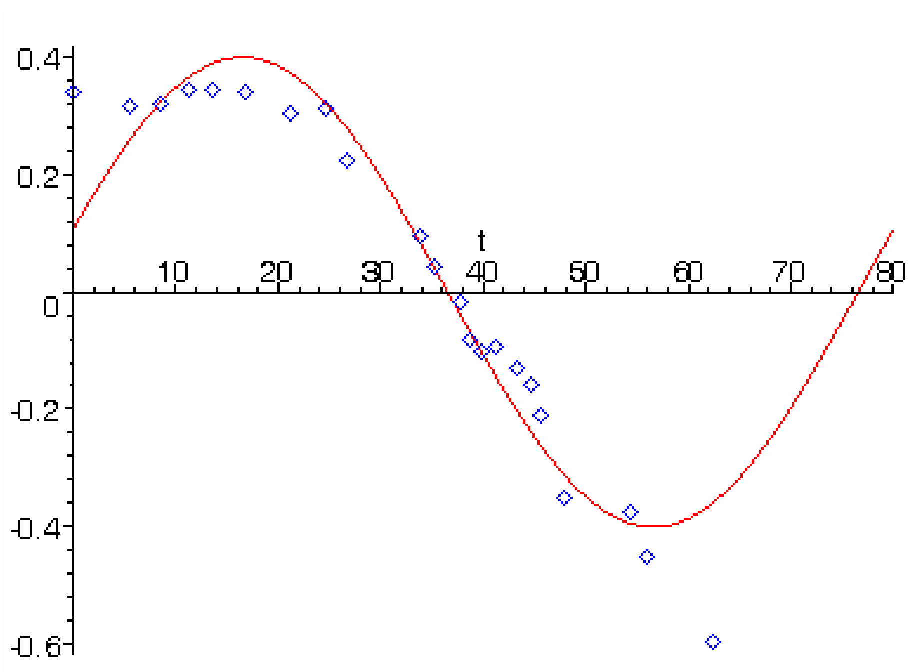
Computed True Orbit

BU 738

Fourier Fit to x(t)…



Fourier Fit to y(t)…



Xi Boo (ADS 9413)

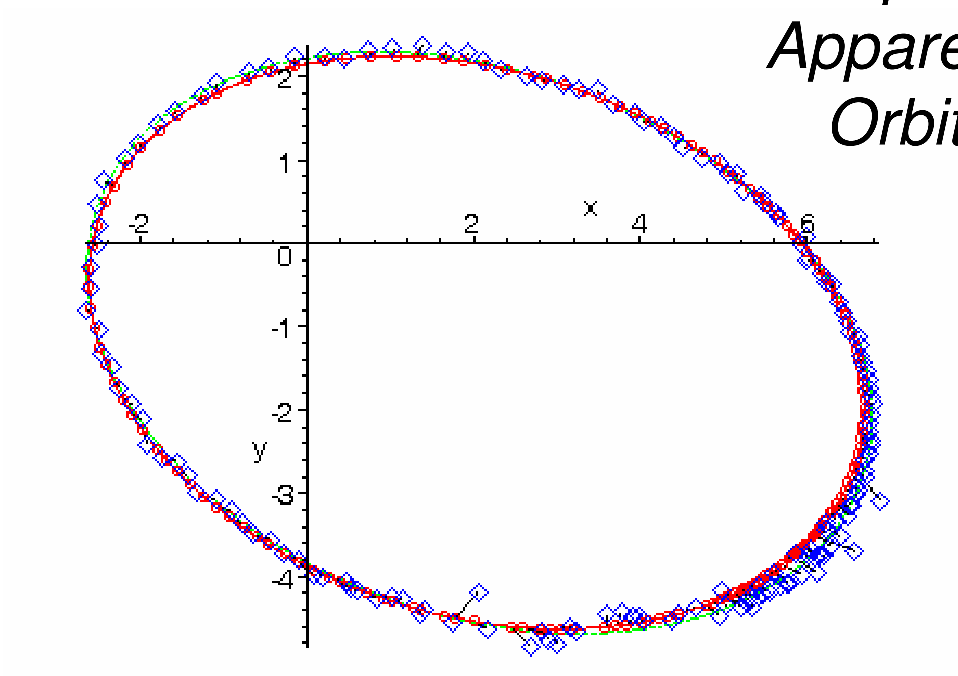
n_{max} = 1

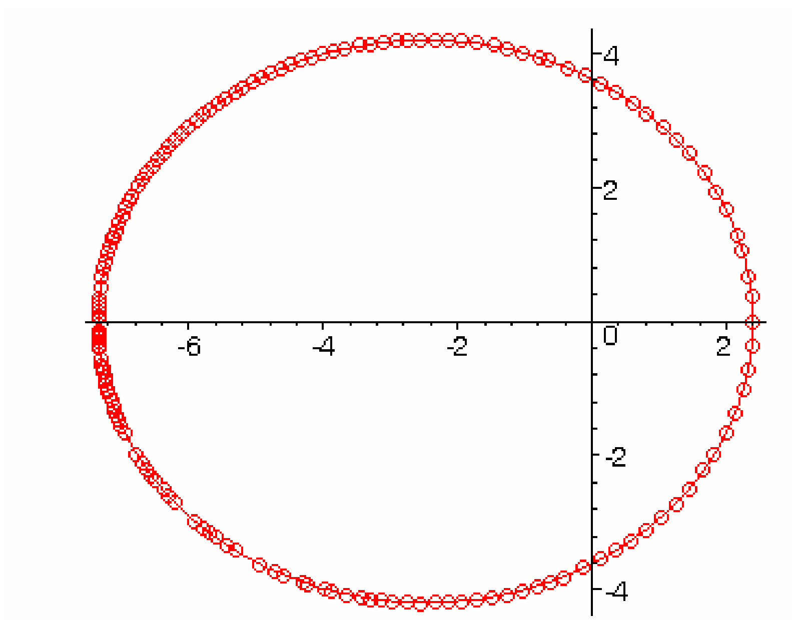
P = 151.5

ω = 0.49
Ω =0.15
i = 0.71
a = 4.88
e = 0.51
Δ = 3.71

Error = 21.12

Computed Apparent Orbit

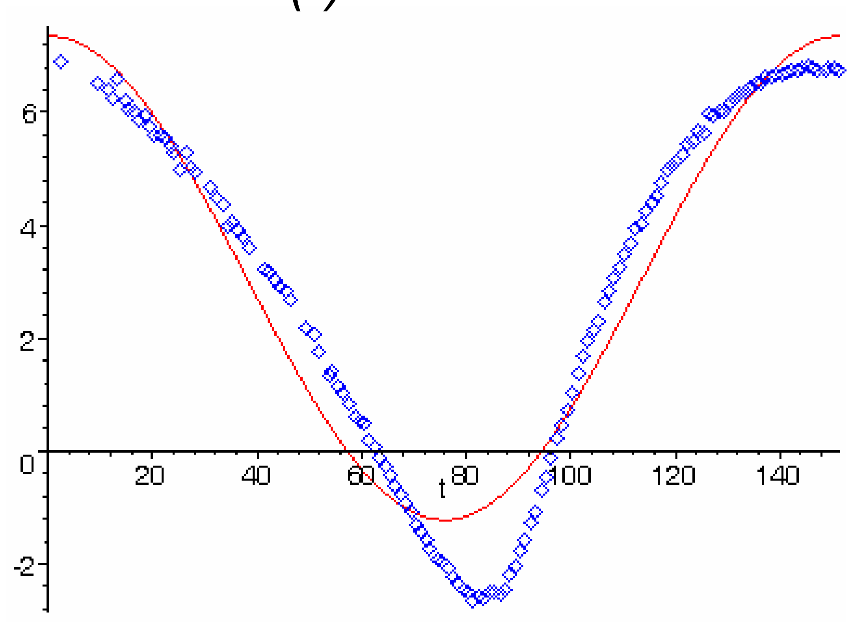




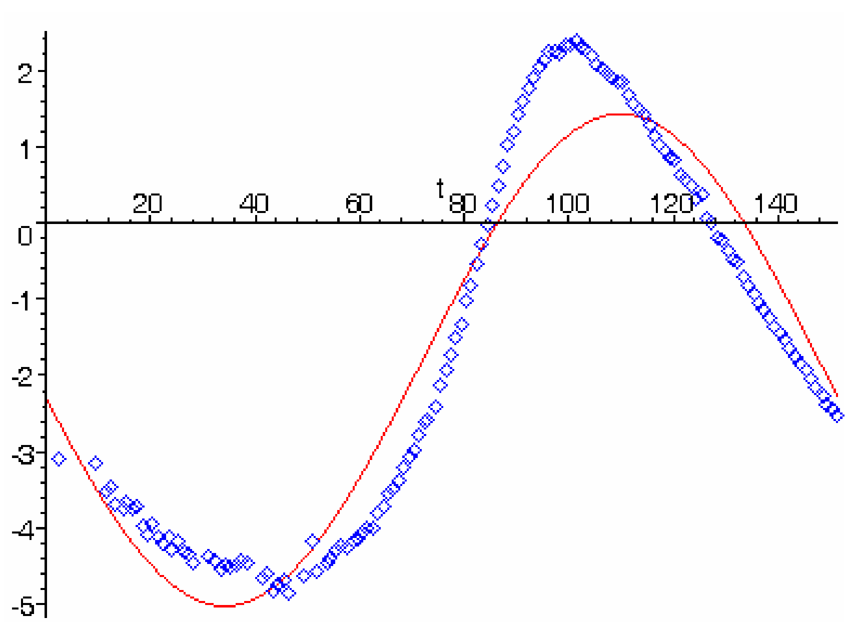
Computed True Orbit

Xi Boo (ADS 9413)

Fourier Fit to x(t)…



Fourier Fit to y(t)…



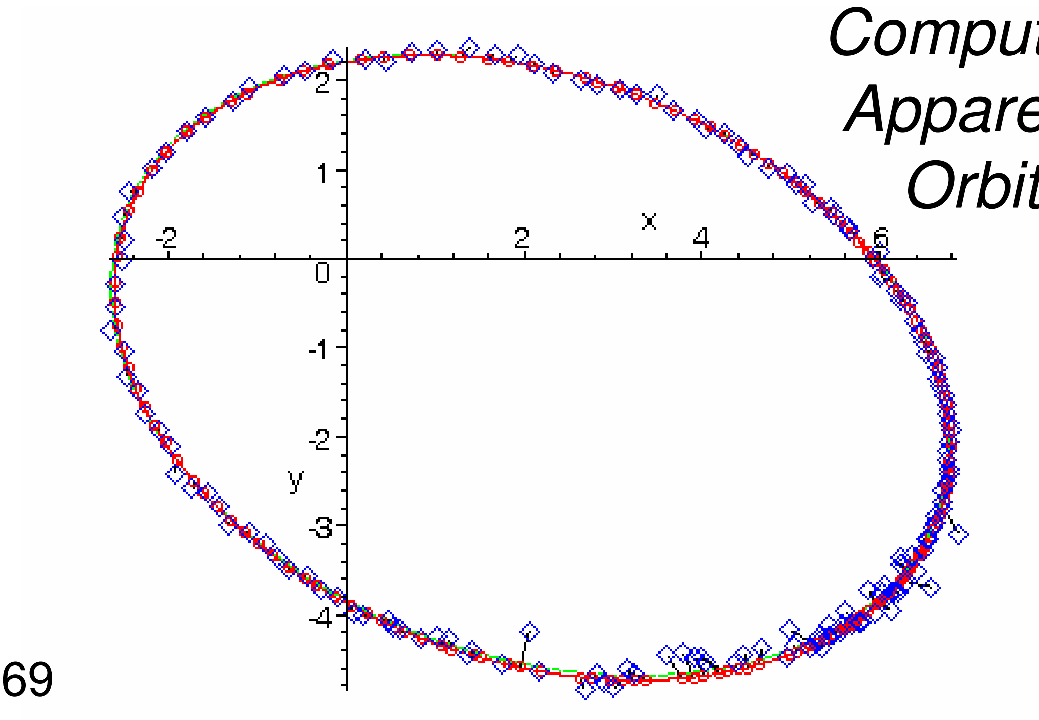
Xi Boo (ADS 9413)

