Pre-Calculus topics you must master for Calculus.

Blue packet pg 1

Factoring

$$(\sin x + \cos x)(\sin x - \cos x)$$

$$\sin^2 x - \cos^2 x$$

$$\sin x + \cos x$$

$$(x - 2)(x^2 + 2x + 4)$$

$$x^3 - 8$$

$$x - 2$$

Factor $8x^3 - 27y^3$

Distance Formula

Equations of lines

Equation of line:

Point: (x_1 , y_1)

Slope: m

Equation:

$$y - y_1 = m(x - x_1)$$

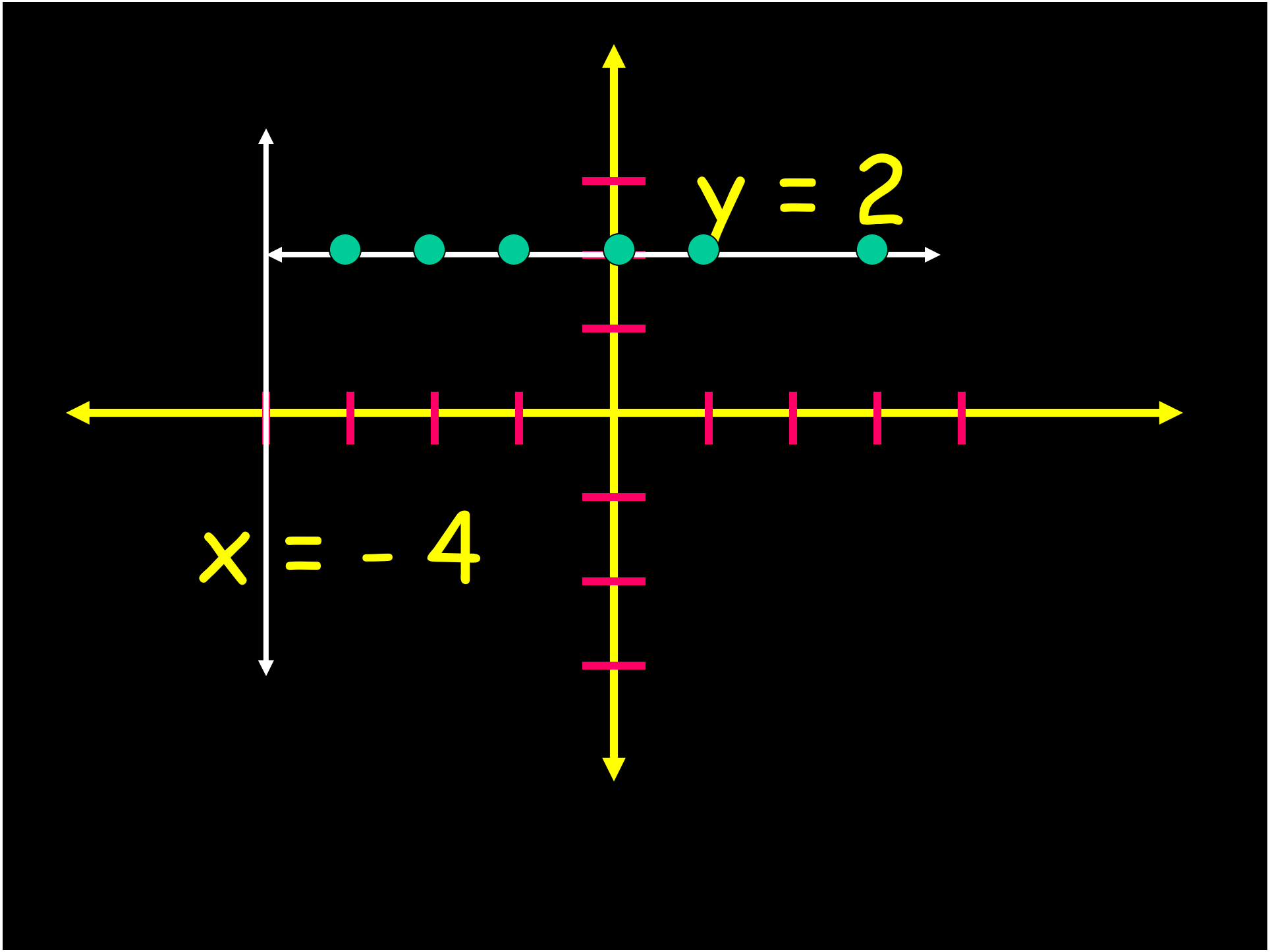
Equation of line:

