Inverse Functio

6. a) Find the inverse:

$$f(x) = \frac{(3x)^2 - 12}{5}$$

$$y = \frac{\sqrt{5x+12}}{3}$$

b) Graph the inverse:

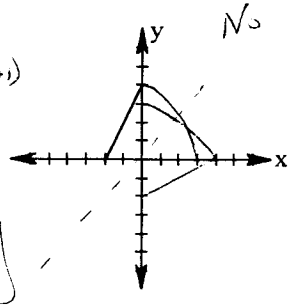
$$g(x) = 3\log_5(x+1) - 2$$

$$\frac{x+2}{3} = \log_5(x+1)$$

$$5^{\frac{x+2}{3}} = y+1$$

$$y = 5^{\frac{x+2}{3}} - 1$$

Is the inverse a function? No



Logarithms

7. Solve the equations for x.

a) $5 = 9^{x-1}$

$\log 5 = (x-1)\log 9$

$x = \frac{\log 5}{\log 9} + 1 = 1.733$

b) $\log_x(1) = 0.5$

$x^{1/2} = 1$

$x = 1$

c) $7.3(2.7)^{x+2} = 60.225$

$2.7^{x+2} = 8.25$

$x+2 = \frac{\log 8.25}{\log 2.7}$

$x = .125$

d) $5 = x^9$

$x = \sqrt[9]{5}$

$x = 1.1960$

Simplifying Alebraic Fractions

8. Simplify the fraction using the fraction-buster technique.

$$\frac{\left(\frac{2}{x}+\frac{1}{y}\right)xy}{\left(\frac{1}{xy}-\frac{2}{x}\right)xy} \quad \frac{2y+x}{1-2y}$$

Equations of Exponential Functions

9. They say all students have a learning curve. For example you retain more information at 8AM than at 2PM. In fact this learning curve could have an exponential function as its mathematical model. Suppose you retain 45% of the information you encounter at 8AM, 32% at 11AM, and you never retain less than 15%.

Find the particular exponential function that would approximate this data.
(Hint: Assume your first ordered pair to be (0,45) & don't forget the asymptote!!)

$y = km^x + 15$

$45 = k + 15$

$k = 30$

$y = 30m^x + 15$

$32 = 30m^3 + 15$

$\frac{17}{30} = m^3$

$m = .828$

$y = 30(.828)^x + 15$

Algebra II--Final Review
Units 5-6

Name

Systems of Equations-3-variables (Solve and explain what the answer means!!)

1.
$x + y + z = 1$
$2x - 3y + 6z = -24$
$5x + y - z = 21$

$3+2+1=1$

a+c) $6x + 2y = 22$

b+c) $32x + 3y = 102$

$18x + 6y = 66$

$64x + 6y = 204$

$46x = 138$

$18 + 2y = 22$

$x = 3 \quad y = 2 \quad z = -4$

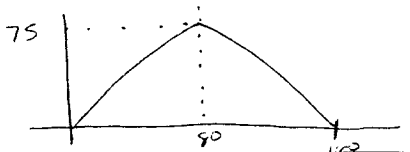
2.
$x + 3y - 2z = 0$
$3x + 9y - 6z = 4$
$5x - 2y - 3z = 7$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 3 & 9 & -6 & 4 \\ 5 & -2 & -3 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Equation of Parabola given 3 points

3. A football is punted and is caught by a player 160 feet from the kicker. The ball attains a maximum height of 75 feet. If the path of the ball is parabolic, find an equation to model the path of this football. After 15 feet, how high is the football?



$y = a(x-80)^2 + 75$

$0 = a(-80)^2 + 75$

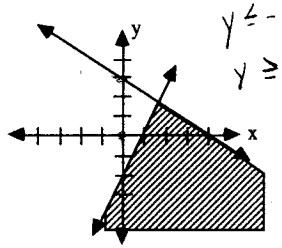
$-75 = a6400$

$a = \frac{-75}{6400} = -.0117$

$y = \frac{-75}{6400}(x-80)^2 + 75$

Inequalites

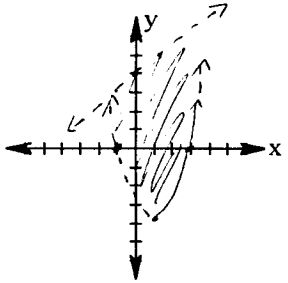
4. a) What inequalities describe this region?



