ECE 308 -8

Solution of Linear Constant-Coefficient Difference Equations

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Solution of Linear Constant-Coefficient Difference Equations

Two methods
- **Direct method**
- **Indirect Method (z-transform)**

Direct solution Method:

The total solution is the sum of two parts

Part 1 homogeneous solution

Part 2 particular solution

The Homogeneous solution

Assuming that the input . Since , this gives us the ***zero-input response*** of the system

$$\sum_{k=0}^{N} a_k y(n-k) = 0$$

Solution of Linear Constant-Coefficient Difference Equations

The solution is the form of an exponential

$$y_h(n) = \lambda^n$$

substitute this in the previous equation.

$$\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \ldots + a_{N-1}\lambda^{n-(N-1)} + a_N\lambda^{n-N} = 0$$

$$\lambda^{n-N}\left(\lambda^N + a_1\lambda^{N-1} + a_2\lambda^{N-2} + \ldots + a_{N-1}\lambda + a_N\right) = 0$$

This is called characteristic polynomial of the system. It has N roots and denotes by $\lambda_1, \lambda_2, \ldots, \lambda_N$

The roots can be real or complex or some roots are identical.

Solution of Linear Constant-Coefficient Difference Equations

Let assume that roots are real and not identical, the solution becomes

$$y_h(n) = C_1\lambda_1^n + C_2\lambda_2^n + \ldots + C_N\lambda_N^n$$

The coefficients $C_i, i = 1, 2, \ldots, N$ are determined from the initial conditions.

If there are two identical roots, the solution becomes

$$y_h(n) = C_1\lambda_1^n + C_1 n\lambda_1^n + C_2\lambda_2^n + \ldots + C_N\lambda_N^n$$

Solution of Linear Constant-Coefficient Difference Equations

Example:

Find the zero-input response for the second-order difference equation

$$y(n) - 3y(n-1) - 4y(n-2) = 0$$

The homogeneous solution form $y_h(n) = \lambda^n$

$$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 3\lambda - 4) = 0$$

$$\lambda_1 = -1, \ \lambda_2 = 4$$

The homogenous solution is

$$y_h(n) = C_1\lambda_1^n + C_2\lambda_2^n = C_1(-1)^n + C_2 4^n$$

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The Particular solution:

Causal system is the output is depends only on present and past input signal

Input Signal $x(n)$	Particular Solution $y_p(n)$
A (constant)	K
AM^n	KM^n
An^M	$K_0 n^M + K_1 n^{M-1} + \ldots + K_M$
$A^n n^M$	$A^n\left(K_0 n^M + K_1 n^{M-1} + \ldots + K_M\right)$
$\begin{Bmatrix} A\cos\omega_0 n \\ A\sin\omega_0 n \end{Bmatrix}$	$K_1\cos\omega_0 n + K_2\sin\omega_0 n$

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Example:

Find the particular solution for for

$$y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$$ $$x(n) = 2^n,\ n \ge 0$$

The particular solution form

$$y_p(n) = K2^n u(n)$$ $$y_p(n) = K2^n,\ n \ge 0$$

$$K2^n u(n) = \frac{5}{6}K2^{n-1}u(n-1) - \frac{1}{6}K2^{n-2}u(n-2) + 2^n u(n)$$

Evaluate the equation for $n \ge 2$

$$4K = \frac{5}{6}2K - \frac{1}{6}K + 4 \Rightarrow K = \frac{8}{5}$$

Therefore, the particular solution is

$$y_p(n) = \frac{8}{5}2^n,\ n \ge 0$$

Solution of Linear Constant-Coefficient Difference Equations

The total solution of the difference equation

$$y(n) = y_h(n) + y_p(n)$$