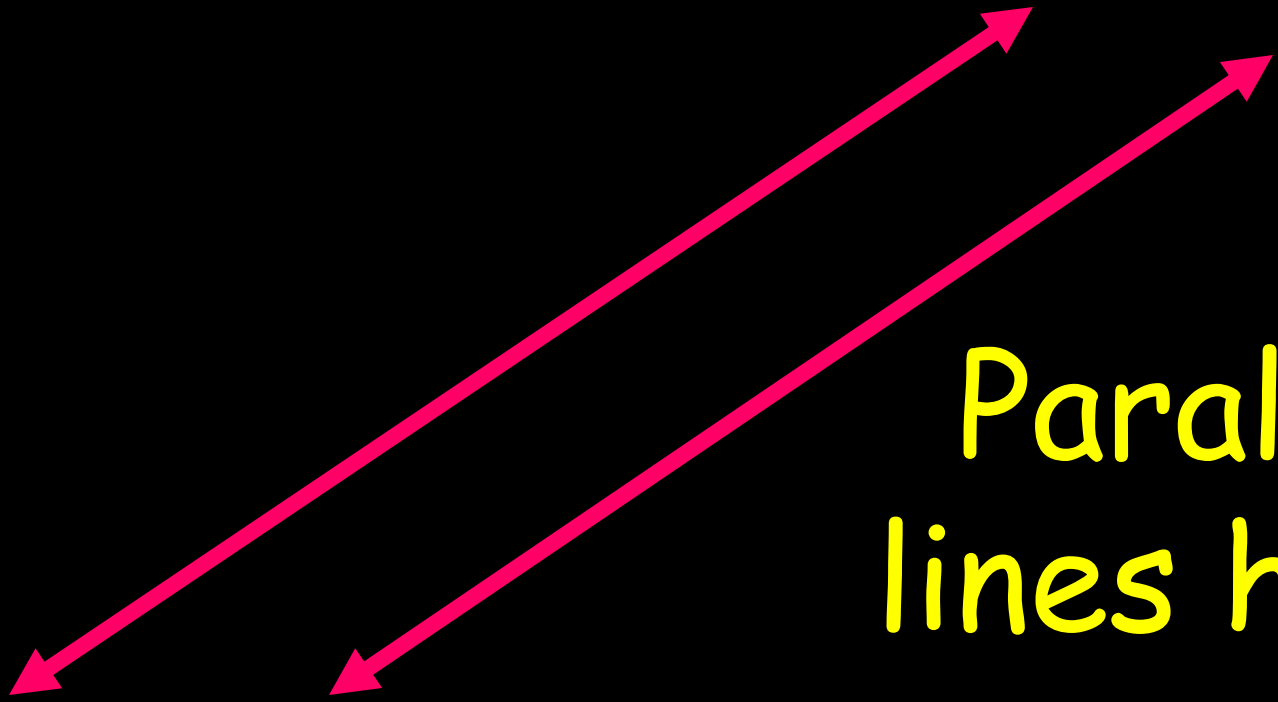
Point: (2 , -3)

Slope: 7

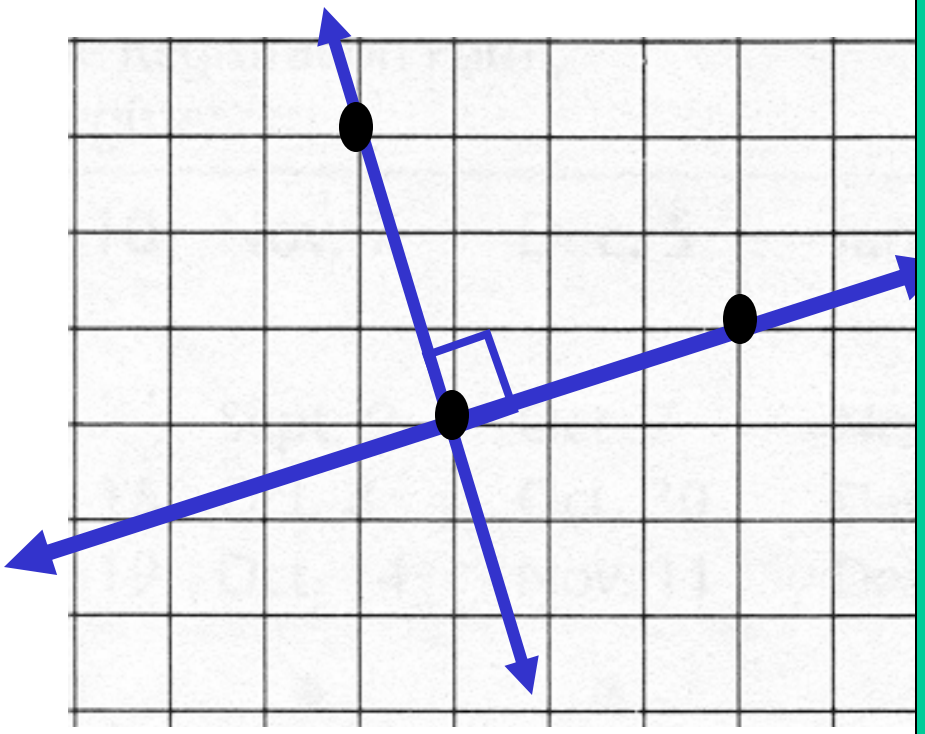
Equation:

$$y - -3 = 7(x - 2)$$


y = 2
x = - 4



Parallel lines have equal slopes!



Perpendicular lines have neg reciprocal slopes

slope	$\perp$ slope
2	-1/2
3/5	-5/3
-5	1/5
-1/3	3
0	und

Log Properties

Log properties

$$e^{3\ln x} = e^{\ln x^3} = x^3$$

$$\ln(3e^x) = \ln 3 + \ln e^x$$

$$= \ln 3 + x$$

Simplify:

1. $(e^x)^2$
2. $\frac{1}{e^x}$
3. $(e^x)(e^x)$
4. $\sqrt{e^x}$
5. $(e^x)(e^{-x})$
6. $\frac{1}{\sqrt{e}}$

Simplify:

1. $\ln e^{3x}$
2. $\ln (e^{3x})^2$
3. $\ln \sqrt{e^x}$
4. $(e^{x} + e^{-x})(e^{x} + e^{-x})$

Trig Properties

$$\sec x = 1 / \cos x$$
$$\csc x = 1 / \sin x$$
$$\cot x = 1 / \tan x$$
$$= \cos x / \sin x$$

$\sin^2 x = 1 - \cos^2 x$

$\cos^2 x = 1 - \sin^2 x$

$\tan^2 x = \sec^2 x - 1$

$\cot^2 x = \csc^2 x - 1$

$\sec^2 x = 1 + \tan^2 x$

$\csc^2 x = 1 + \cot^2 x$

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 1 - 2\sin^2 x$$

$$= 2\cos^2 x - 1$$

Simplify:

1. $\sqrt{1 - \cos^2 x}$

2. $\dfrac{4 \sin 2x}{2 \sin x}$

3. $\dfrac{\cos^2 x - \sin^2 x}{\sin 2x}$

Intercepts and Roots

Domain

Domain: For what values of x is the function defined?

Range: For what values of y is the function defined?