$y \leq -\frac{3}{4}x + 3$

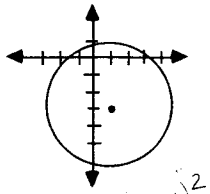
$y \geq 2x - 2$

b) Graph: $y > (x+1)(x-3)$ & $y < x + 4$



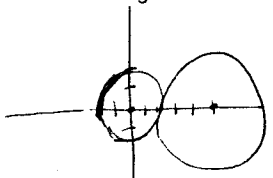
Equations of Circles

5. a) Find the equation of the circle below.



$(x-1)^2 + (y+3)^2 = 16$

b) Sketch & find the equations of 2 circles with different diameters that are tangent to each other.



$x^2 + y^2 = 4$ $(x-5)^2 + y^2 =$

Permutations / Combinations / Other:

15. a) Thirteen students want to go on a field trip, but there is only room for five in the car. How many different ways could the group making the trip be composed?

$_{13}C_5$ $\quad \frac{13!}{8!5!} =$

b) There are eight teaching assistants available for grading papers in a college course. The first exam consists of four questions, and the professor wants a different assistant grading each question (one assistant per question). In how many ways can assistants be chosen to grade the exam?

$_8P_4 = \frac{8!}{4!} = 8 \cdot 7 \cdot 6 \cdot 5 = 1680$

16. a) How many ways are there to choose the 1st, 2nd, and 3rd place winners in the annual Bake-Off Tournament if there are 50 contestants?

$_{50}P_3 \quad \frac{50!}{47!} \quad 50 \cdot 49 \cdot 48 = 117600$

b) How many 4-digit numbers less than 7000 could be made if the last digit must be an even number?

$8 \cdot 8 \cdot 7 \cdot 5 = 2240$

c) If four cards are dealt form a standard deck of playing cards, what is the probability that all four cards will be clubs?

.00264 or .2641% or $\frac{11}{4165}$ $\quad \frac{_{13}C_4}{_{52}C_4} = \frac{\frac{13!}{9!4!}}{\frac{52!}{48!4!}} = \frac{13!\,48!\,4!}{9!4!\,52!} = \frac{13 \cdot 12 \cdot 11 \cdot 10}{52 \cdot 51 \cdot 50 \cdot 49} = \frac{17160}{6497400}$

Factorials:

17. a) Write the first three factors of 82!

$82 \cdot 81 \cdot 80$

b) Write the first three factors of $(n+1)!$

$(n+1)(n)(n-1)$

c) Simplify: $\frac{n!}{(n-3)!}$

$\frac{n!}{(n-3)(n-4)\ldots}$ $\quad n(n-1)(n-2)$

Completing Square Method:

18. Complete the square to write the following in graphing form. State the vertex and intercepts.

$f(x) = x^2 - 6x + 5 = (x-5)(x-1)$ $\quad y = 3x^2 + 12x - 5$

$y = (x-3)^2 - 4$ $\quad 3(x^2 + 4x - 5/3)$

$(3, -4)$ $\quad 3[(x+2)^2 - 5\frac{1}{3}]$

$x = 5, x = 1$ $\quad 3(x+2)^2 - 17$

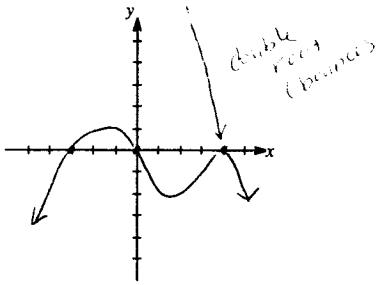
Polynomial Functions:

10. a) If you are shown the graph of a polynomial, how can you tell if the roots are real or imaginary? What can you tell about the degree?

