ECE 308 -5

ECE 308
Discrete-Time Signals and Systems

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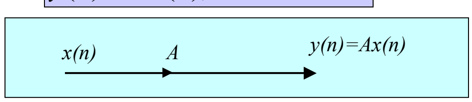
Addition, Multiplication, and Scaling of Sequences

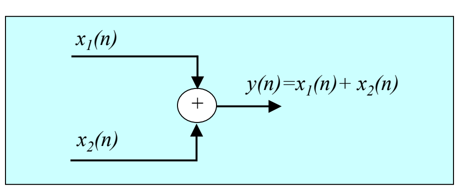
Amplitude Scaling: (A Constant Multiplier)

$$y(n) = Ax(n), \quad -\infty < n < \infty$$



Addition of two signals (An Adder)

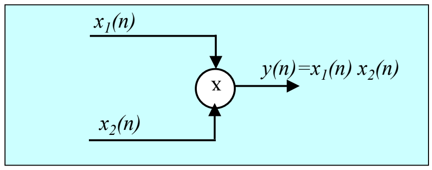
$$y(n) = x_1(n) + x_2(n), \quad -\infty < n < \infty$$



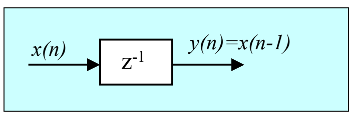
Addition, Multiplication, and Scaling of Sequences

The product of two signals (A signal Multiplier)

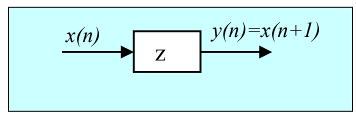
$$y(n) = x_1(n)x_2(n), \quad -\infty < n < \infty$$



A unit delay element

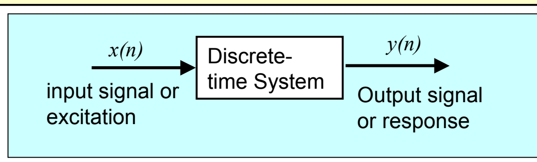


A unit advance element



Input-Output Description of Systems

The relation between the input and output signals are known input-output relationship



Mathematical representation of the transformation is

$$y(n) = \tau\left[x(n)\right]$$

τ denotes the transformation. In general input-output relationship can be also shown as

$$x(n) \xrightarrow{\tau} y(n)$$

Input-Output Description of Systems

Example: The input signal is

$$x(n)=\begin{cases}|n| & -3\le n\le 3\\ 0 & \text{otherwise}\end{cases}$$

a $y(n)=x(n)$

b $y(n)=x(n-1)$

c $y(n)=x(n+1)$

d $y(n)=\frac{1}{3}[x(n+1)+x(n)+x(n-1)]$

e $y(n)=\max[x(n+1),x(n),x(n-1)]$

f $y(n)=\sum_{k=-\infty}^{n}x(k)=x(n)+x(n-1)+x(n-2)+\ldots$

Solution

$$x(n)=\{\ldots,0,3,2,1,\underset{\uparrow}{0},1,2,3,0,\ldots\}$$

a $y(n)=x(n)$ $\quad y(n)=\{\ldots,0,3,2,1,\underset{\uparrow}{0},1,2,3,0,\ldots\}$

b $y(n)=x(n-1)$ $\quad y(n)=\{\ldots,0,3,2,\underset{\uparrow}{1},0,1,2,3,0,\ldots\}$

Input-Output Description of Systems

Solution (cont)

c $y(n)=x(n+1)$ $\quad y(n)=\{\ldots,0,3,2,1,0,\underset{\uparrow}{1},2,3,0,\ldots\}$

d $y(n)=\frac{1}{3}[x(n+1)+x(n)+x(n-1)]$ $\quad y(0)=\frac{1}{3}[x(1)+x(0)+x(-1)]=\frac{1}{3}[1+0+1]=\frac{2}{3}$

$$y(-1)=\frac{1}{3}[x(0)+x(-1)+x(-2)]=\frac{1}{3}[0+1+2]=1$$

....