$n_{max} = 2$

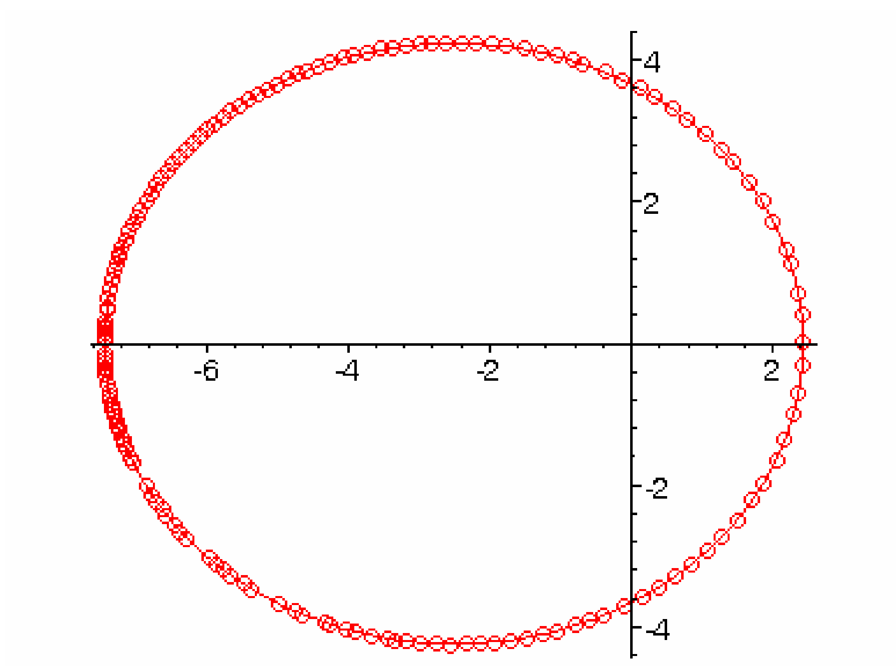
P = 151.5

$\omega = 0.40$
$\Omega = 0.22$
$i = 0.71$
$a = 4.93$
$e = 0.51$
$\Delta = 3.71$

Error = 12.69



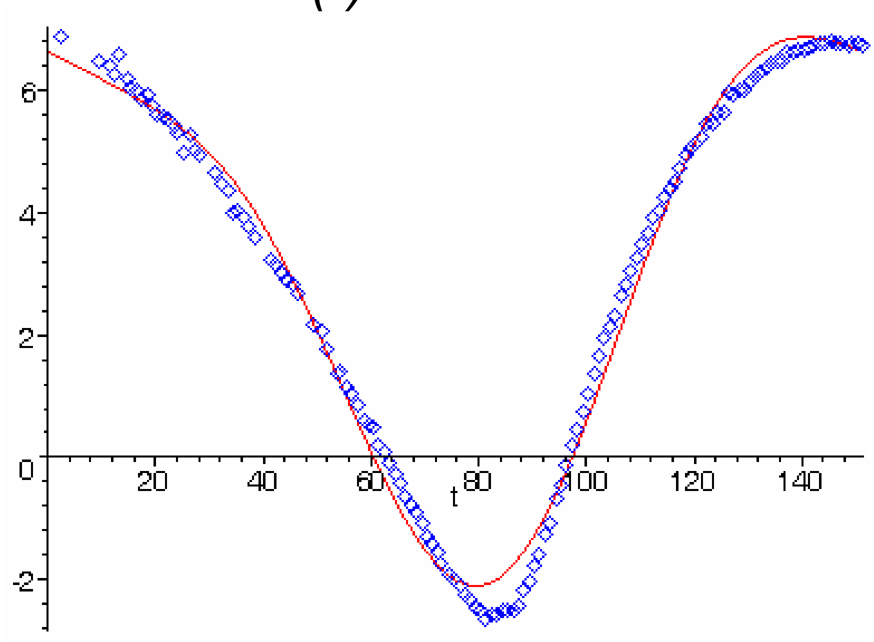
Computed Apparent Orbit



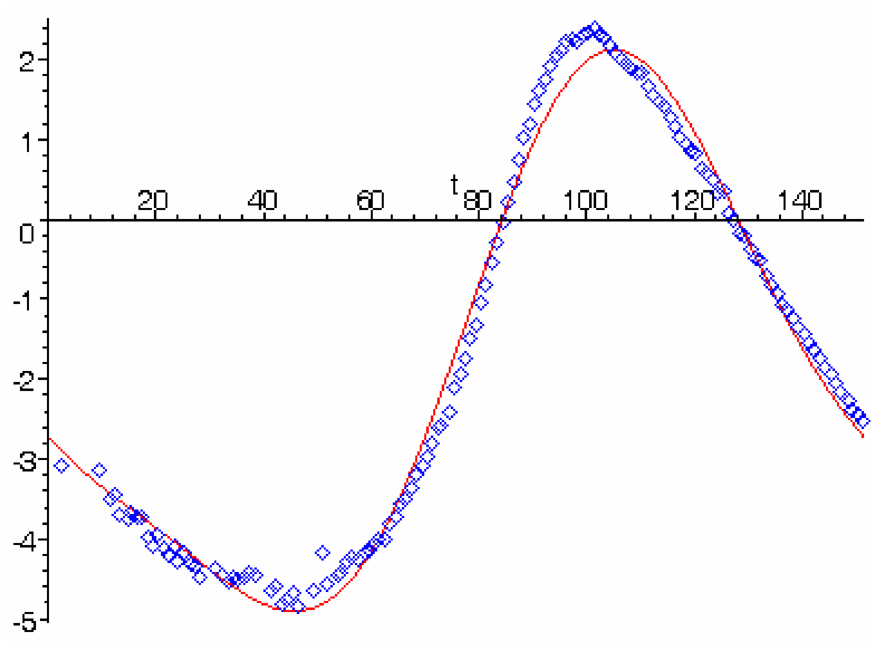
Computed True Orbit

Xi Boo (ADS 9413)

Fourier Fit to x(t)…



Fourier Fit to y(t)…



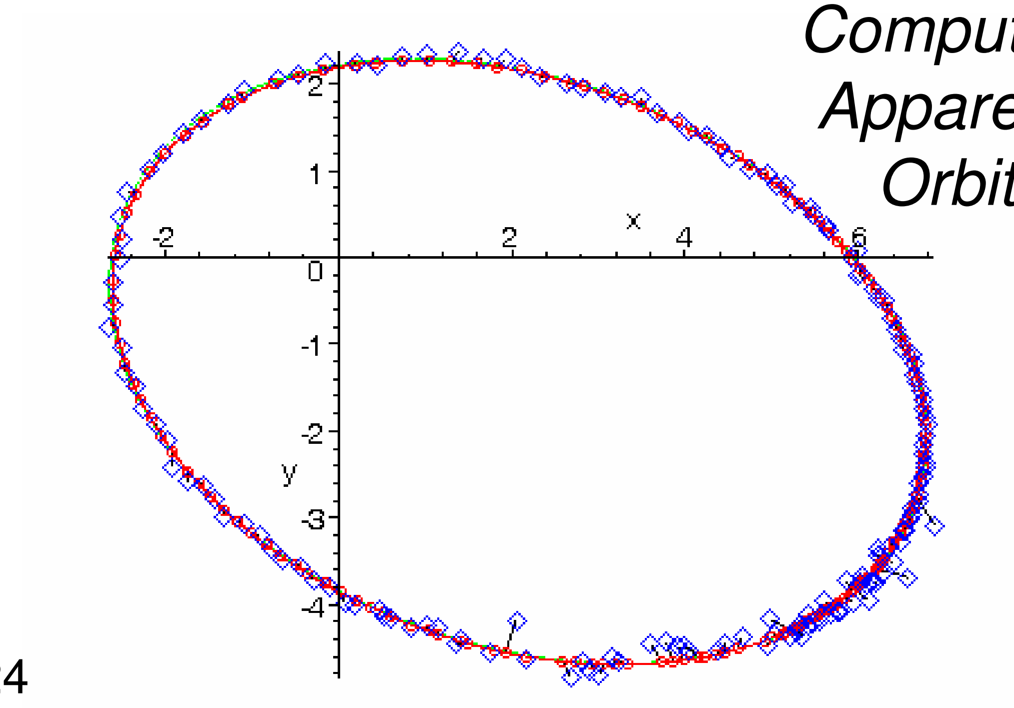
Xi Boo (ADS 9413)