Domain is all real numbers except:

set denominator $\neq 0$

set inside even roots ≥ 0

set inside logs or ln > 0

Domain of

$$y = \frac{1}{2x + 3}$$

Set $2x + 3 \neq 0$

Domain of

$$y = \sqrt{2x + 3}$$

Set $2x + 3 \geq 0$

Domain of

$$y = \sqrt[3]{2x + 3}$$

All real numbers

Domain of

$$y = \ln(2x + 3)$$

Set $2x + 3 > 0$

Domain of

$$y = \frac{1}{\sqrt{2x + 3}}$$

Set $2x + 3 > 0$

Domain of

$$y = \frac{1}{x} + \sin x$$

Set $x \neq 0$

Domain of

$$y = 6x - 7 + \sqrt{2x}$$

Set $2x \geq 0$

Domain of

$$y = \ln(x + 3) - x^2 - \ln x - 1$$

Set $x + 3 > 0$

and $x > 0$

Domain of

$$y = \frac{1}{\sqrt{x^2 + 1}}$$

Set $x^2 + 1 > 0$

Domain of

$$y = \frac{1}{\sqrt{x^2 - 1}}$$

Set $x^2 - 1 > 0$

Solving by Calculator

Using Calculator to solve equations:

1) make the eq = 0

2) graph (use domain if any)

3) 2nd calc the zeros

Solve by calculator:

1. $x^4 - x^3 - 5x^2 + 3x + 6 = 0$
2. $6\ln(4x) - 1 = 14$
3. $-5e^{-x} + 18 = 7$

Solve by calculator:

4. $x e^{x} = e^{x}$

5. $x^{5} - x = 0 \quad -4 < x < 0$

6. $2\sin^{2}x - \sin x = 1 \quad [0, 2\pi]$

Answers:

1. $\pm 1.732, -1, 2$
2. 3.046
3. -0.788
4. 1
5. -1
6. $1.57, 3.665, 5.76$

Interval Notation

$x < 3$ $(-\infty, 3)$

$x \leq 3$ $(-\infty, 3]$

$x > 3$ $(3, \infty)$

$x \geq 3$ $[3, \infty)$

$x \neq 3$ $(-\infty, 3), (3, \infty)$

$-2 < x \leq 3$	$(-2, 3]$
$-2 \leq x < 3$	$[-2, 3)$
$-2 < x < 3$	$(-2, 3)$
$-2 \leq x \leq 3$	$[-2, 3]$
All reals	$(-\infty, \infty)$

