Exam 1

Prob.	1	2	3	4	5	6	7	EC	
Value	10	10	10	15	15	25	15	5	100
Points									

Show all work for credit. Answers with little or no supporting work will receive little or no credit.

1. Find the equation of the sphere with center $(1, 2, 4)$ and passes through the point $(5, 2, 7)$.

2. Find all values of x such that $\langle -3x, 2x\rangle$ and $\langle 4, x\rangle$ are orthogonal.

3. Find a parametric equation for the line of intersection of the planes $2x + 5z + 3 = 0$ and $x - 3y + z + 2 = 0$.

4. Find an equation of the plane that contains the points $(0, 5, 2)$, $(2, 1, 4)$, and $(1, 1, 2)$.

5. Let $\mathbf{r}(t) = \begin{bmatrix} \sqrt{t+3} \\ \frac{t-1}{t^2-1} \\ \frac{\tan^2 t\pi}{t+1} \end{bmatrix}$

(a) Evaluate $\lim_{t \to 1} \mathbf{r}(t)$. Give an exact value.

(b) Is $\mathbf{r}(t)$ continuous at $t = 1$? Justify your answer.

6. Let $\mathbf{r}(t) = \begin{bmatrix} t\sqrt{3} \\ \sin t \\ \cos t \end{bmatrix}$

(a) Find the arc length for $0 \leq t \leq \pi$.

(b) Find the unit tangent vector, the unit normal vector, and the bi-normal vector.

7. A particle starts at $(1, 2, 5)$ with initial velocity $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$. Its acceleration is $\mathbf{a} = t\mathbf{i} + \mathbf{j} + t^2\mathbf{k}$. Find its position vector.

8. (Bonus) Given that $||\mathbf{a} \cdot \mathbf{b}|| \leq ||\mathbf{a}|| \, ||\mathbf{b}||$ (the Cauchy-Schwarz Inequality), prove that $||\mathbf{a} + \mathbf{b}|| \leq ||\mathbf{a}|| + ||\mathbf{b}||$ for all vectors $\mathbf{a}$ and $\mathbf{b}$. This is called the Triangle Inequality.
Hint: Remember that $||\mathbf{a} + \mathbf{b}||^2 = (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b})$.