$n_{max} = 4$

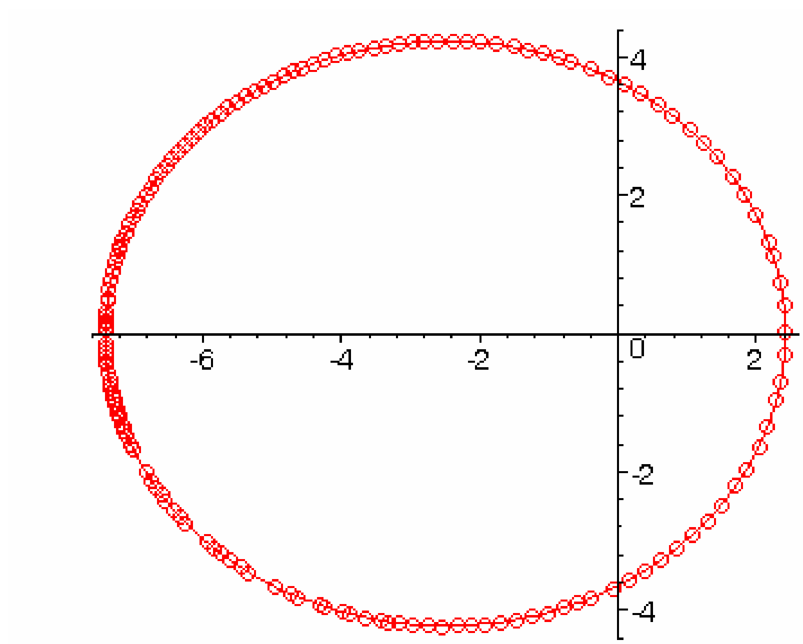
P = 151.5

ω = 0.42
Ω =0.21
i = 0.71
a = 4.92
e = 0.51
Δ = 3.70

Error = 13.24



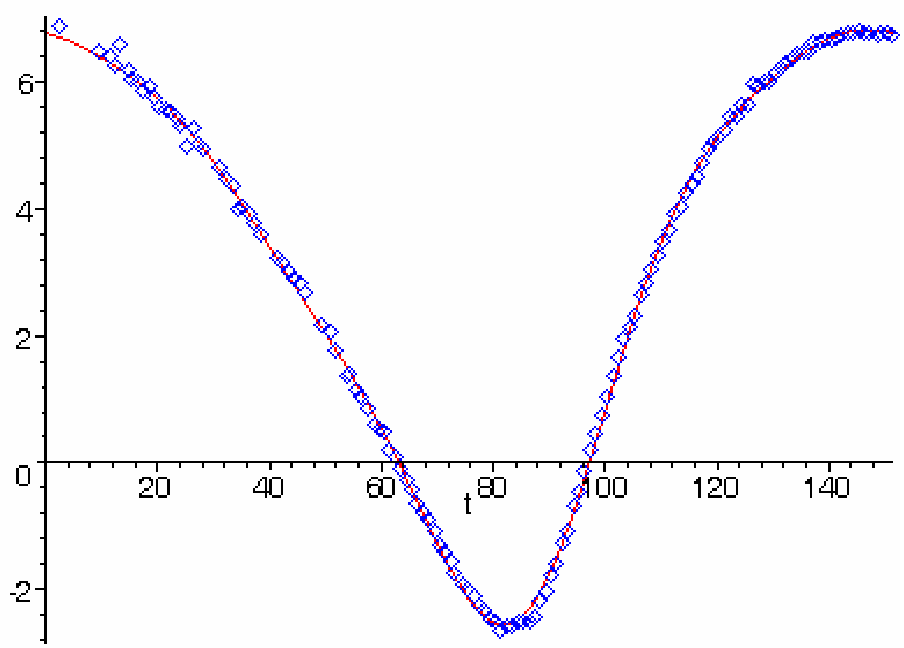
Computed Apparent Orbit



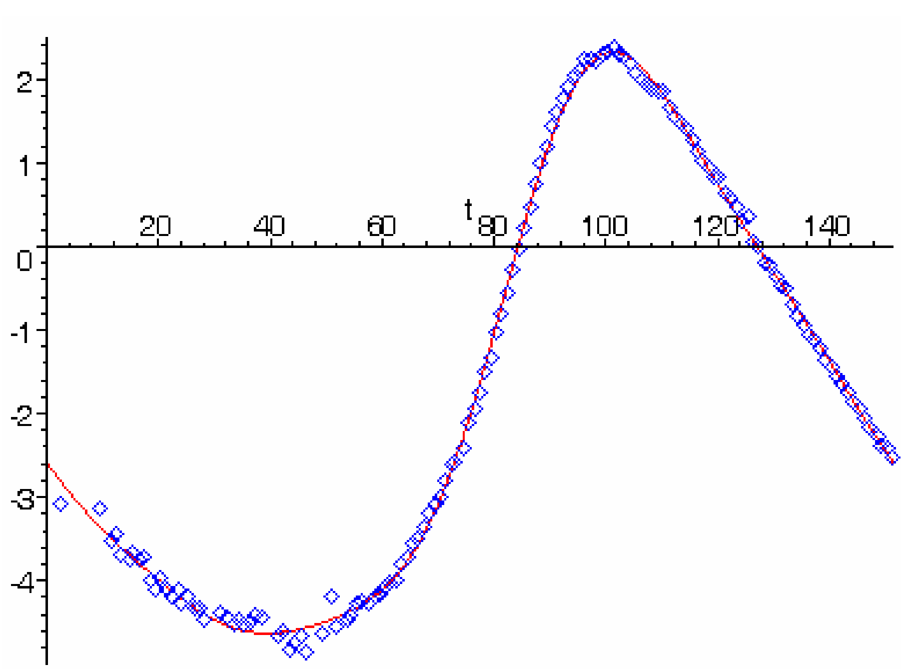
Computed True Orbit

Xi Boo (ADS 9413)