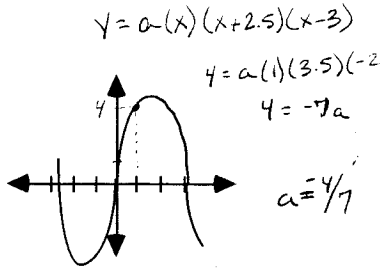
You can find the degree by the changes in direction roots are real iff the graph crosses the x-axis.

b) Sketch the graph on the axis provided.

$y = -2x(x+3)(x-4)^2$



c) Write an equation for the graph.



11. What are the roots of the following function: $y = x^3 - 2x^2 + 10x$

$\frac{+2 \pm \sqrt{4 - 4 \cdot 1 \cdot 10}}{2}$ $\quad y = x(x^2 - 2x + 10)$

$\frac{2 \pm \sqrt{-36}}{2} = \frac{2 \pm 6i}{2} = 1 \pm 3i$ $\quad + x = 0$

Complex Numbers:

12. Simplify the following expressions

a) i^{34} $\quad i^2 = -1$

b) $(5 - 6i)(4 - 3i)$ $\quad 20 - 15i - 24i + 18i^2$ (what are these called??) $\quad 2 - 39i$

c) $(4 - 7i)(4 + 7i)$ $\quad$ complex conjugates $\quad 16 - 49i^2 \quad 65$

d) $9 - 6i - (2 - 3i)^2$ $\quad 9 - 6i - 4 - 6i + 9i^2 \quad 9 - 6i - 4 + 6i + 9 \quad 16$

Probability:

13. A standard deck of playing cards is shuffled, unfortunately, the queen of spades is missing from the deck. If a card is chosen at random, what is the probability that the card chosen is

a) a heart? $\quad 13/51$

b) a queen? $\quad 3/51$

c) a queen and a heart? $\quad 1/51$

d) a queen or a spade? $\quad 15/51$

14. A penny, a nickel, and a dime are all flipped

a) List all elements of the sample space (i.e. HHH, . . .).

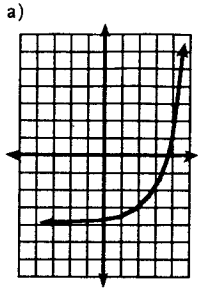
HHH HTH TTT THT
HHT HTT TTH THH

b) What is the probability of getting no tails? $\quad 1/8$

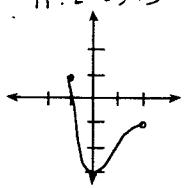
c) What is the probability of getting at least one tail? $\quad 7/8$

Old Stuff you re should know

25. State the domain and range of the following functions:

a) 

D: $(-\infty, \infty)$
R: $(-4, \infty)$

b) 

D: $[-1, 2)$
R: $[-3, 1]$

c) $y = \sin x$

D: $(-\infty, \infty)$
R: $[-1, 1]$

26. If $f(x) = 2x + 7$ and $g(x) = 5x - x^2$, find the following:

a) $f(-4)$
$2(-4) + 7$
$= -1$

b) $f(g(1))$ $= 15$
$g(1) = 5 \cdot 1 - 1^2 = 4$
$f(4) = 8 + 7 = 15$

c) x when $f(x) = 4$
$2x + 7 = 4$
$2x = -3$
$x = -3/2$

27. Does the number -128 belong to the sequence 4, -8, . . .

a) if the sequence is arithmetic?
$g(n) = -12n + 4$
$-128 = -12n + 4$
$-132 = -12n$
$n = 11$
Yes (whole #)

b) if the sequence is geometric?
$g(n) = (4)(-2)^n$
$-128 = 4 \cdot (-2)^n$
$-32 = (-2)^n$
$n = 5$ Yes (odd)

28. Jennifer had $1250 in a savings account three years ago. The interest rate at the time of deposit was a 6% annual rate compounded quarterly. $6/4 = 1.5\%$

a) Write an equation that would represent the amount of money in the account.
$1250(1.015)^{4n} = \$$

b) How much money is in the account today?
$1250(1.015)^{12} = 1494.53$

c) How much money will be in the account 12 years from now assuming no deposits or withdrawals?
$1250(1.015)^{60} = \$3054.05$

