ECE 308 -4

ECE 308
Discrete-Time Signals and Systems

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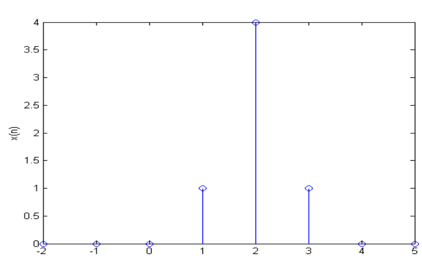
Discrete-Time Signals

- A Discrete-time signal $x(n)$ is a function of an independent integer variable *n*. The signal $x(n)$ is not defined for non-integer values of *n*.

We can represent a discrete-time signal different ways;

1. Graphical representation

Such as



Discrete-Time Signals

2. Functional representation

Such as

$$x(n)=\begin{cases}1, & \text{for } n=1,3\\ 4, & \text{for } n=2\\ 0, & \text{elsewhere}\end{cases}$$

3. Tabular representation

Such as

n	… -2	-1	0	1	2	3	4	5 …
$x(n)$	… 0	0	0	1	4	1	0	0 …

4. Sequential representation

Such as

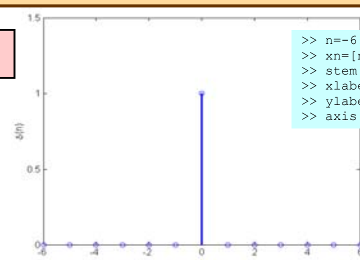
$$x(n)=\{\ldots 0,0,1,4,1,1,0,0,\ldots\}$$
$$\uparrow$$

The time origin (n=0) is indicated by the symbol ↑.

Some Elementary Discrete-Time Signals

1. The unit sample sequence:

$$\delta(n)=\begin{cases}1, & \text{for } n=0,\\ 0, & \text{for } n\neq 0\end{cases}$$

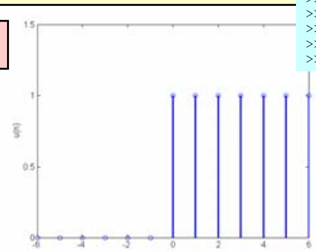


```
>> n=-6:6;
>> xn=[n==0];
>> stem (n,xn)
>> xlabel('n')
>> ylabel('\delta(n)')
>> axis ([-6 6 0 1.5])
```

The unit sample sequence is often referred to as a discrete-time impulse or an impulse.

2. The unit step signal :

$$u(n)=\begin{cases}1, & \text{for } n\geq 0,\\ 0, & \text{for } n<0\end{cases}$$

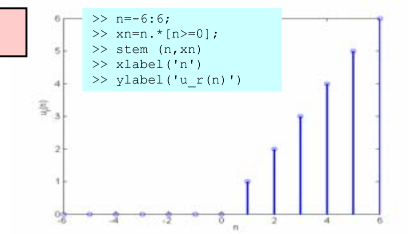


```
>> n=-6:6;
>> xn=[n>=0];
>> stem (n,xn)
>> xlabel('n')
>> ylabel('u(n)')
>> axis ([-6 6 0 1.5])
```

Some Elementary Discrete-Time Signals

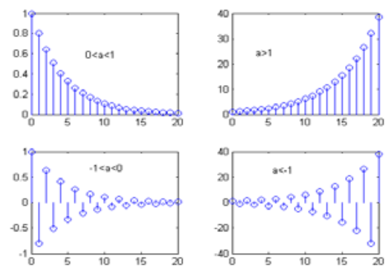
3. The unit ramp signal :

$$u_r(n) = \begin{cases} n, & \text{for } n \ge 0, \\ 0, & \text{for } n < 0 \end{cases}$$



4. The exponential signal :

$$x(n) = a^n \quad \text{for all } n$$



If *a* is real, then $x(n)$ is a real signal.

Some Elementary Discrete-Time Signals

If a is complex, $x(n)$ can be expressed as

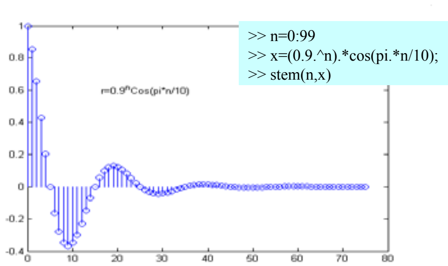
$$x(n) = a^n = \left(re^{j\theta}\right)^n = r^n e^{j\theta n}$$
$$= r^n(\cos\theta n + j\sin\theta n)$$