$$y(n)=\left\{\ldots,0,1,\frac{5}{3},2,1,\underset{\uparrow}{\frac{2}{3}},1,2,\frac{5}{3},1,0,\ldots\right\}$$

f

$$y(n)=\max[x(n+1),x(n),x(n-1)]$$

$$y(0)=\max[x(1),x(0),x(-1)]=\max[1,0,1]=1$$

$$y(-1)=\max[x(0),x(-1),x(-2)]=\max[0,1,2]=2$$

$$y(1)=\max[x(2),x(1),x(0)]=\max[2,1,0]=2$$

$$y(n)=\{0,3,3,3,2,\underset{\uparrow}{1},2,3,3,3,0,\ldots\}$$

Input-Output Description of Systems

Solution (cont)

e $$y(n) = \sum_{k=-\infty}^{n} x(k) = x(n) + x(n-1) + x(n-2) + \ldots$$

This system is called an accumulator

$$y(0) = \sum_{k=-\infty}^{0} x(k) = x(0) + x(-1) + x(-2) + x(-3)$$
$$= 0 + 1 + 2 + 3 = 6$$

$$y(1) = \sum_{k=-\infty}^{1} x(k) = x(1) + x(0) + x(-1) + x(-2) + x(-3)$$
$$= 1 + 0 + 1 + 2 + 3 = 7$$

$$y(n) = \{\ldots, 0, 3, 5, 6, \underset{\uparrow}{6}, 7, 9, 12, 12, \ldots\}$$

A simple algebraic manipulation the input-output relation of the accumulator can be written as

$$y(n) = \sum_{k=-\infty}^{n} x(k) = \sum_{k=-\infty}^{n-1} x(k) + x(n)$$

$$y(n) = y(n-1) + x(n)$$

Input-Output Description of Systems

Example: Input function is $x(n) = nu(n)$

Find the output $y(n) = \sum_{k=-\infty}^{n} x(k)$ under the following conditions:

a $y(-1) = 0$ b $y(-1) = 1$

Solution: a

$$y(n) = \sum_{k=-\infty}^{n} x(k) = \sum_{k=-\infty}^{-1} x(k) + \sum_{k=0}^{n} x(k)$$

$$y(n) = y(-1) + \sum_{k=0}^{n} x(k)$$

$$y(n) = 0 + \sum_{k=0}^{n} k = \frac{n(n+1)}{2}$$ for $n \geq 0$

b

$$y(n) = y(-1) + \sum_{k=0}^{n} x(k)$$

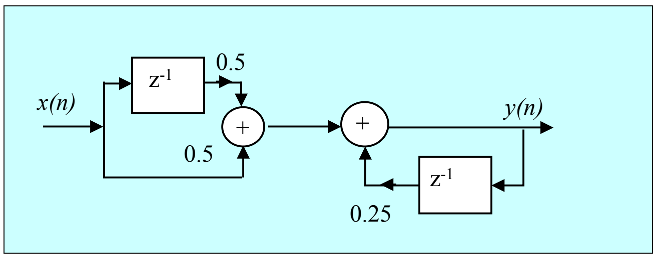
$$y(n) = 1 + \sum_{k=0}^{n} k = 1 + \frac{n(n+1)}{2} = \frac{n^2 + n + 2}{2}$$ for $n \geq 0$

Input-Output Description of Systems

Example:

Sketch the block diagram representation of the discrete-time system described by the input-output relation $-N < n < N$

$$y(n) = \frac{1}{4}y(n-1) + \frac{1}{2}x(n) + \frac{1}{2}x(n-1)$$



Classification of Discrete-Time Systems

Static System

If the output depends at most on the input sample at the same time, but not on past or future samples of the input, is called static or memoryless system

$$y(n) = ax(n) \qquad y(n) = nx(n) + bx^3(n)$$

They don't need to store any of the past input or output in order to compute the present output. It can be described in the following form

$$y(n) = \tau\left[x(n), n\right]$$

Classification of Discrete-Time Systems

Dynamic System

If the output depends on the any past or future samples of the input, the system is called dynamic or system with memory.

$$y(n) = ax(n) + 3x(n-1)$$

$$y(n) = \sum_{k=0}^{n} x(n-k)$$

$$y(n) = \sum_{k=0}^{\infty} x(n-k)$$

Classification of Discrete-Time Systems

Time-invariant Systems:

A system is called time-invariant if its input-output characteristics do not change with time.

$$x(n) \xrightarrow{\tau} y(n)$$

$$x(n-k) \xrightarrow{\tau} y(n-k)$$

If It is also called shift invariant

We can write the output

$$y(n,k) = \tau\left[x(n-k)\right]$$

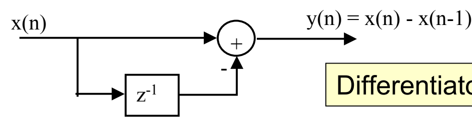
So, if the output $y(n,k) = y(n-k)$, for all possible values of k, the system is time invariant

Classification of Discrete-Time Systems

Time-variant systems:

If the output $y(n,k) \neq y(n-k)$, even for one value of k, the system is called time variant.