Fourier Fit to x(t)…



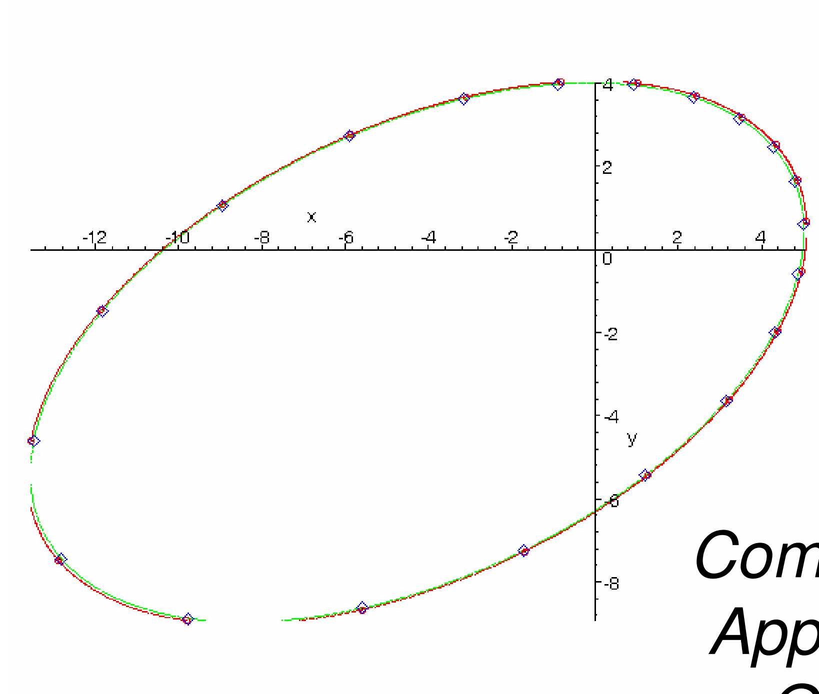
Fourier Fit to y(t)…



Synthetic Orbit

INPUT

P = 199
ω = 0.20
Ω = 0.40
i = 0.90
a = 10
e = 0.5
Δ = 0



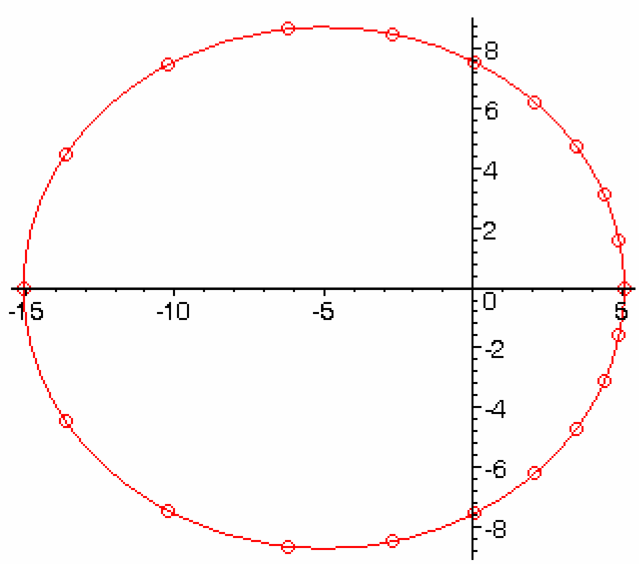
Computed Apparent Orbit

n_{max} = 2

P = 199

ω = 0.20
Ω =0.40
i = 0.90
a = 10.1
e = 0.50

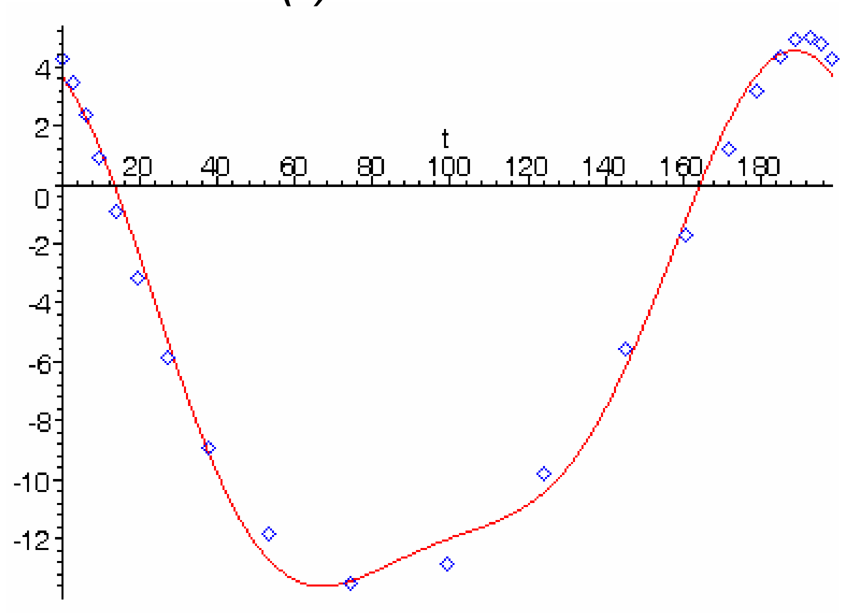
Error = 1.50



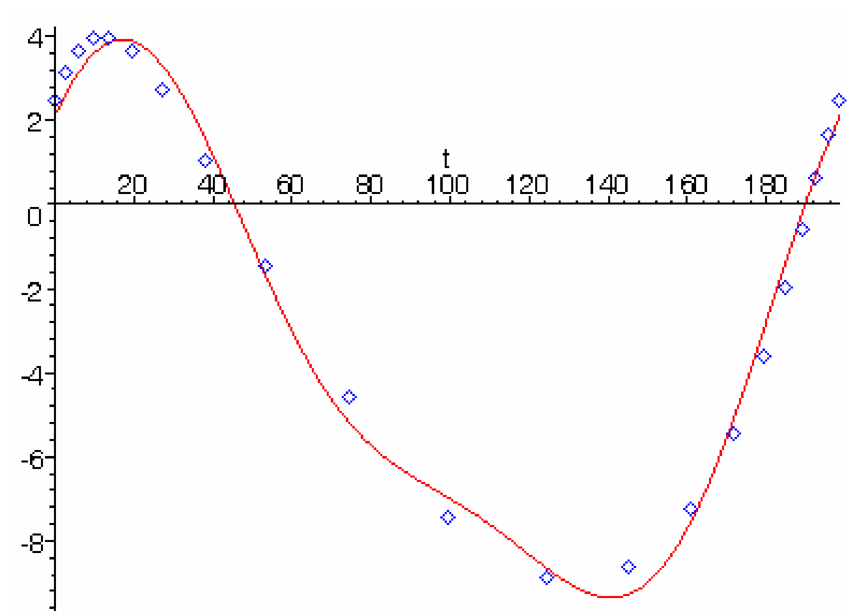
Computed True Orbit

Synthetic Orbit

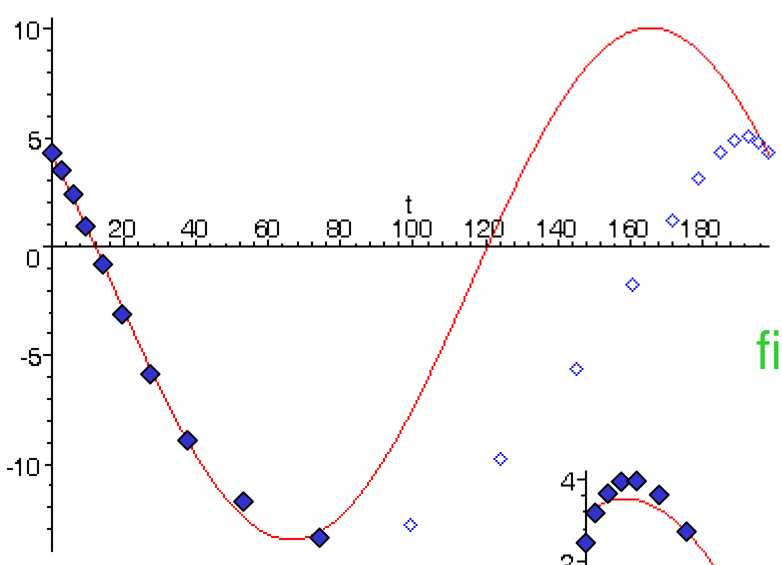
Fourier Fit to x(t)…



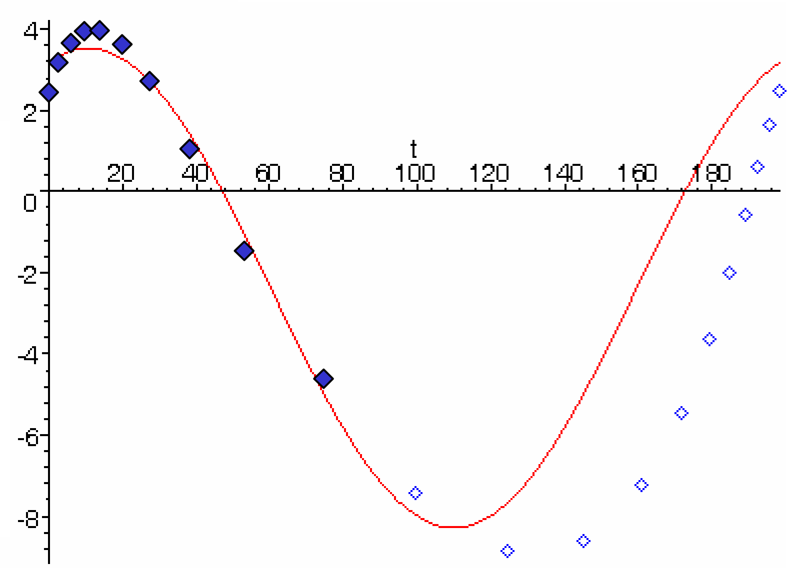
Fourier Fit to y(t)…

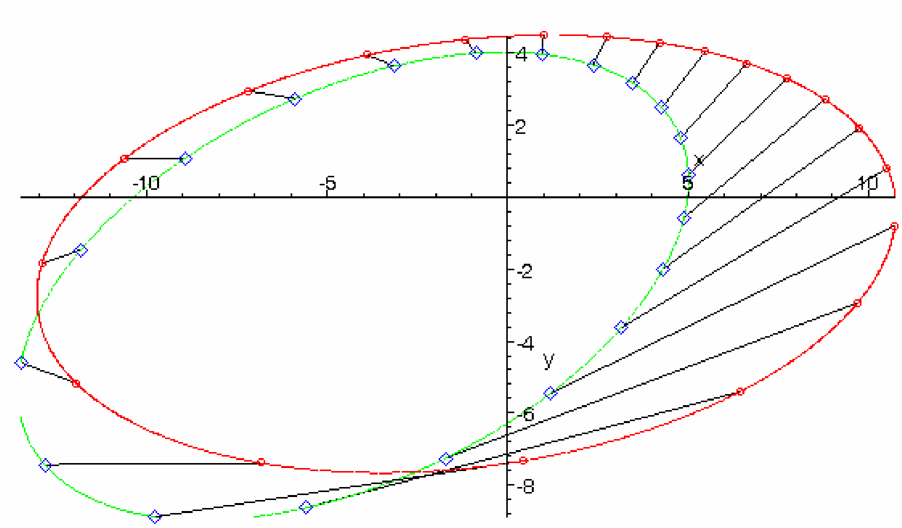


Synthetic Orbit

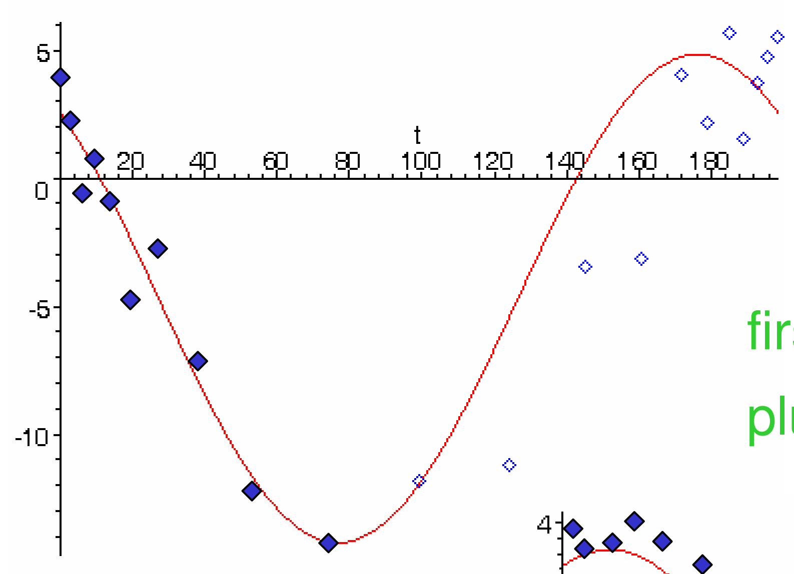


first 10 data points only



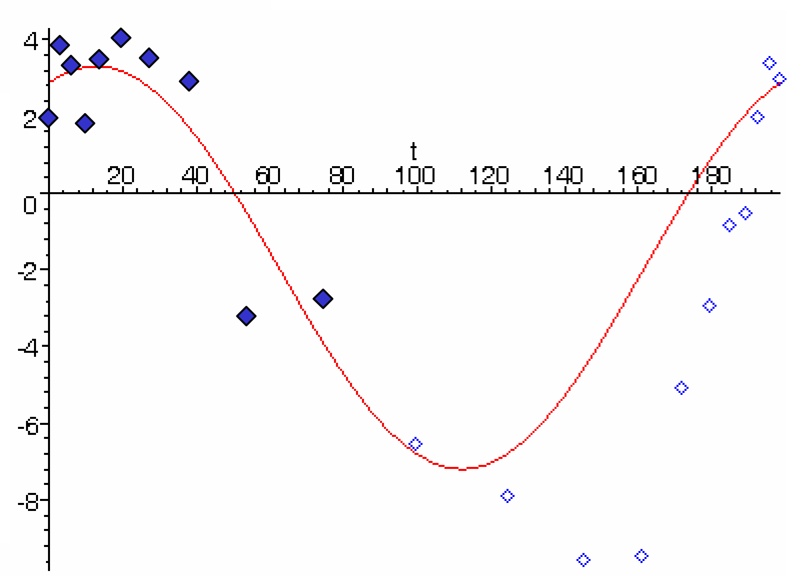


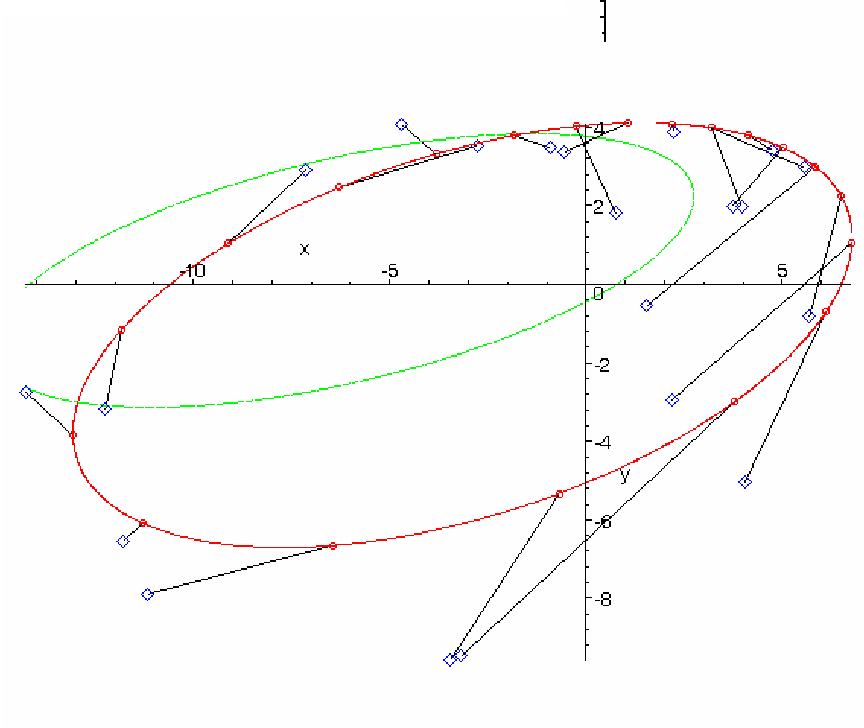
Synthetic Orbit



first 10 data points only

plus random “noise”

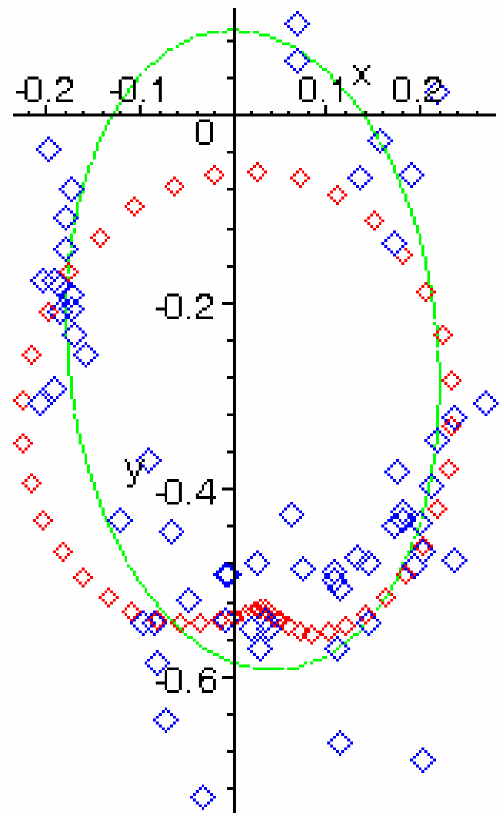




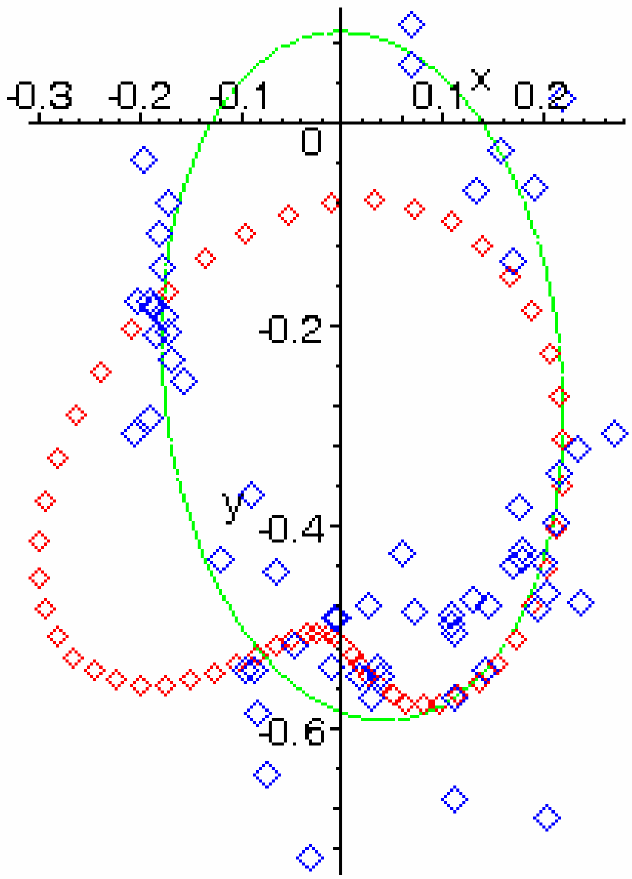
$$x(t) = \rho \cos\theta = \alpha_0 + \sum_{n=1}^{\infty} \alpha_n \cos(nM) + \sum_{n=1}^{\infty} \beta_n \sin(nM)$$

$$y(t) = \rho \sin\theta = \gamma_0 + \sum_{n=1}^{\infty} \gamma_n \cos(nM) + \sum_{n=1}^{\infty} \delta_n \sin(nM)$$

$$\begin{aligned}
\alpha_n &= \alpha_n(a_n, b_n, \Delta) \\
\beta_n &= \beta_n(a_n, b_n, \Delta) \\
\gamma_n &= \gamma_n(c_n, d_n, \Delta) \\
\delta_n &= \delta_n(c_n, d_n, \Delta)
\end{aligned}$$