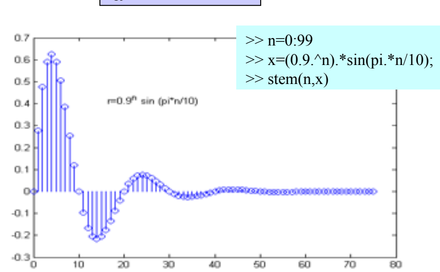
The real part is

$$x_R(n) = r^n \cos\theta n$$

The imaginary part

$$x_R(n) = r^n \sin\theta n$$





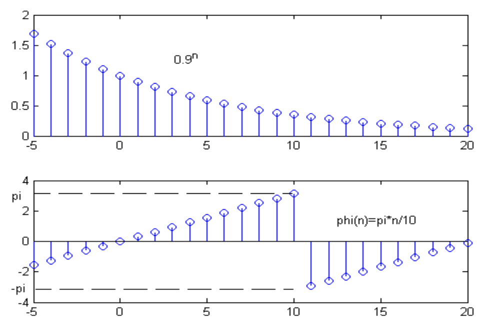
Some Elementary Discrete-Time Signals

Or we can represent the amplitude function

$$|x(n)| = r^n$$

and the phase function

$$\angle x(n) = \phi(n) = \theta n$$



Classification of Discrete-Time Signal

Energy signals and Power signals

The energy E of a signal $x(n)$ is defined as

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

If E is finite ($0 < E < \infty$) then $x(n)$ is called an energy signal

The average power of $x(n)$ is defined as

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Classification of Discrete-Time Signal

Energy signals and Power signals (cont)

The energy of $x(n)$ is be found over the finite interval $-N < n < N$ as

$$E_N = \sum_{n=-N}^{N} |x(n)|^2$$

The signal energy E is

$$E = \lim_{N \to \infty} E_N$$

The average power of $x(n)$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} E_N$$

Classification of Discrete-Time Signal

Periodic Signals

A signal $x(n)$ is periodic with period *N(N>0)* if and only if

$$x(n+N) = x(n)$$ for all *n*.

A sinusoidal signal

$$x(n) = A \sin 2\pi f_0 n$$

is periodic when f_0 is a rational number, that can be expressed as

$$f_0 = \frac{k}{N}$$ where k and N are integer

The average power in periodic is given by

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

Classification of Discrete-Time Signal

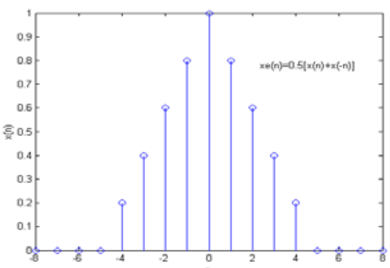
Symmetric (even) and anti-symmetric (odd) signals

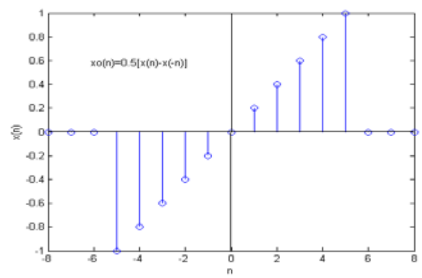
A real-valued signal $x(n)$ is called symmetric (even) if

$$x(-n) = x(n)$$

A signal $x(n)$ is called anti-symmetric (odd) if

$$x(-n) = -x(n)$$





Simple Manipulation of $x(n)$

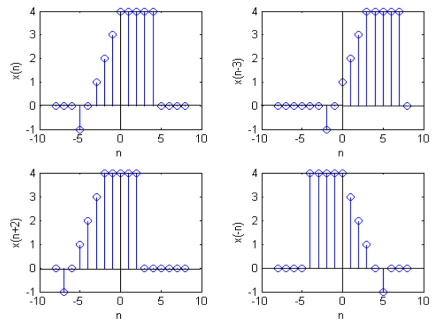
Transformation of the time

A signal may be shifted in time replacing the time variable n by n-k, where k is integer.

- If k is a positive integer, the results in a delay of the signal by k unit of time
- If k is a negative integer, the results in a advance of the signal by k unit of time

If the time base is to replace the independent variable n by $-n$, It is called folding or a reflection of the signal the time origin n=0

Simple Manipulation of $x(n)$



Let's denote the Time-delay operation by *TD* and the folding operation by *FD*

$$TD_k\left[x(n)\right]=x(n-k),\quad k>0$$

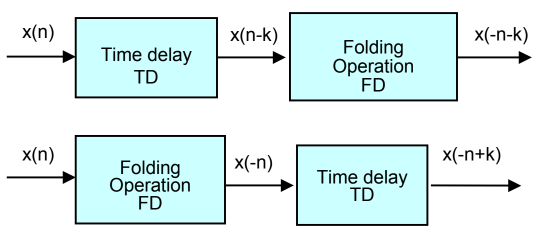
$$FD\left[x(n)\right]=x(-n)$$

$$TD_k\left\{FD\left[x(n)\right]\right\}=TD_k\left[x(-n)\right]=x(-n+k)$$

$$FD\left\{TD_k\left[x(n)\right]\right\}=FD\left[x(n-k)\right]=x(-n-k)$$

Simple Manipulation of $x(n)$

The analog signal $x_a(t)$ is almost linear between quantization levels. The quantization error



Outputs of these two systems are not the same

Simple Manipulation of $x(n)$

Example:

$x(n)$=[0,0,0,-3,-2,-1,0,1,2,3,4,4,4,4,4,4,4,0,0,0]

$y(n) = x(2n)$

Find $y(n)$.

$y(0) = x(0),\ y(-1) = x(-2),\ y(1) = x(2),\ y(-2) = x(-4),\ y(2) = x(4)$