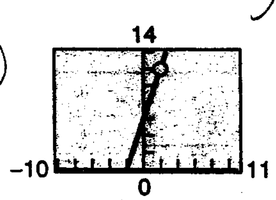
Pre Cal
Chpt 12
practice test

Name Key

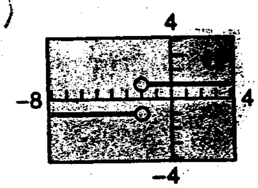
1) Use the graph to find the indicated limit:

a)



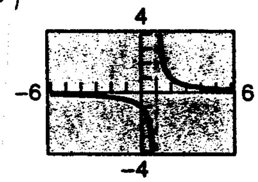
$\lim_{x \to 2}$ 12

b)



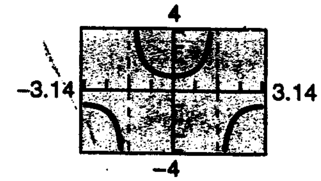
$\lim_{x \to -2}$ DNE

c)



$\lim_{x \to 1}$ DNE

d)



$\lim_{x \to \pi/2^-}$ $+\infty$

2) Find the limits

a) $\lim_{x \to 1} x^2 + 3x - 4$

0

b) $\lim_{x \to 2} f(x)$ $\quad f(x) = \begin{cases} 2x+1; & x<2 \\ 2x+2; & x \geq 2 \end{cases}$

DNE

c) $\lim_{x \to -3} \dfrac{\sqrt{x+7} - 2}{x+3}$

$\dfrac{\overbrace{x+7-4}^{=x+3}}{(x+3)(\sqrt{x+7}+2)} = \dfrac{1}{\sqrt{x+7}+2} = \boxed{\dfrac{1}{4}}$

d) $\lim_{x \to 9} \dfrac{x-9}{\sqrt{x}-3}$ $\quad \dfrac{(x-9)(\sqrt{x}+3)}{x-9}$

$\boxed{=6}$

e) $\lim_{x \to 0} \dfrac{\frac{1}{x+1} - 1}{x}$

$\dfrac{1-(x+1)}{x(x+1)} = \dfrac{-1}{x+1} = \boxed{-1}$

f) $\lim_{x \to 6^+} \dfrac{|x-6|}{6-x} = -1$

g) $\lim_{x \to 1} \dfrac{x-1}{x^2-4x+3}$

$(x-3)(x-1)$

$\boxed{\dfrac{1}{x-3}} = -\dfrac{1}{2}$

h) $\lim_{x \to \infty} 7 + \dfrac{2x^2}{(x+3)^2} + \dfrac{x}{(x+1)^2}$

$7 + 2 + 0 = 9$

3. Find the slope of the graph at the given point:

a) $f(x) = x^2 - 4x$ at $(3,-3)$

$(x+h)^2 - 4(x+h) - x^2 + 4x$

$x^2 + 2xh + h^2 - 4x - 4h - x^2 + 4x$

$(2xh - 4h)/h$

$2x - 4$ $\boxed{f'(3) = 2}$

b) $f(x) = \dfrac{1}{x-2}$ at $(4, 1/2)$

$$\lim_{h \to 0} \frac{\frac{1}{x+h-2} - \frac{1}{x-2}}{h} \qquad \frac{(x-2)-(x+h-2)}{(x-2)(x+h-2)h}$$

$\cancel{\frac{-h}{(x+2)^2}}$ $\boxed{\dfrac{-1}{(x-2)^2}}$

c) $f(x) = \sqrt{x-1}$ at $(5,2)$

$$\lim_{h \to 0} \frac{\sqrt{x+h-1} - \sqrt{x-1}}{h} = \frac{x+h-1-(x-1)}{h(\sqrt{x+h-1}+\sqrt{x-1})}$$

$$= \frac{1}{\sqrt{x+h-1}+\sqrt{x-1}} \Rightarrow \frac{1}{2\sqrt{x-1}}$$

d) Find when the tangent line is horizontal:

$f(x) = 3x^3 - 9x$ $\quad 3(x+h)^3 - 9(x+h) - 3x^3 + 9x$

$1 \ 3 \ 3 \ 1$

$3x^3 + 9x^2h + 9xh^2 + 3h^3 - 9x - 9h - 3x^3 + 9x$

$(9x^2h + 9xh^2 + 3h^3 - 9h) \div h = 9x^2 + 9xh + 3h^2 - 9$

$\lim_{h \to 0}$ $9x^2 - 9 = 0$ $\quad 9(x^2 - 1) = 0$

$\boxed{x = \pm 1}$ $(1,-6)$ $(-1,12)$

4) Find the sum

a) $\displaystyle\sum_{1}^{10} j^3 - 3j^2$

$$\frac{n^2(n+1)^2}{4} - 3\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$\cancel{\frac{100 \cdot 121}{4}}$ $\cancel{3240}$.

$$\frac{\overset{25}{\cancel{100}} \cdot 121}{4} - \frac{\cancel{3} \cdot \overset{5}{\cancel{10}} \cdot 11 \cdot 21}{\underset{2}{\cancel{6}}}$$

$\boxed{1870}$

b) $\displaystyle\sum_{1}^{n} \frac{2i+3}{n^2}$ for $n = 10$

$$\frac{2}{n^2}\left(\frac{n \cdot (n+1)}{2}\right) + \frac{3}{n^2}(n)$$

$$\frac{11}{10} + \frac{3}{10} = \frac{14}{10} = \boxed{1.4}$$

5) estimate the area under the curve:

a) $y = 8 - 2x^2$ $[0, 2]$

w/ 4 rectangles

$$\sum_{1}^{4} \frac{2}{4}\left(8 - 2\left(\frac{i}{2}\right)^2\right) \qquad \sum_{1}^{4} 4 - \frac{i^2}{4}$$

$$16 - \frac{1}{4}\left(\frac{4(5)(9)}{6}\right) \qquad 16 - \frac{45}{6}$$

$\boxed{8\frac{1}{2}}$

b) $y = 8 - x^3$ $[0, 2]$

n rectangles $n \to \infty$

$$\sum_{1}^{n} \frac{2}{n}\left(8 - \left(\frac{2i}{n}\right)^3\right)$$

$$\sum_{1}^{n} \frac{16}{n} - \frac{16i^3}{n^4}$$

$$16 - \frac{16}{n^4}\sum i^3 = 16 - \frac{16}{n^4}\left(\frac{n^2(n+1)^2}{4}\right)$$

$16 - 4 = \boxed{12}$

c) $y = 2 - x^2$ $[-1, 1]$

$n \to \infty$

$$\sum_{1}^{n} \frac{2}{n}\left(2 - \left(-1 + \frac{2i}{n}\right)^2\right)$$

$$\frac{2}{n}\left(2 - \left(1 - \frac{4i}{n} + \frac{4i^2}{n^2}\right)\right)$$

$$\frac{2}{n}\left(1 + \frac{4i}{n} - \frac{4i^2}{n^2}\right)$$

$$\sum_{1}^{n} \frac{2}{n} + \frac{8i}{n^2} - \frac{8i^2}{n^3}$$

$$\sum \frac{2}{n} + \frac{8}{n^2}\sum i - \frac{8}{n^3}\sum i^2$$

$$2 + \frac{8(n(n+1))}{2n^2} - \frac{8(n)(n+1)(2n+1)}{6n^3} = 3\frac{1}{3}$$

$2 + 4 - \frac{8}{6}$