Fractional Powers

$$8^{1/3} = \sqrt[3]{8} = 2$$

$$\frac{1}{\sqrt[3]{8}} = \frac{1}{2}$$

$$8^{2/3} = \sqrt[3]{8}^{2} = 4$$

$$\frac{1}{\sqrt[3]{8}^{2}} = \frac{1}{4}$$

Evaluate without a Calculator

$\left(\frac{16}{9}\right)^{1/2}$

$\left(\frac{8}{27}\right)^{4/3}$

$\left(\frac{9}{16}\right)^{-3/2}$

$\left(\frac{25}{49}\right)^{-1/2}$

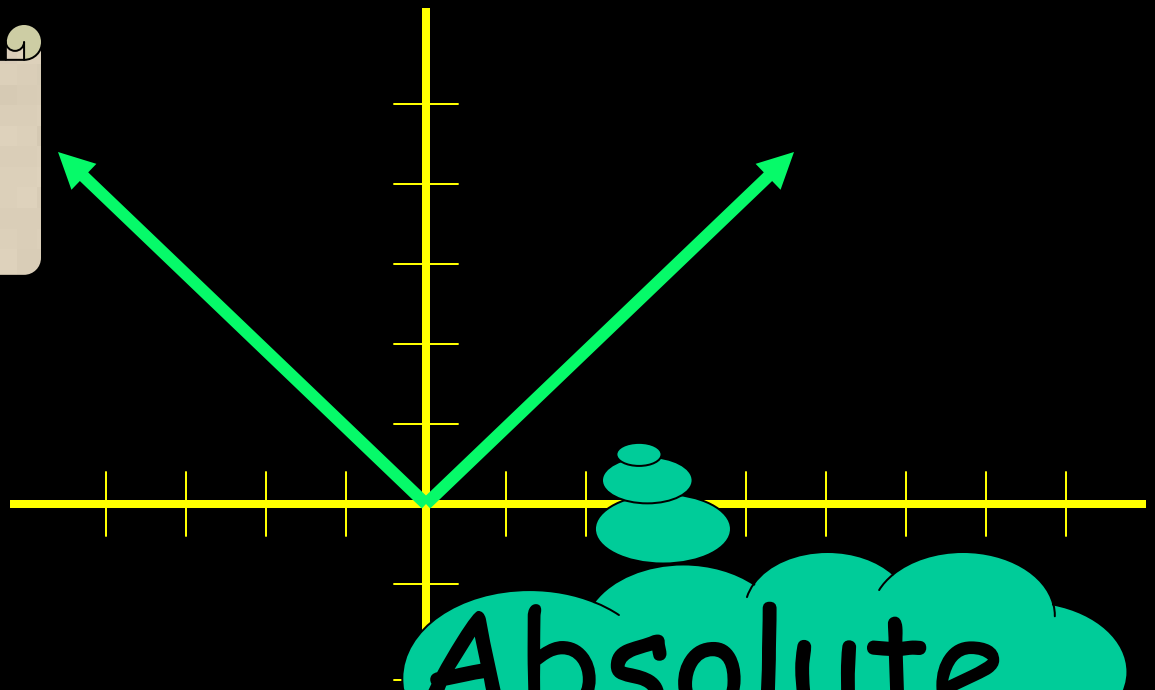
$\left(\frac{9}{100}\right)^{-3/2}$

$\left(\frac{1}{8}\right)^{-4/3}$

Graphs

Basic Graphs