2
47
3.6397
0.4814
1.7445
-0.1701
0.5817
0.3690
0.8281
4.5767



2

55

4.2683

0.5482

1.5298

0.0068

1.0063

0.5627

0.8984

3.9333

QQ1 := -15.19441610

QQ2 := -4.455155063

OmEll := 1.713401610

QQ3 := 15.83407424

QQ4 := 78.36004346

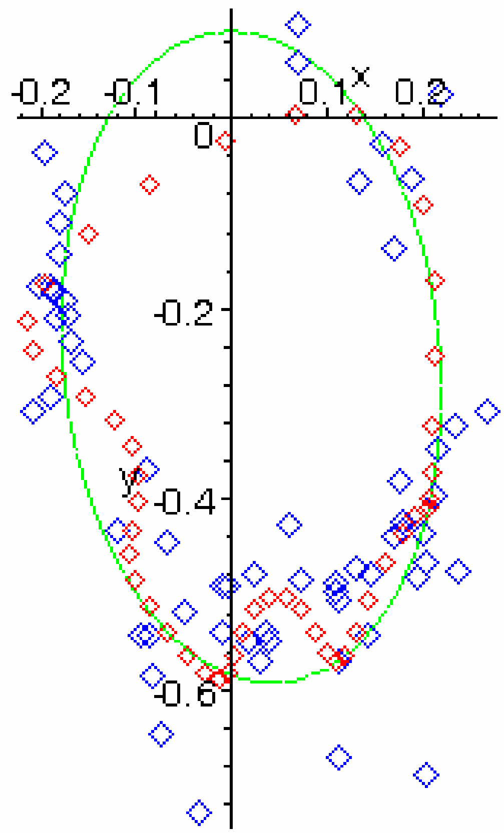
QQp := .1597598572

IEll := .5662690141

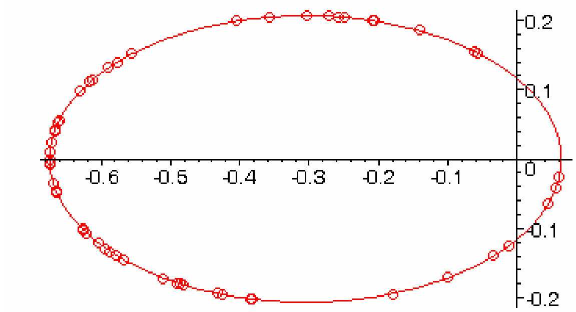
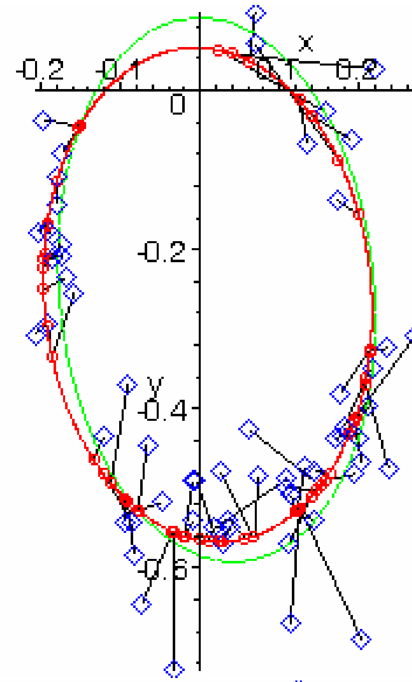
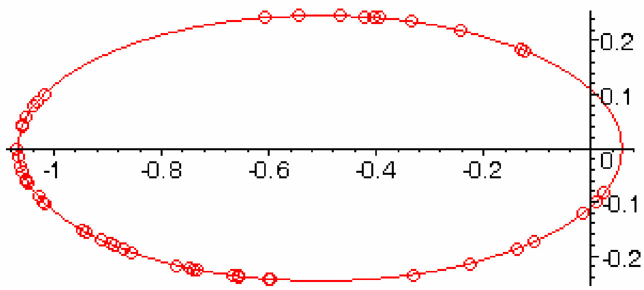
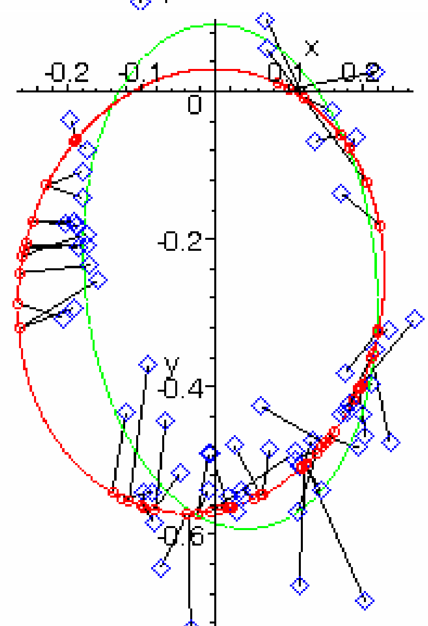
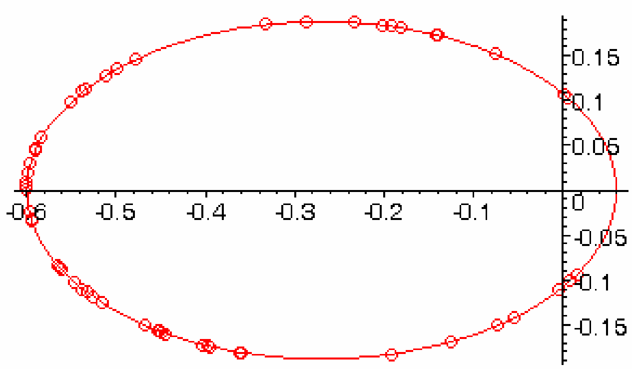
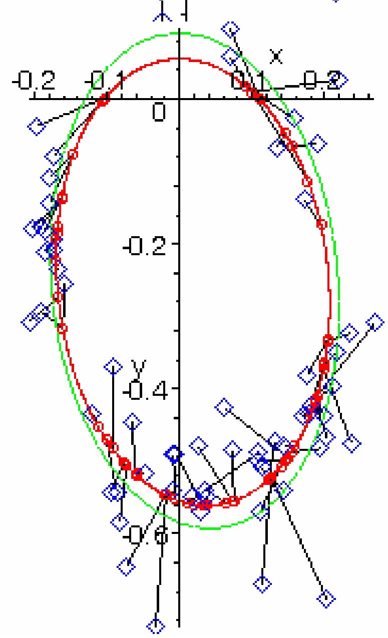
omEll := -.07159649311