Examples of time-invariant and time-variant systems



Differentiator

Differentiator systems are a time-invariant, because;

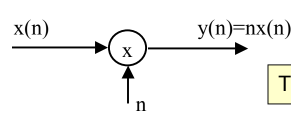
The system equation: $y(n) = \tau\left[x(n)\right] = x(n) - x(n-1)$

If the input delay by k unit $y(n,k) = x(n-k) - x(n-k-1)$

The other side if we delay y(n) by k unit $y(n-k) = x(n-k) - x(n-k-1)$

Classification of Discrete-Time Systems

Examples of time-invariant and time-variant systems



Time multiplier

Time multiplier systems are time variant, because;

The system equation is $y(n) = \tau\left[x(n)\right] = nx(n)$

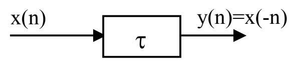
If the input delay k unit $y(n,k) = \tau\left[x(n-k)\right] = nx(n-k)$

If we delay y(n) by k unit in time
$$\begin{aligned} y(n-k) &= (n-k)x(n-k) \\ &= nx(n-k) - kx(n-k) \end{aligned}$$

The system is time variant, since $y(n,k) \neq y(n-k)$

Classification of Discrete-Time Systems

Examples of time-invariant and time-variant systems



Folder

Folder systems are time variant, because

The system equation is $y(n) = \tau[x(n)] = x(-n)$

If the input delay k unit $y(n,k) = \tau[x(n-k)] = x(-n-k)$

If we delay y(n) by k unit in time $y(n-k) = x(-n+k)$

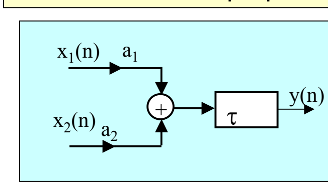
The system is time variant, since $y(n,k) \neq y(n-k)$

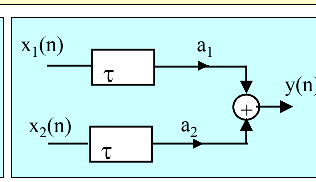
Classification of Discrete-Time Systems

Linear System: A system is linear if and only if

$$\tau[a_1 x_1(n) + a_2 x_2(n)] = a_1 \tau[x_1(n)] + a_2 \tau[x_2(n)]$$
$$= a_1 y_1(n) + a_2 y_2(n)$$

for any arbitrary $x_1(n)$ and $x_2(n)$, and any arbitrary constant a_1 and a_2. It satisfies the superposition principle.





This relation demonstrates the additivity property of a linear system.

In general, we can write that $x(n) = \sum_{k=1}^{M-1} a_k x_k(n) \xrightarrow{\tau} y(n) = \sum_{k-1}^{M-1} a_k y_k(n)$

where $y_k(n) = \tau[x_k(n)], \; k = 1, 2, \ldots, M-1$

Classification of Discrete-Time Systems

Causal Systems:

A system is called *causal* if the output of the system at any time n depends only on present and past inputs, but not depends on future inputs.

$$y(n) = F[x(n), x(n-1), x(n-2), \ldots]$$

If the system does not satisfy this definition, it is called *noncausal*.

Stable Systems:

An arbitrary relaxed system is said to be bounded input-bounded output (BIBO) stable if and only if every bounded input produces a bounded output.

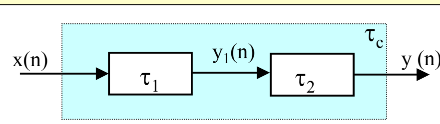
$$|x(n)| \le M_x < \infty \qquad |y(n)| \le M_y < \infty$$

for all n. M_x and M_x are some finite numbers

Classification of Discrete-Time Systems

Interconnection of Discrete-Time Systems

Discrete-time systems can be interconnected to form larger systems. They can be interconnected serial or parallel.



$$y_1(n) = \tau_1[x(n)]$$

$$y(n) = \tau_2[y_1(n)] = \tau_2[\tau_2[x(n)]]$$

If we combine τ_1 and τ_2 to , then $y(n) = \tau_c[x(n)]$

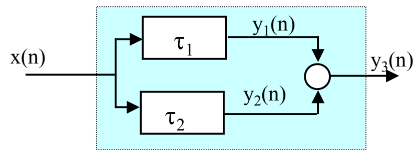
If the systems τ_1 and τ_2 are linear and time invariant $\tau_2\tau_1 = \tau_1\tau_2$

otherwise $\tau_2\tau_1 \ne \tau_1\tau_2$

Classification of Discrete-Time Systems

Interconnection of Discrete-Time Systems

In the parallel interconnection systems,



$$\begin{aligned} y_3(n) &= y_1(n) + y_2(n) \\ &= \tau_1[x(n)] + \tau_2[x(n)] \\ &= (\tau_1 + \tau_2)[x(n)] \\ &= \tau_p[x(n)] \end{aligned}$$

where $\tau_p = \tau_1 + \tau_2$