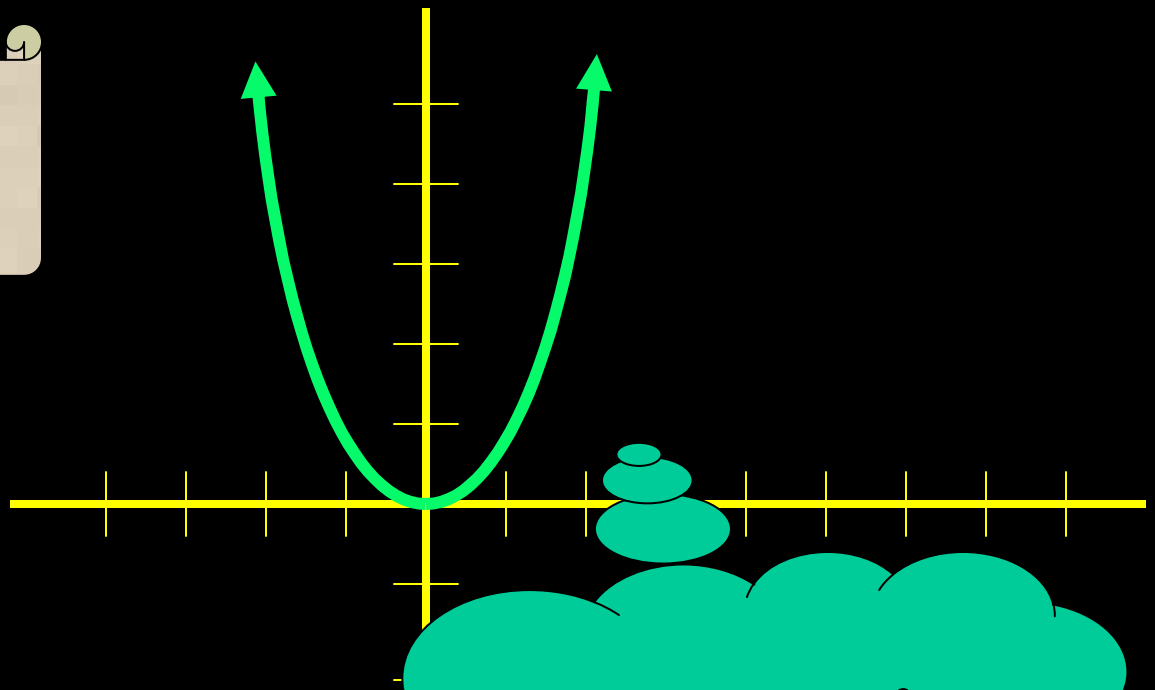
$y = |x|$



Absolute value

Basic Graphs

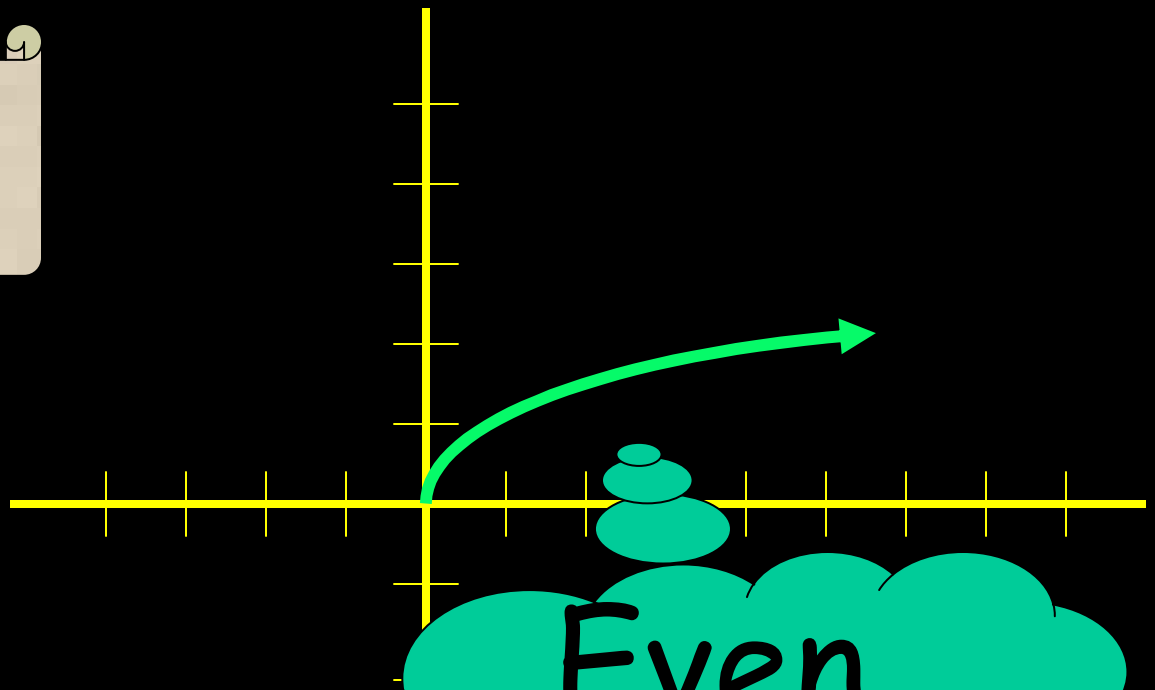
$y = x^2$



parabola

Basic Graphs

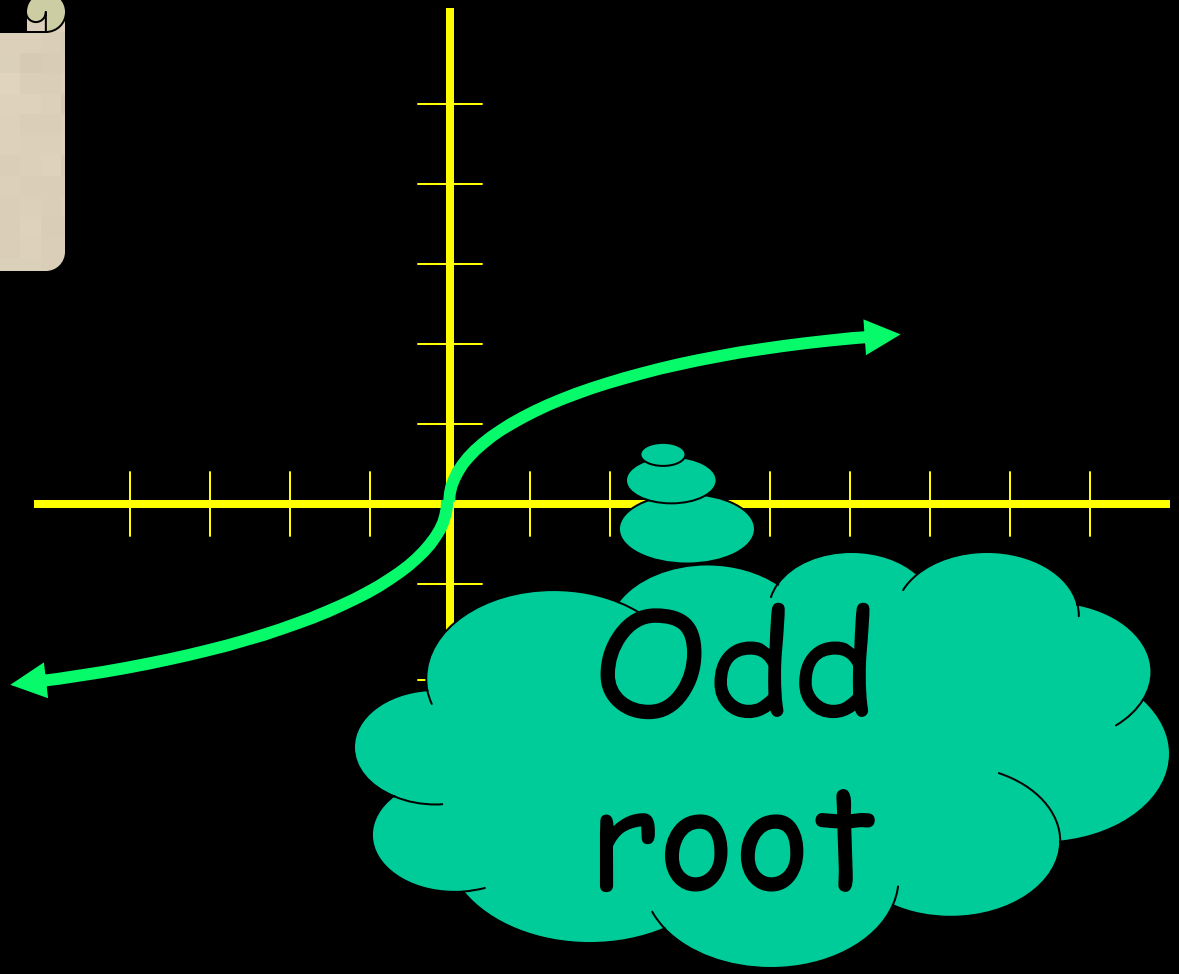
$$y = \sqrt{x}$$



Even root

Basic Graphs

