ECE 308 -2

ECE 308
Continuous-Time and Discrete-Time Signal
Sampling of Analog Signals

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Continuous Time Signal

Let's have the following continuous-time sinusoidal signal:

$$x_a(t) = A\cos(\Omega t + \theta), -\infty < t < \infty$$

where

A: the amplitude of the signal

Ω: the frequency in radians per second

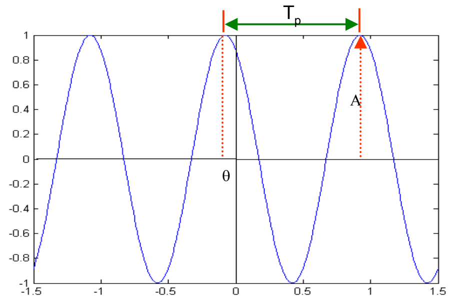
θ: the phase in radians

The frequency can be expressed in cycles/s or Hertz (Hz)

$$F = \frac{\Omega}{2\pi}$$

The period is define as $T_p = \frac{1}{F}$

Continuous Time Signal



The analog sinusoidal signal can repeat every period

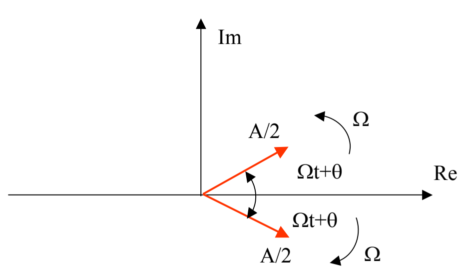
$$x_a(t+T_p) = x_a(t)$$

- Increasing the frequency means decreasing the period of the signal, so that increase the rate of oscillation of the signal

Continuous Time Signal

The analog sinusoidal signal can be expressed in complex exponent for as

$$x_a(t) = A\cos(\Omega t + \theta) = \frac{A}{2}e^{j(\Omega t+\theta)} + \frac{A}{2}e^{-j(\Omega t+\theta)}$$



Discrete-Time Sinusoidal Signal

A discrete-time sinusoidal signal may be expressed as

$$x(n) = A\cos(\omega n + \theta),\ -\infty < n < \infty$$

where *n :* integer variable
A : the amplitude of the signal
ω : the frequency in radians per sample
θ : the phase in radians

The frequency can be expressed in cycles per sample

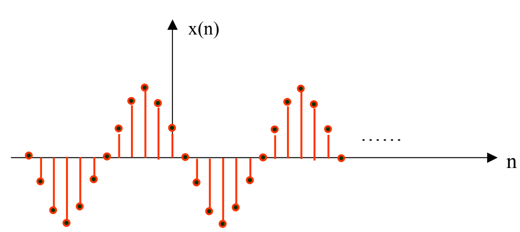
$$f = \frac{\omega}{2\pi}$$

and the signal is

$$x(n) = A\cos(2\pi fn + \theta),\ -\infty < n < \infty$$

Discrete-Time Sinusoidal Signal

Example: A sinusoidal signal with the amplitude A, frequency $\omega=\pi/6$ radians per sample (f=1/12) and phase $\theta=\pi/3$



A discrete-time sinusoidal is periodic only if its frequency is rational number

$$x(n+N) = x(n) \qquad \cos[2\pi f_0(N+n)+\theta] = \cos[2\pi f_0 n + \theta]$$

It is true if and only if

$$2\pi f_0 N = 2k\pi \quad \text{or} \quad f_0 = k/N$$

Discrete-Time Sinusoidal Signal

Discrete-time sinusoids whose frequencies are separated by an integer multiple of 2π are identical

$$\cos[(\omega_0 + 2\pi)n + \theta] = \cos(\omega_0 n + 2\pi n + \theta) = \cos(\omega_0 n + \theta)$$

$$x_k(n) = A\cos(\omega_k n + \theta) = A\cos[(\omega_0 + 2k\pi)n + \theta],\ for\, k = 0,1,2...$$

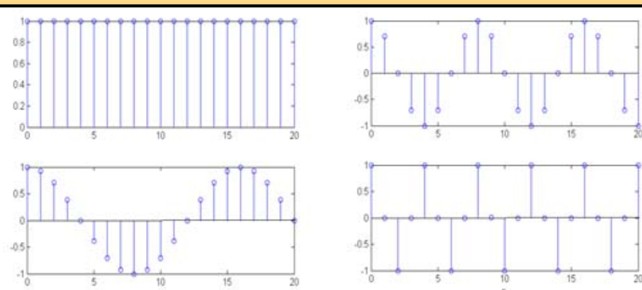
are identical and where $-\pi \le \omega_0 \le \pi$

The highest rate of oscillation in a discrete-time sinusoidal is attained when ω=π (or ω=-π) equivalent to f=1/2 (or f=-1/2)

For $x(n) = A\cos\omega_0 n$

ω	0	π/8	π/4	π/2	π
f	0	1/16	1/8	1/4	1/2
N	∞	16	8	4	2

Discrete-Time Sinusoidal Signal



If $\pi \le \omega_0 \le 2\pi$, it creates an aliasing. How?