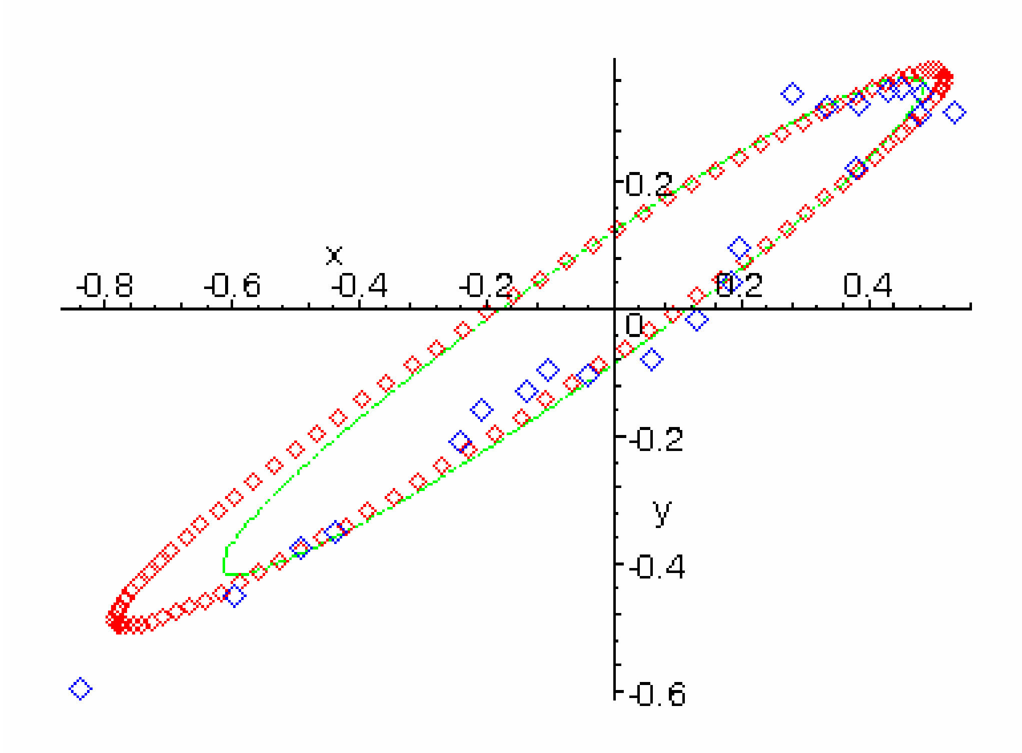
EEll := .7318519920

AEll := .3440189097



5
47
3.7892
0.3153
1.8895
-0.2688
0.3766
0.3295
0.8203
4.6665


-0.6 -0.5 -0.4 -0.3 -0.2 -0.1 0
0.2 0.1 -0.1 -0.2

x
-0.2 -0.1 0.1 0.2
0
-0.2 -0.4 -0.6
y

-1 -0.8 -0.6 -0.4 -0.2 0
0.2 0.1 -0.1 -0.2

x
-0.2 -0.1 0.1 0.2
0
-0.2 -0.4 -0.6
y

-0.6 -0.5 -0.4 -0.3 -0.2 -0.1 0
0.15 0.1 0.05 -0.05 -0.1 -0.15

x
-0.2 -0.1 0.1 0.2
0
-0.2 -0.4 -0.6
y



1
100
1.8202
0.1045
0.9763
-0.5885
1.4651
0.8003
0.2129
2.0077

QQ1 := 46.86660710

QQ2 := 131.9720710

OmEll := .6147814755

QQ3 := 140.0468007

QQ4 := 4.62049998

QQp := .6579160959

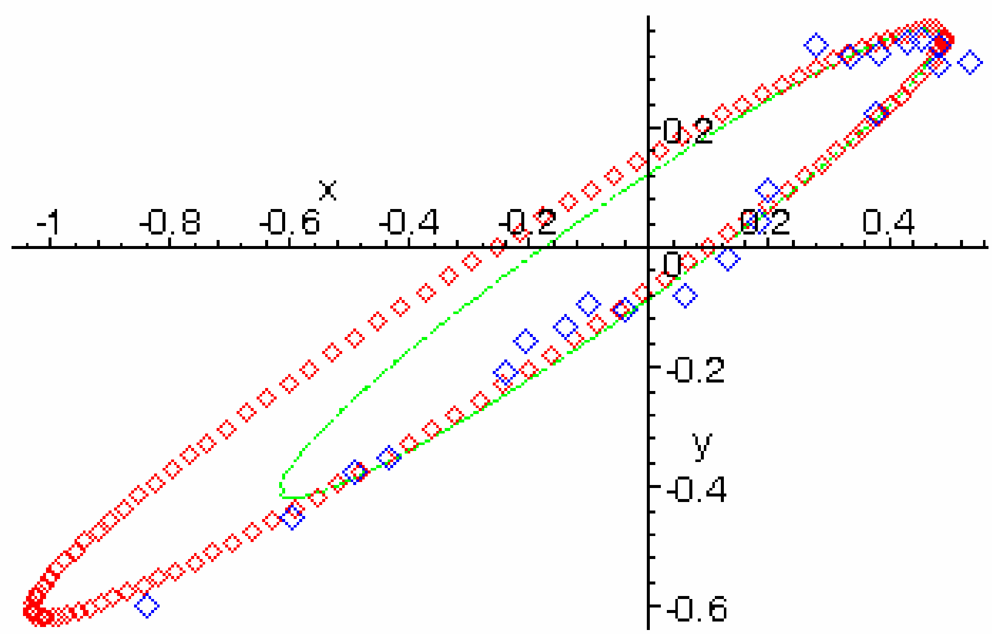
IEll := 1.443057856

omEll := -1.083418520

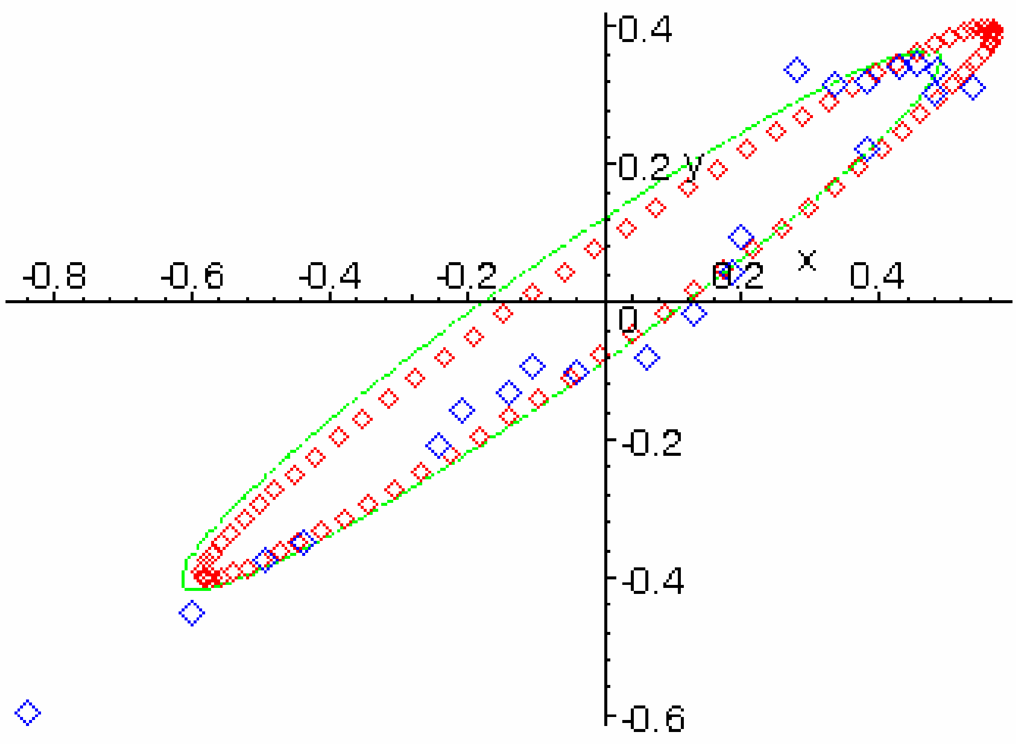
EEll := .2089946430

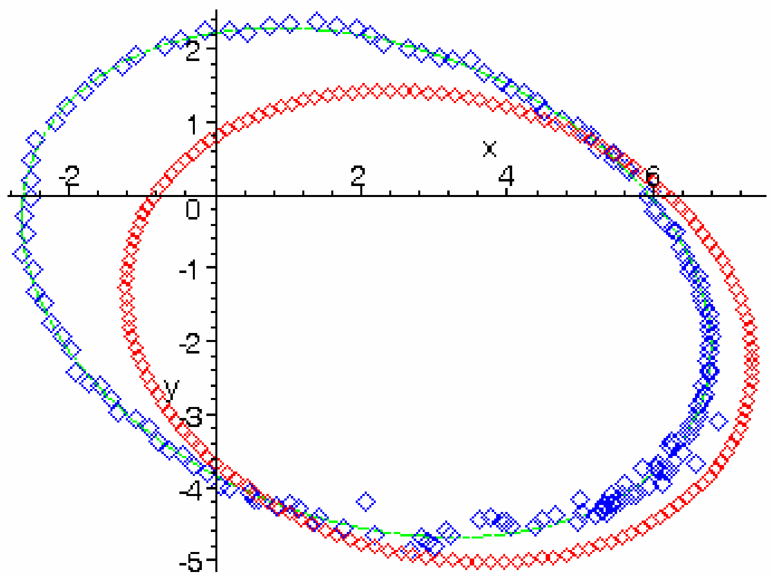
AEll := .6879655799

1
120
2.0831
0.0739
0.8645
-0.5707
1.4588
0.9510
0.3184
1.6504



1
80
2.0401
0.3438
1.4144
-0.6044
1.4756
0.6994
0.0508
2.8496

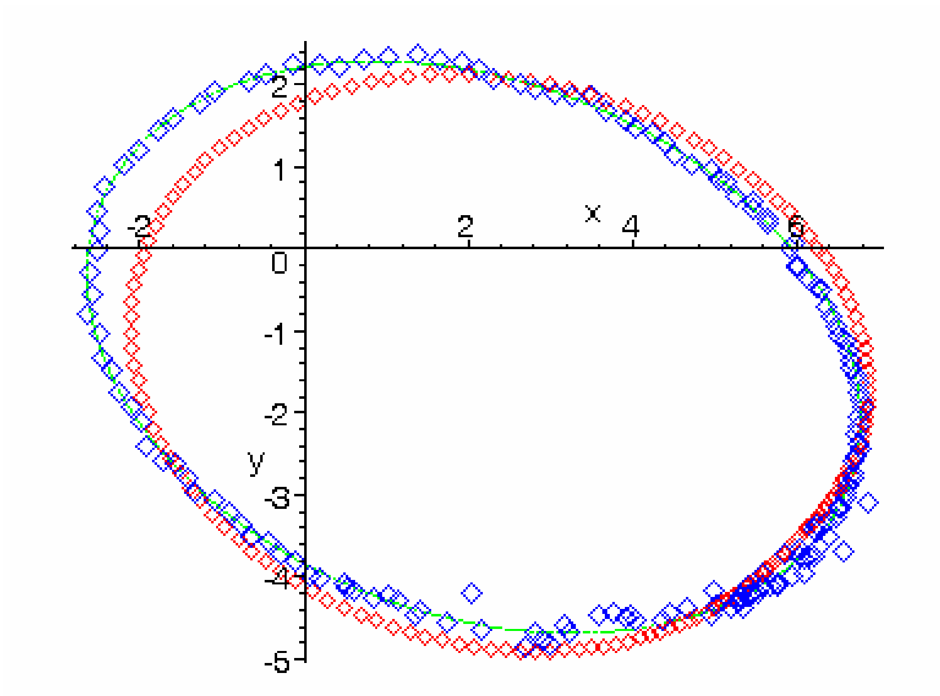


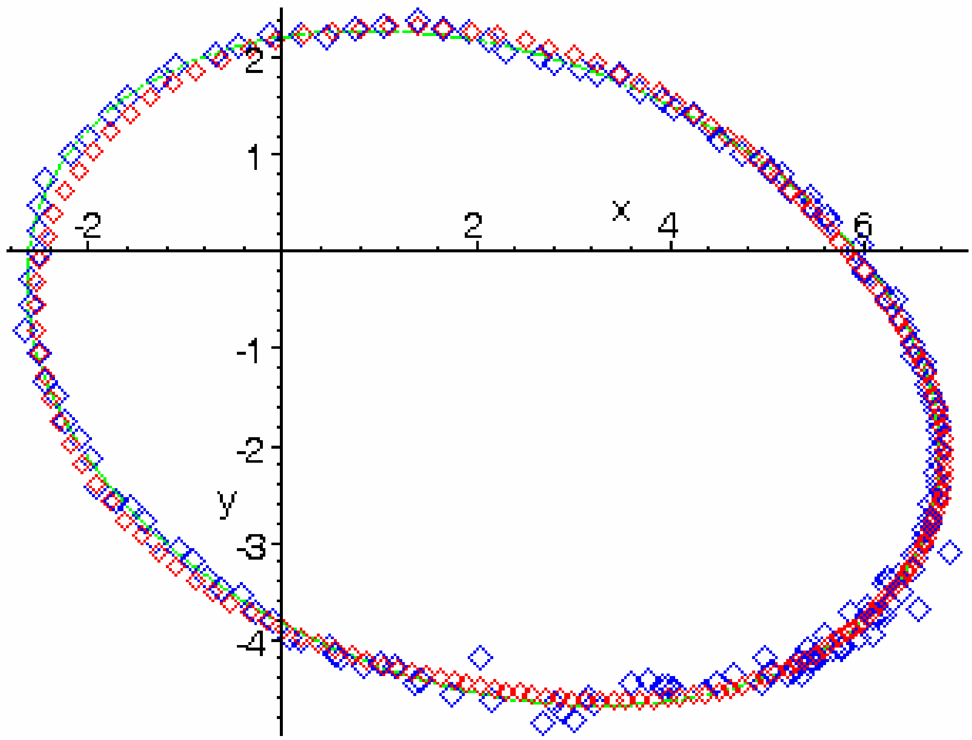


1.0000
151.5000
0.4865
0.148418046
0.707073508
4.876123901
0.5078125
3.710364508
> DD;
> FF;
21.11668587
147.460603

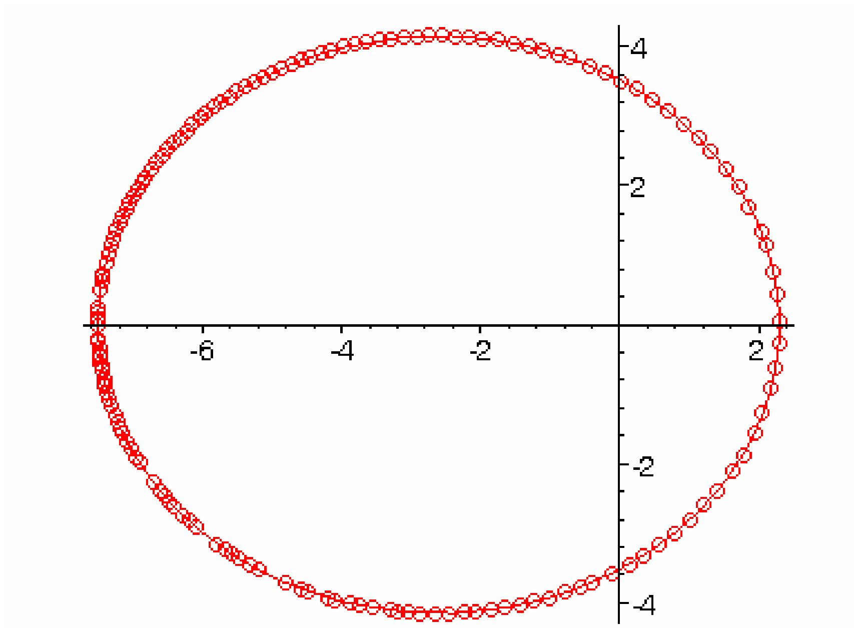
QQ1 := .05091480872
QQ2 := -.02393155327
OmEll := 2.921896742
QQ3 := .05625880179
QQ4 := .1458846664
QQp := 3.702628797
IEll := .7206531954
omEll := -.3965104122
EEll := .5005970848
AEll := 4.940774159