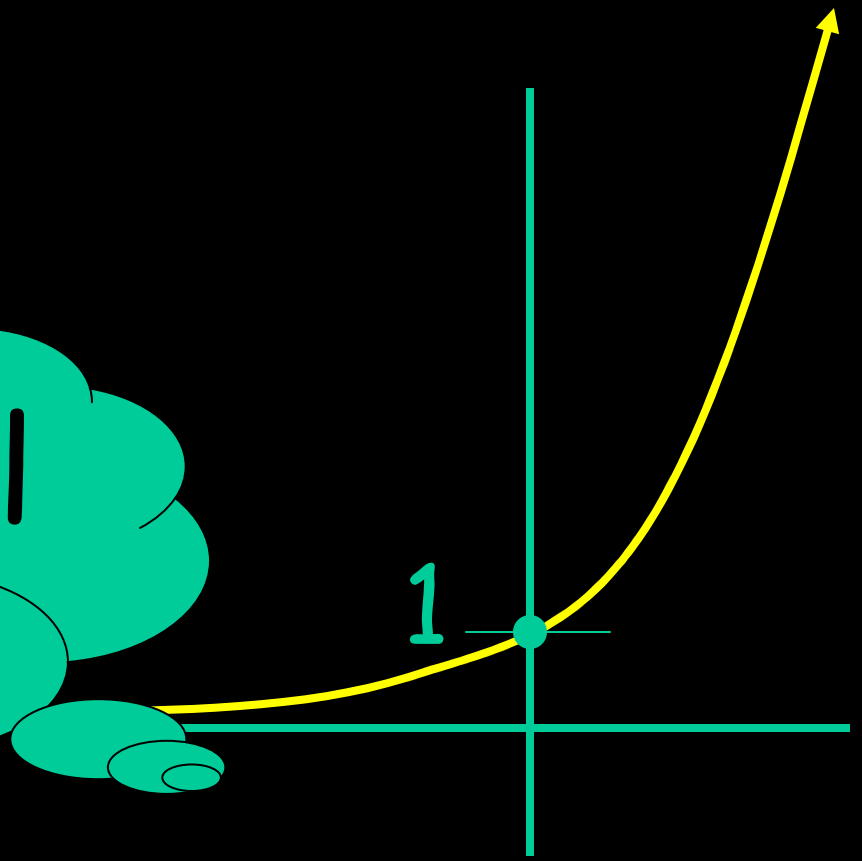
$$y = \sqrt[3]{x}$$



Basic Graphs

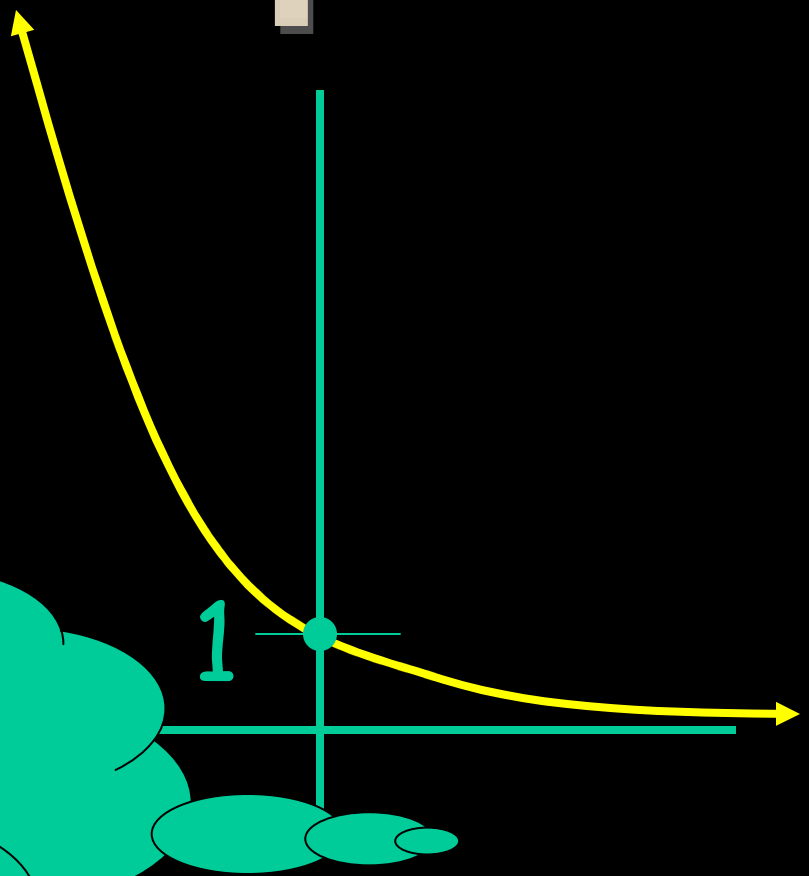
$y = e^x$

Exponential growth



Basic Graphs

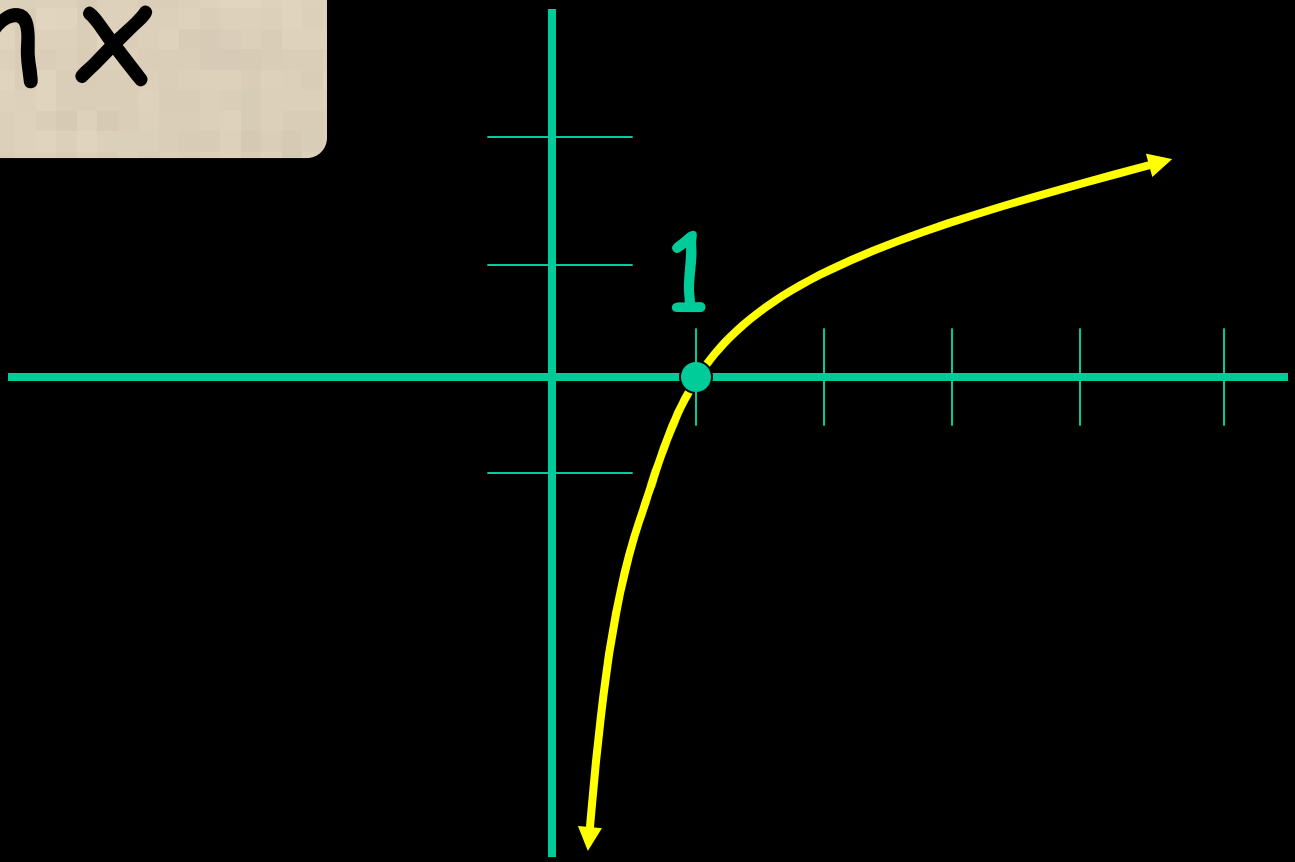
$y = e^{-x}$



Exponential decay