Let's $\omega_1 = \omega_0$ which $\pi \le \omega_0 \le 2\pi$ and $\omega_2 = 2\pi - \omega_0$ which $\pi \le \omega_0 \le 2\pi$

$$x_1(n) = A\cos\omega_1 n = A\cos\omega_0 n$$

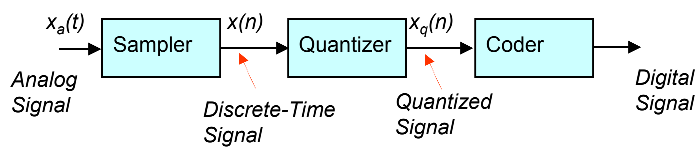
$$x_2(n) = A\cos\omega_2 n = A\cos(2\pi - \omega_0)n$$
$$= A\cos\omega_0 n$$
$$= x_1(n)$$

Hence, ω_2 is an alias of ω_1 .

Analog-to-Digital Conversion (ADC)

- In many real-world application, the signals are analog.
- To process analog signal by digital, we need to convert them into digital signal
- This process is called Analog-to-Digital conversion and devices are A/D Converter (ADCs).

- A/D Conversion has three steps:



Sampling :

- Conversion of a continuous-time signal into a discrete-time signal
- Taking "samples" of the continuous-time signal at discrete-time instants.
- Sampling interval is T. $x(n) = x_a(nT)$

Analog-to-Digital Conversion (ADC)

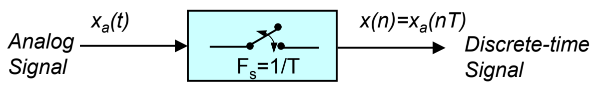
2. Quantization:

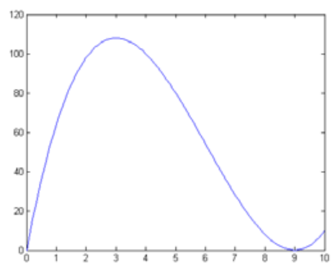
- Conversion of a discrete-time continuous valued signal into a discrete-time, discrete valued digital signal $x_q(n)$
- Digital signal values are a finite set of possible values.
- The differences between $x_q(n)$ and $x(n)$($x_q(n) - x(n)$) is called the quantization error.
- Discrete–time Signals are defined only at certain specific values of time or variable.

3. Coding:

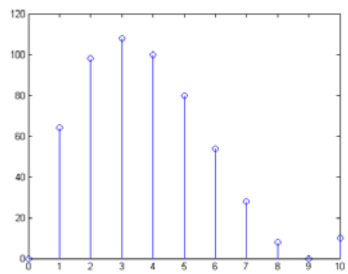
- Each discrete value is represented by a b-bit binary sequence.

Sampling of Analog Signals






```
>> x=0:0.1:10;
>> y=x.^3-18*x.^2+81*x;
>> plot(x,y)
```




```
>> x=0:10;
>> y=x.^3-18*x.^2+81*x;
>> stem(x,y)
```


Sampling of Analog Signals

The discrete-time signal $x(n)$ is obtained by "taking-samples" of the analog signal $x_a(t)$ every T second.

$$x(n) = x_a(nT)$$

The time interval T is called the *sampling period* or *sampling interval*

The sampling rate or the sampling frequency is found as

$$F_s = \frac{1}{T} \quad [Hz]$$

The relationship between the variable *t* of analog signal and the variable *n* of discrete-time signal is

$$t = nT = \frac{n}{F_s}$$

Sampling of Analog Signals

Consider an analog sinusoidal signal

$$x_a(t) = A\cos(2\pi Ft + \theta)$$

Sampling frequency is $F_s = 1/T$, so that

$$x(n) = x_a(nT) = A\cos(2\pi FnT + \theta)$$
$$= A\cos(2\pi F\frac{n}{F_s} + \theta)$$

or $x(n) = A\cos(\omega n + \theta)$

We call relative or normalize frequency that $f = \dfrac{F}{F_s}$

Equivalently,

$$\omega = \frac{2\pi F}{F_s} = \frac{\Omega}{F_s} = \Omega T$$

Sampling of Analog Signals

Relations between analog signals and Discrete-time signal

Continuous-time signal	Discrete-time signal
$\Omega = 2\pi F$ Ω [radians/s] F [Hz]	$\omega = 2\pi f$ ω [radians/sample] f [cycles/sample]
$\omega = \Omega T,\ f = F/F_s$ →	$-\pi \le \omega \le \pi$ $-\frac{1}{2} \le f \le \frac{1}{2}$
← $\Omega = \omega/T,\ F = f.F_s$	
Range $-\infty < \Omega < \infty$ $-\infty < F < \infty$	Range $-\frac{\pi}{T} \le \Omega \le \frac{\pi}{T}$ $-\frac{F_s}{2} \le F \le \frac{F_2}{2}$