2
151.5
0.399922077
0.219377071
0.706706127
4.933219099
0.5078125
3.705101218
> DD;
> FF;
12.68930151
23.61221329

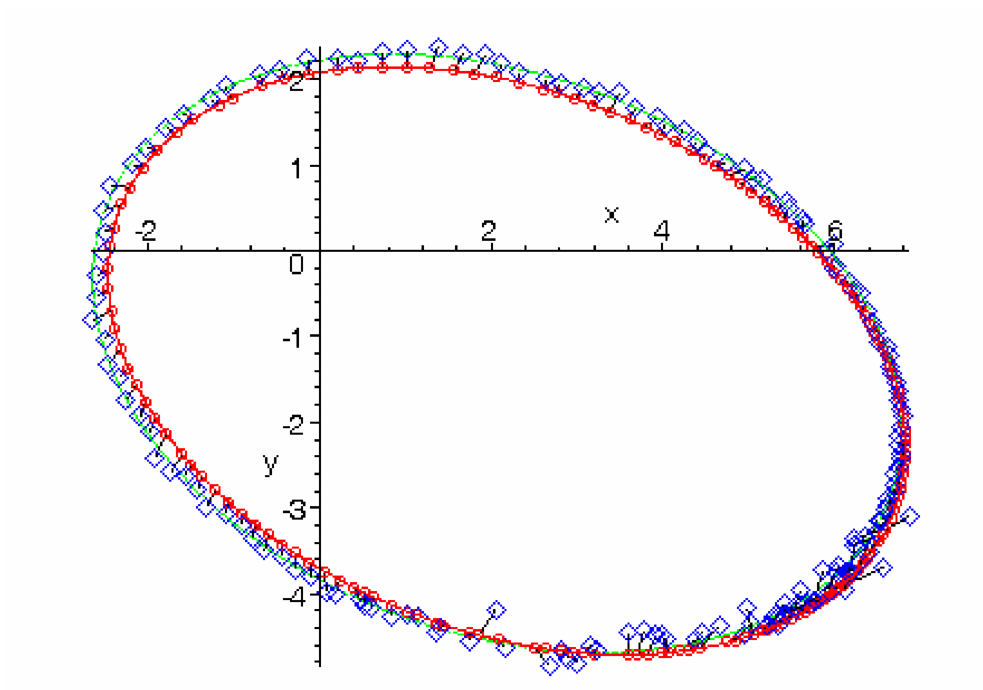


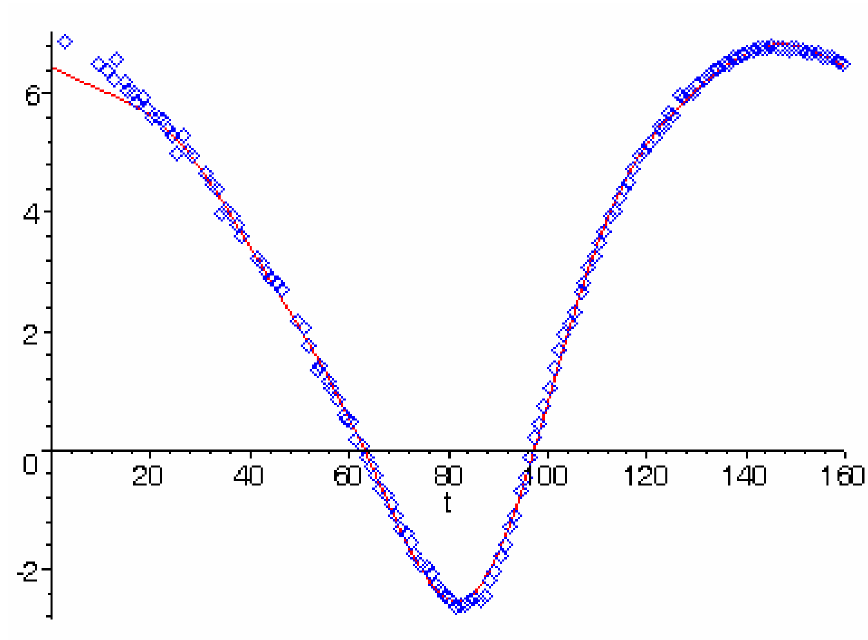


4
151.5
0.416276149
0.205250868
0.711072963
4.916265362
0.5078125
3.704125054
> DD;
> FF;
13.23962248
2.570782821

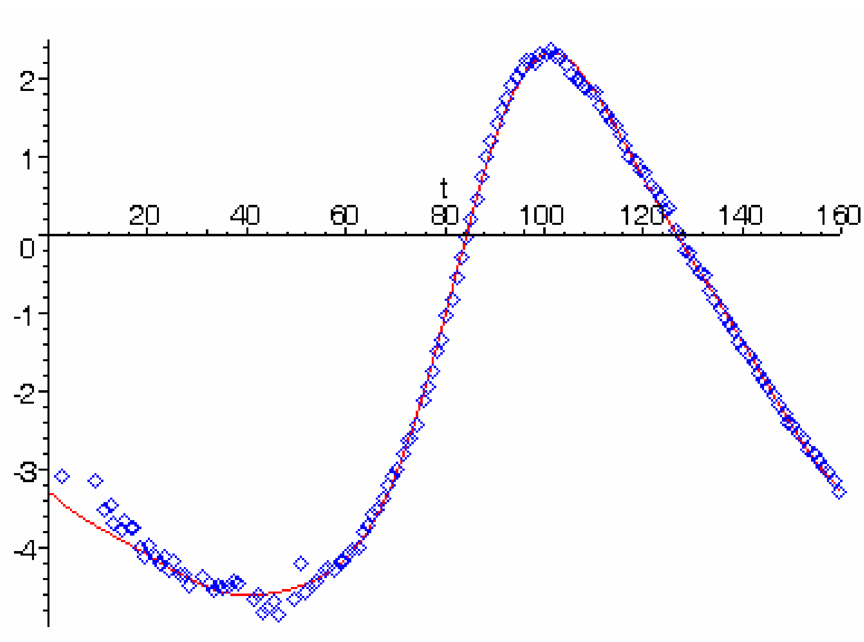


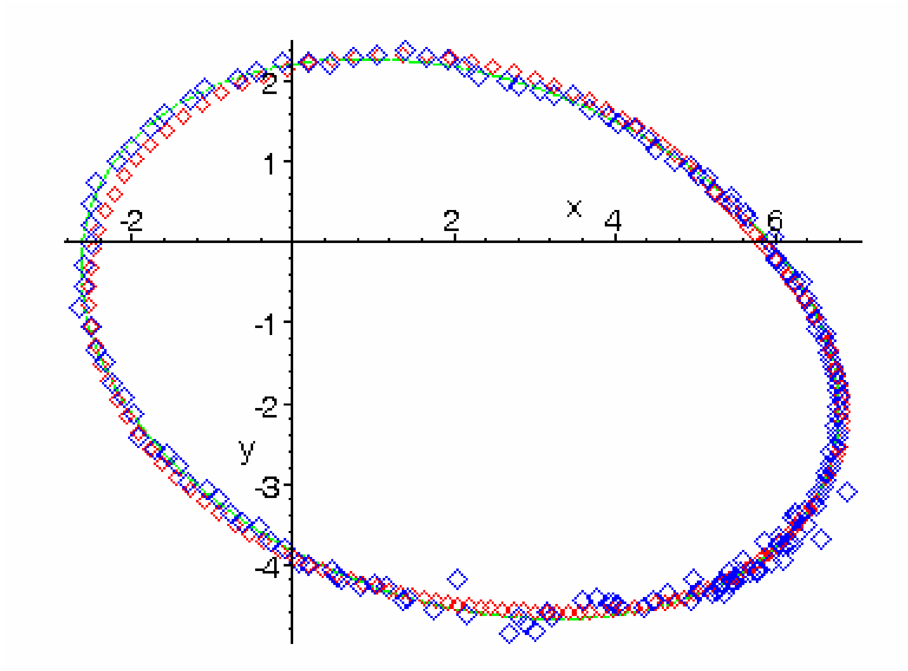
4
160
0.368305011
0.251829596
0.728364009
4.905091697
0.53125
3.502930071
> DD;
> FF;
34.42144583
6.780186053





4
160
0.368305011
0.251829596
0.728364009
4.905091697
0.53125
3.502930071
> DD;
> FF;
34.42144583
6.780186053

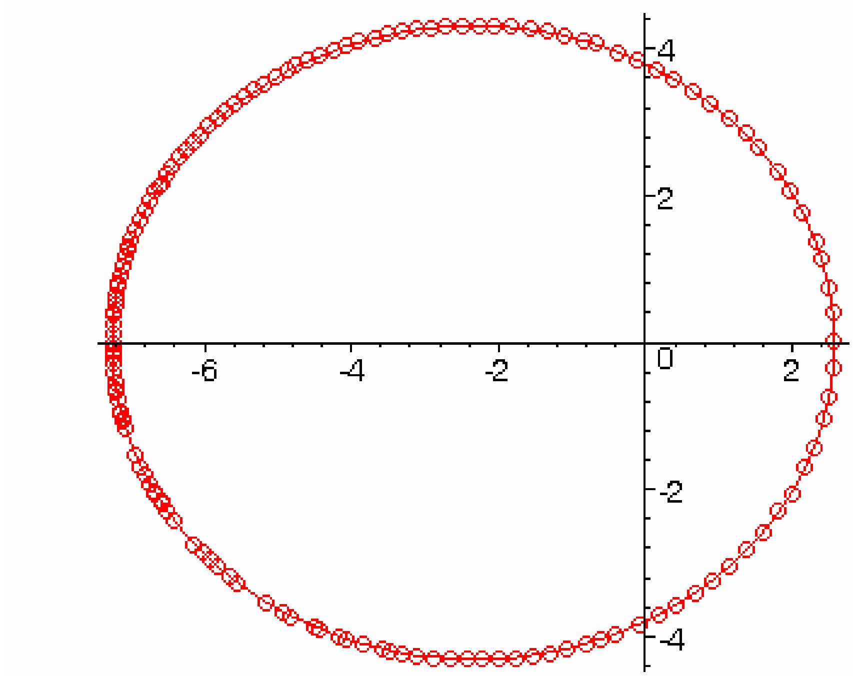




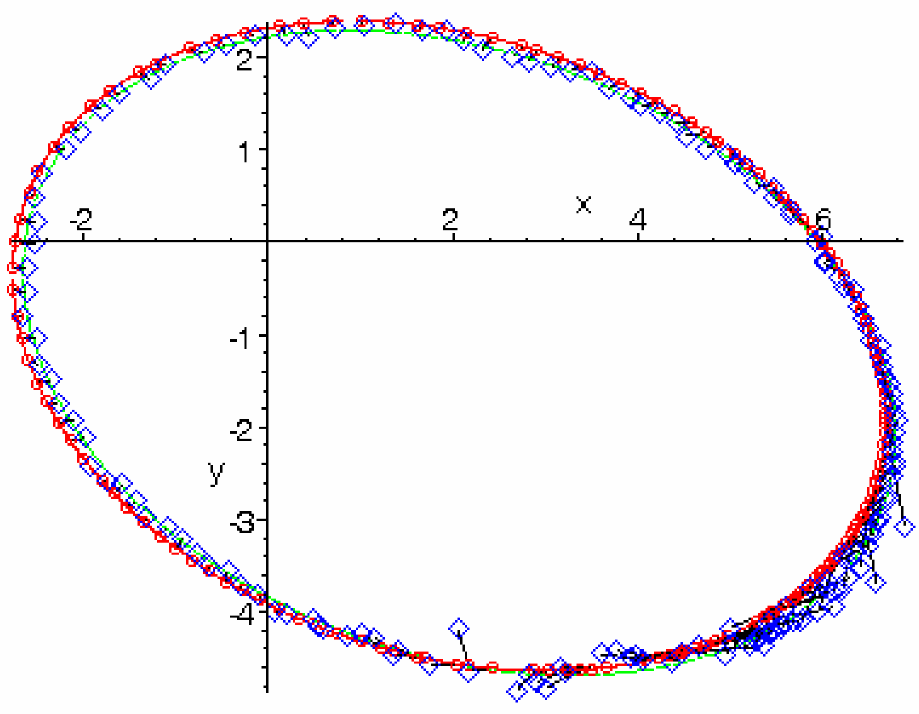
4
160
0.368305011
0.251829596
0.728364009
4.905091697
0.53125
3.502930071