29. In an arithmetic sequence, t(6) = 33 and t(41) = 138. $g(n) = g \cdot n + i$ $33 = g \cdot 6 + i$ $138 = g \cdot 41$

a) Find the common difference (generator). 3

b) Find the initial value, t(0). $= 15$

c) Write the rule for this sequence. $g(n) = 3n + 15$

a) $138 = g \cdot 41 + i$
$33 = g \cdot 6 + i$
$105 = 35g$
$g = 3$

$g(n) = 3 \cdot n + i$
$33 = 3 \cdot 6 + i$
$33 - 18 = i = 15$

Final Review Unit 8 & etc.

Name Key

Radian / Degree Conversion:

19. Convert each of the following:

a) 6π radians to degrees
$\frac{6\pi}{1} \cdot \frac{180}{\pi} = 1080$

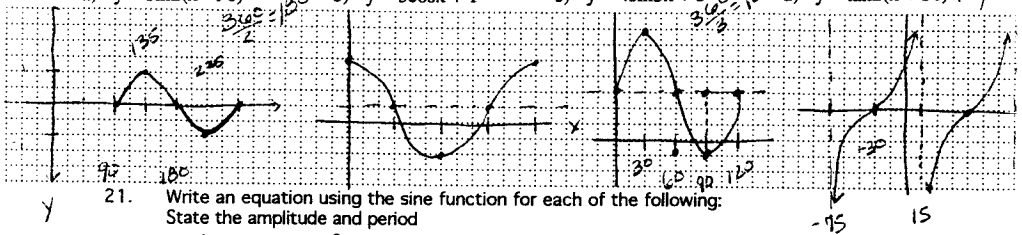
b) 300 degrees to radians
$\frac{300\pi}{180} = \frac{5\pi}{3}$

c) $\frac{4\pi}{3}$ degrees to radians
$\frac{4\pi}{3} \cdot \frac{\pi}{180} = \frac{\pi^2}{135}$

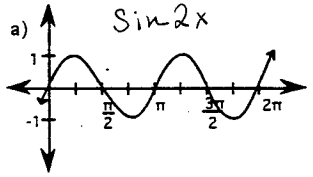
Circular Functions:

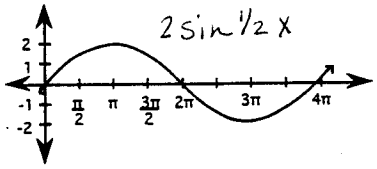
20. Sketch a graph for the following equations:

a) $y = \sin 2(x - 90)$ b) $y = 3\cos x + 1$ c) $y = 4\sin 3x + 3$ d) $y = \tan 2(x + 30)$



21. Write an equation using the sine function for each of the following:
State the amplitude and period

a) 

b) 

22. Write an equation of a sin function that has:

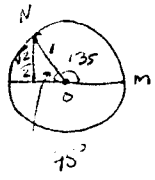
a) a period of 90° $\sin 4x$

b) no x-intercepts $\sin x + 2$

c) has an amplitude of 12 $12 \sin x$

Unit Circle / Special Right Triangles:

23. Sketch a 135° angle in a unit circle. Label it ∠MON

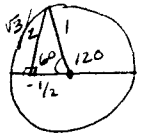


a) What is the reference angle for ∠MON? Show it on your diagram.
45°

b) Show, on your diagram, the sine of ∠MON. $\sin = \frac{opp}{hyp}$
$\sin 135 = \sqrt{2}/2$

c) Convert both ∠MON and its reference angle to radians.
$135 \cdot \frac{\pi}{180} = \frac{3\pi}{4}$ $45 \cdot \pi/180 = \pi/4$

24. Find the exact value of sine, cosine, and tangent of 120°.



$\sin = \sqrt{3}/2$
$\cos 120 = -1/2$
$\tan 120 = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$