Sampling of Analog Signals

The fundamental different between analog signal and discrete-time signal is frequency range.

The highest frequency in the discrete-time signal is $\omega = \pi$ or , $f = 1/2$ the sampling rate F_s , the corresponding highest value of F and Ω are

$$F_{\max} = \frac{F_s}{2} = \frac{1}{2T} \qquad \Omega_{\max} = \pi F_s = \frac{\pi}{T}$$

Example: Consider two analog signal

$$x_1(t) = \cos 2\pi 10 t \qquad x_2(t) = \cos 2\pi 50 t$$

The sampling rate is $F_s = 40\,\mathrm{Hz}$

Find discrete time signal $x_1(n)$ and $x_2(n)$

Sampling of Analog Signals

Example:(cont) Corresponding discrete-time signals are

$$x(n) = \cos 2\pi\left(\frac{10}{40}\right)n = \cos\frac{\pi}{2}n$$

$$x(n) = \cos 2\pi\left(\frac{50}{40}\right)n = \cos\frac{5\pi}{2}n$$

We know that

$$\cos\frac{5\pi}{2}n = \cos\left(2\pi + \frac{\pi}{2}\right)n = \cos\frac{\pi}{2}n$$

Hence $x_1(n) = x_2(n)$

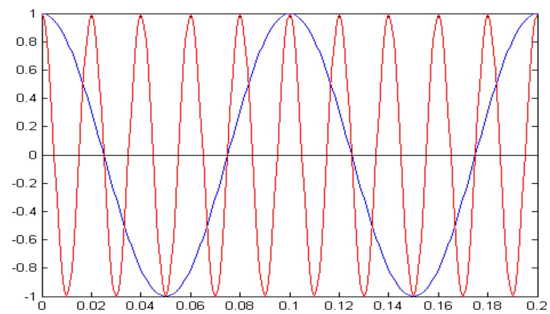
The frequency $F_2 = 50\,\mathrm{Hz}$ is an alias of the frequency $F_1 = 10\,\mathrm{Hz}$ at the sampling rate of $F_s = 40\,Hz$. Even

$F_k = (F_1 + 40k),\ k = 1,2,3,...$ are alias of F_1 at the sampling rate at $F_s = 40\,Hz$.

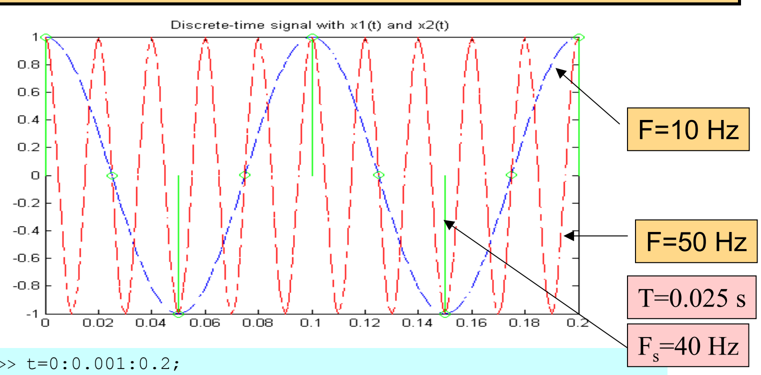
Sampling of Analog Signals

In general form, $F_k = (F_0 + kF_s),\ k = \pm1, \pm2, \pm3, \ldots$ are creates an alias for frequency F_0 of analog signal, which are outside of

$$-\frac{F_s}{2} \le F \le \frac{F_2}{2}$$



Sampling of Analog Signals




```
>> t=0:0.001:0.2;
>> x1=cos(2*pi*10*t);
>> plot (t,x1,'--')
>> hold on
>> x2=cos(2*pi*50*t);
>> plot (t,x2,'r--')
>> n=0:0.025:0.2;
>> y1=cos(2*pi*10*n);
>> stem (t,y1,'g-')
>> Title('Discrete-time signal with x1(t) and x2(t)')
```