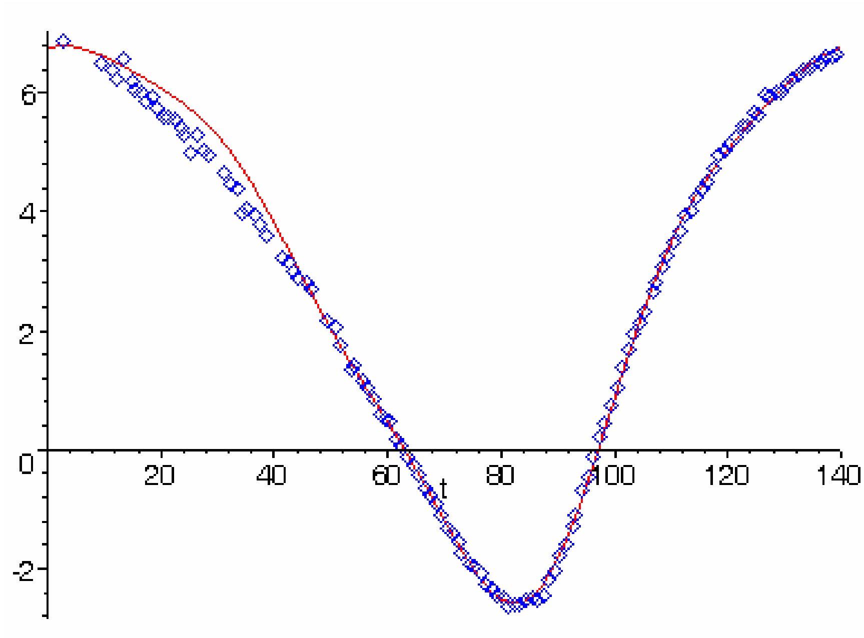
```
> DD;
> FF;
```

34.42144583
6.780186053

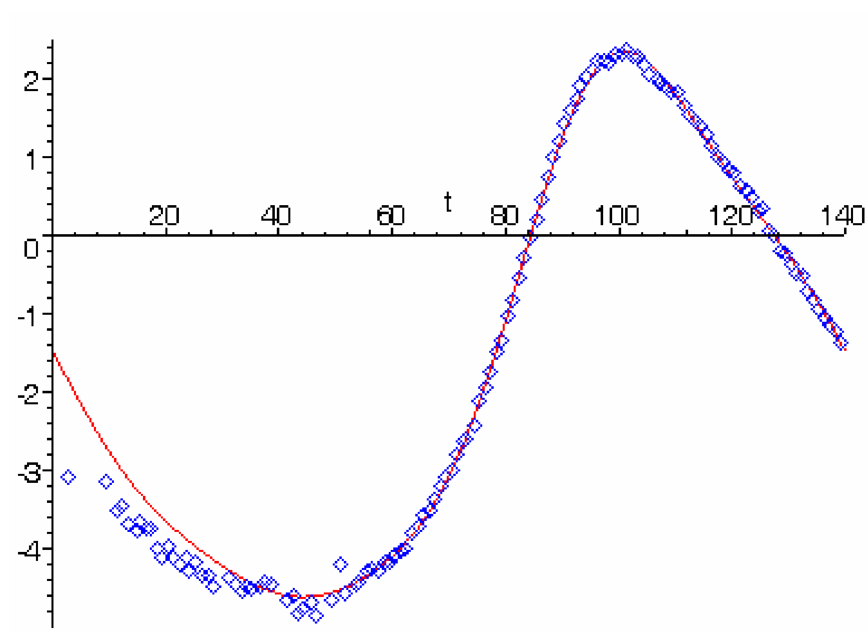


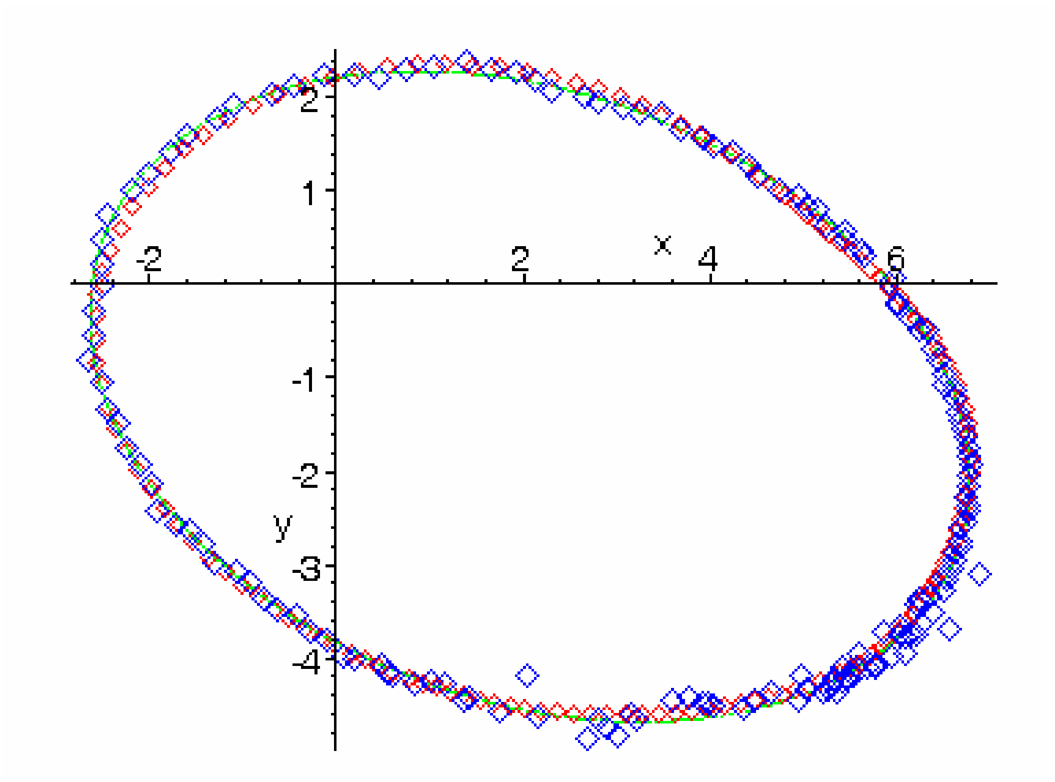
4.0000
140.0000
0.423057155
0.191108206
0.70742865
4.913450835
0.4765625
4.010570682
> DD;
> FF;
48.63840268
20.64967173



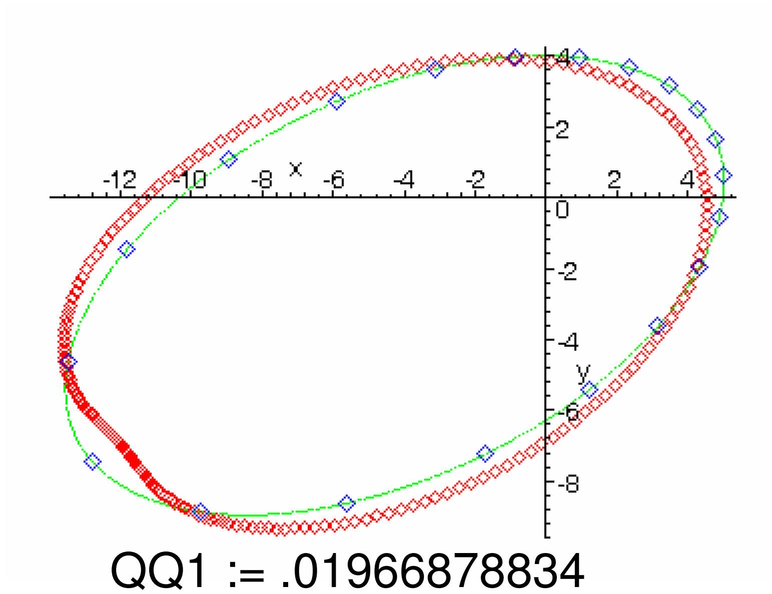


4.0000
140.0000
0.423057155
0.191108206
0.70742865
4.913450835
0.4765625
4.010570682
> DD;
> FF;
48.63840268
20.64967173





4.0000
140.0000
0.423057155
0.191108206
0.70742865
4.913450835
0.4765625
4.010570682
> DD;
> FF;
48.63840268
20.64967173



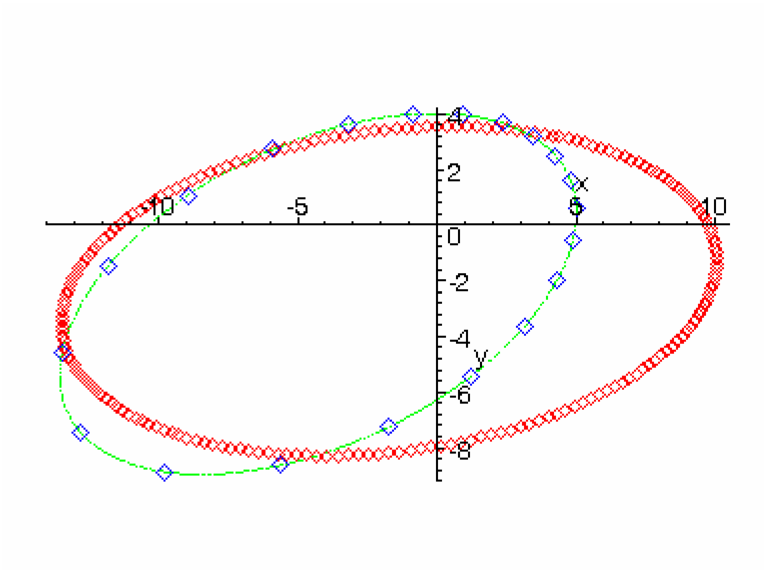
QQ1 := .01966878834

QQ2 := .02025174302
OmEll := .4000000022
QQ3 := .02823108801
QQ4 := .03555555615
QQp := 7.499999937
IEll := .8999999909
omEll := .1999999931
EEll := .4999999984
AEll := 9.999999895

2
198.6917652
0.1977
0.402024331
0.903780007
10.08080955
0.49609375
-8

.1019676100 10

> DD;
> FF;
1.503553394
11.43874037



QQ1 := .01966879329

QQ2 := .02025175027

OmEll := .4000000288

QQ3 := .02823109666

QQ4 := .03555555690

QQp := 7.499999859

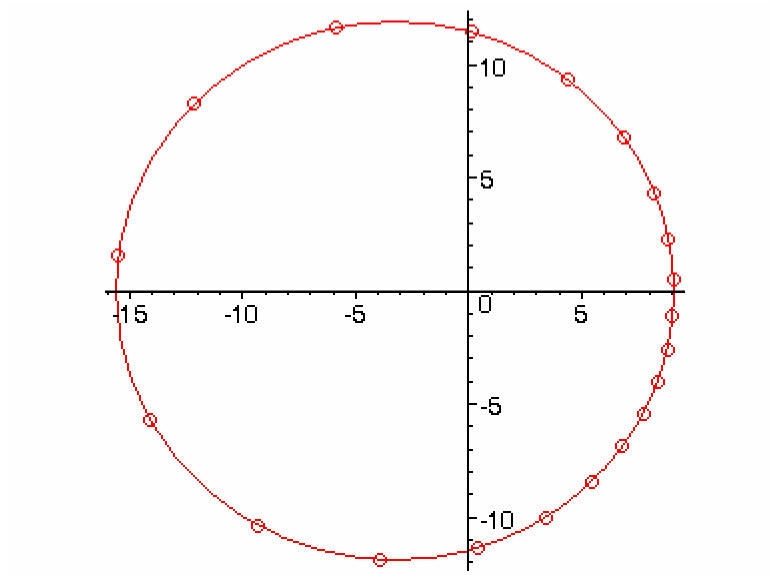
IEll := .9000000605

omEll := .1999997846

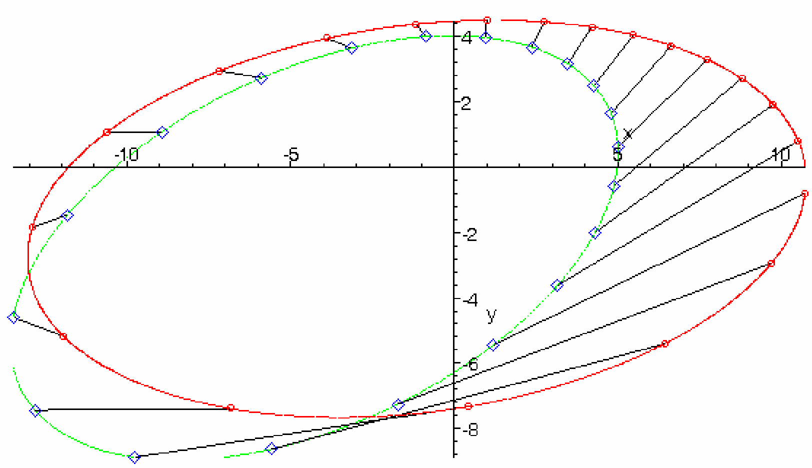
EEll := .4999999890

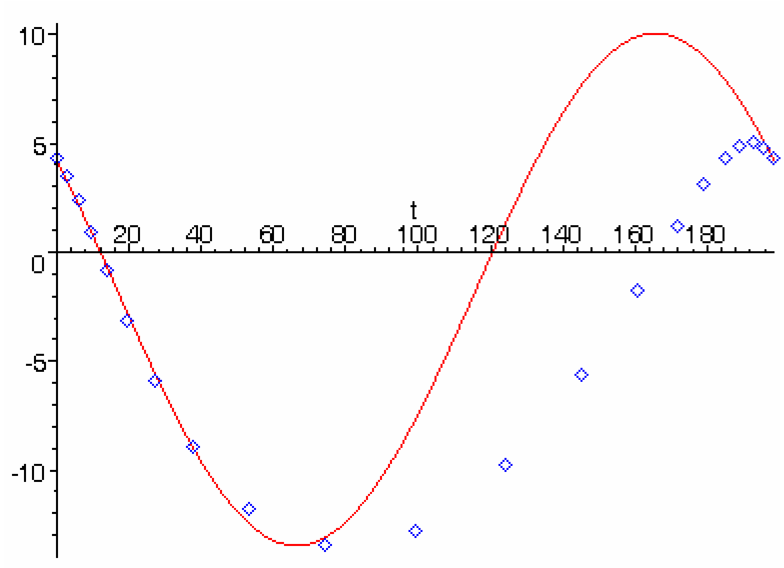
AEll := 9.999999665

1.0000
198.6918
1.1556
0.116005808
1.068248931
12.3449798
0.264648438
0.16109497
> DD;
> FF;
92.14958694
2.593276006



1.0000
198.6918
1.1556
0.116005808
1.068248931
12.3449798
0.264648438
0.16109497
> DD;
> FF;
92.14958694
2.593276006





1.0000
198.6918
1.1556
0.116005808
1.068248931
12.3449798
0.264648438
0.16109497
> DD;
> FF;
92.14958694
2.593276006