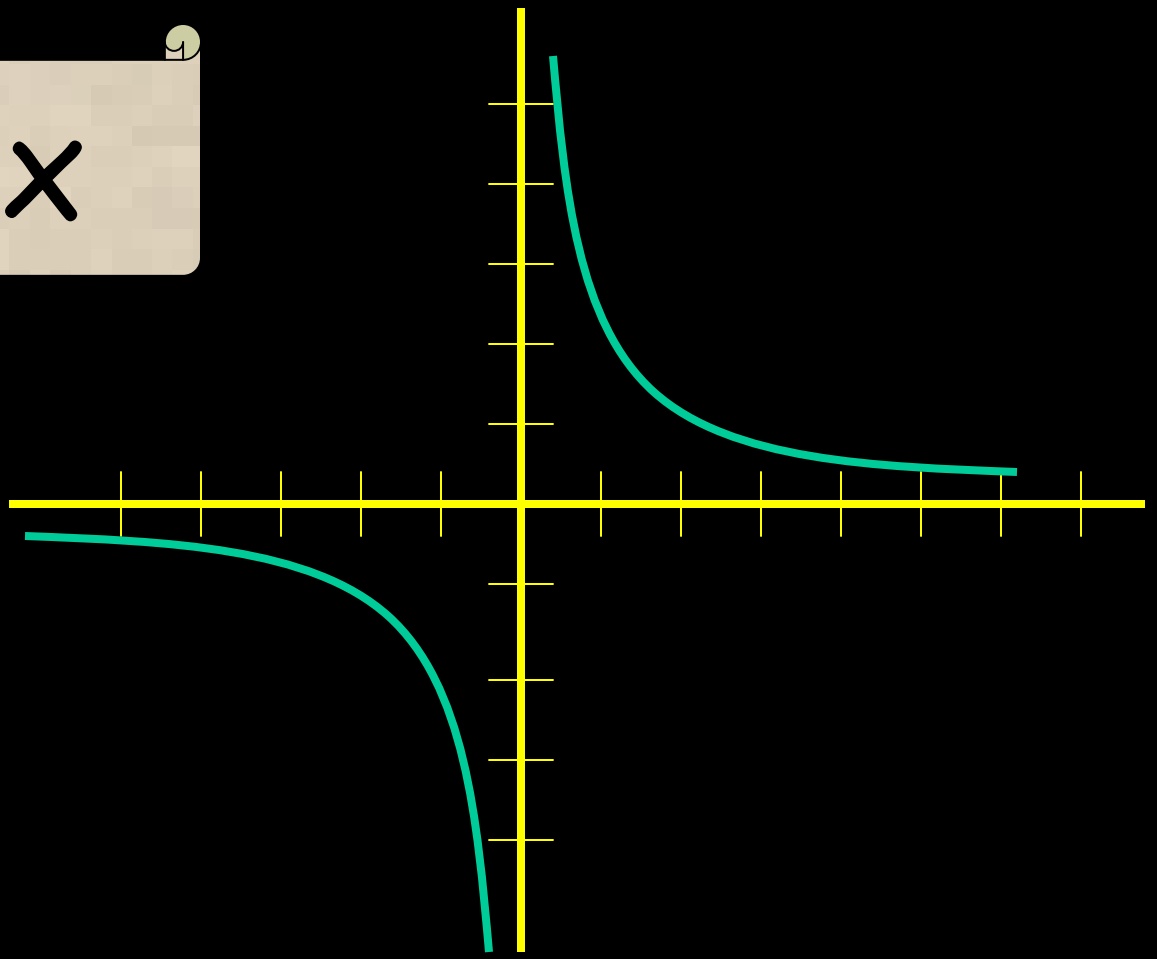
Basic Graphs

$y = \ln x$



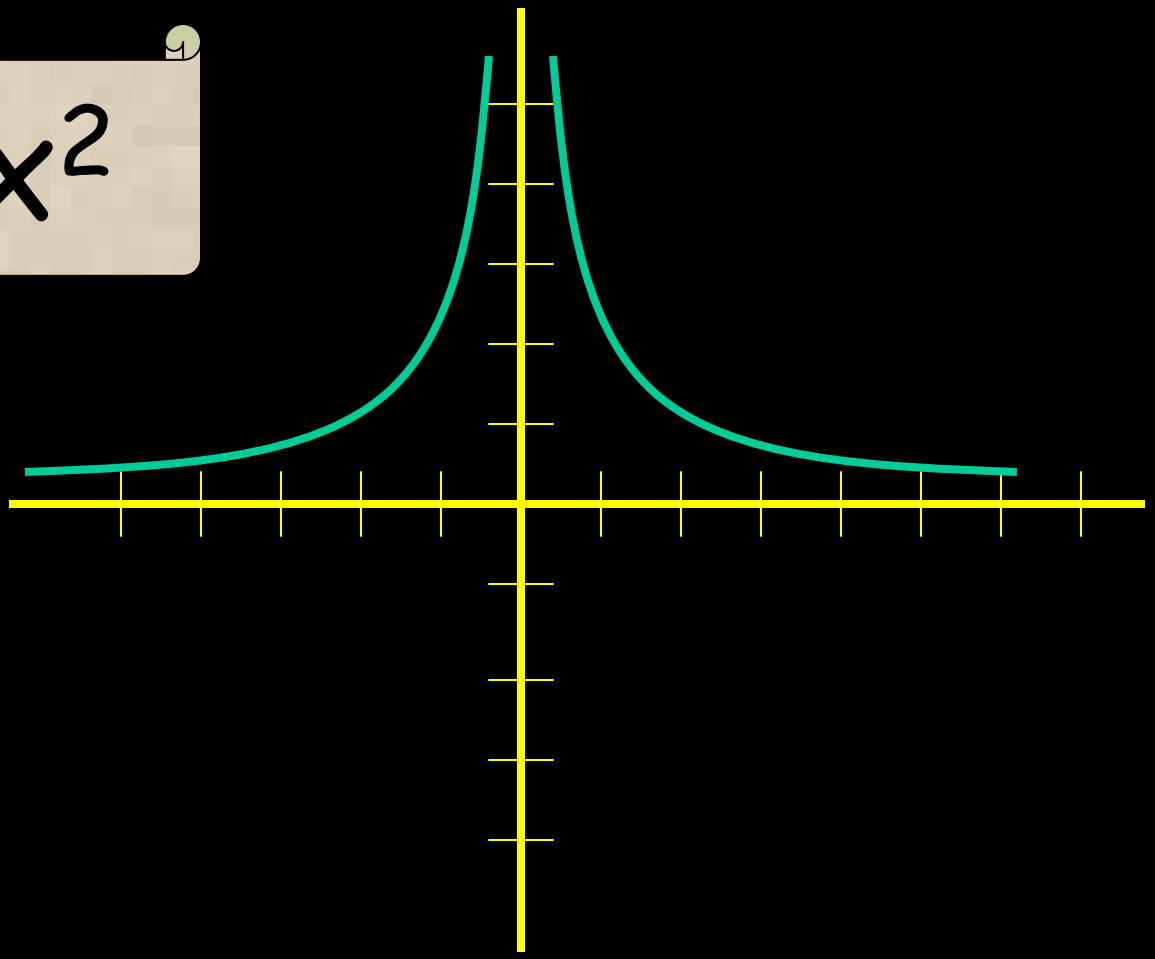
Basic Graphs

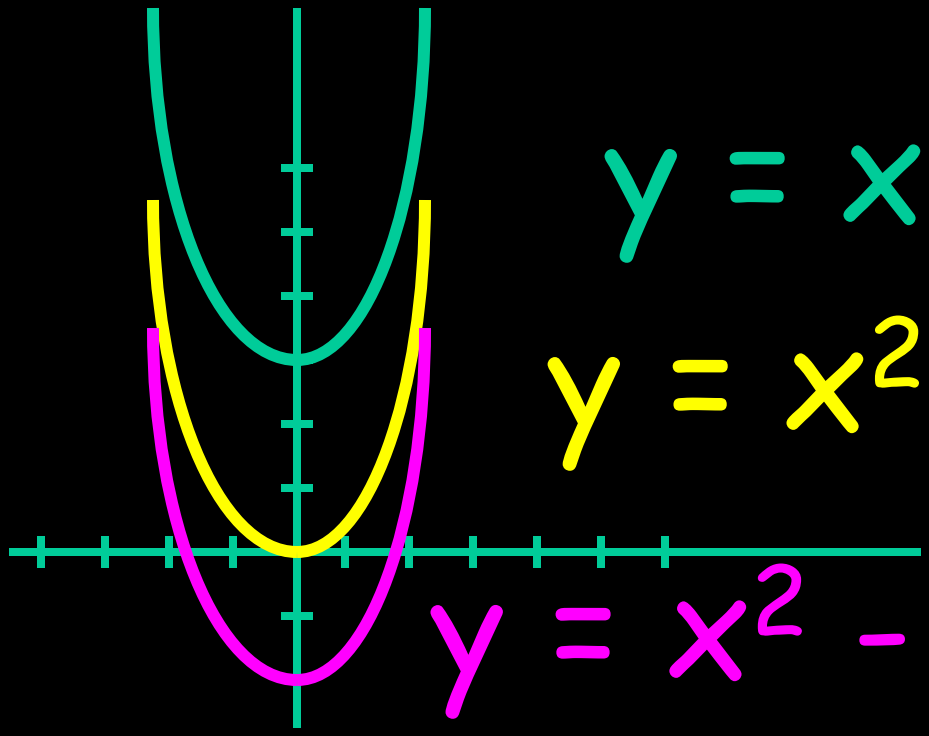
$y = 1/x$



Basic Graphs

$y = 1/x^2$

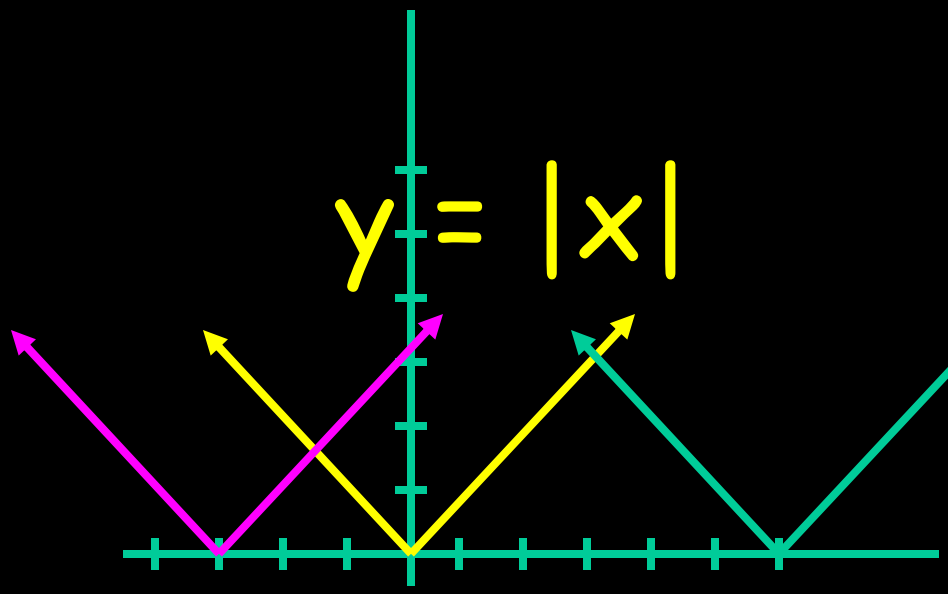




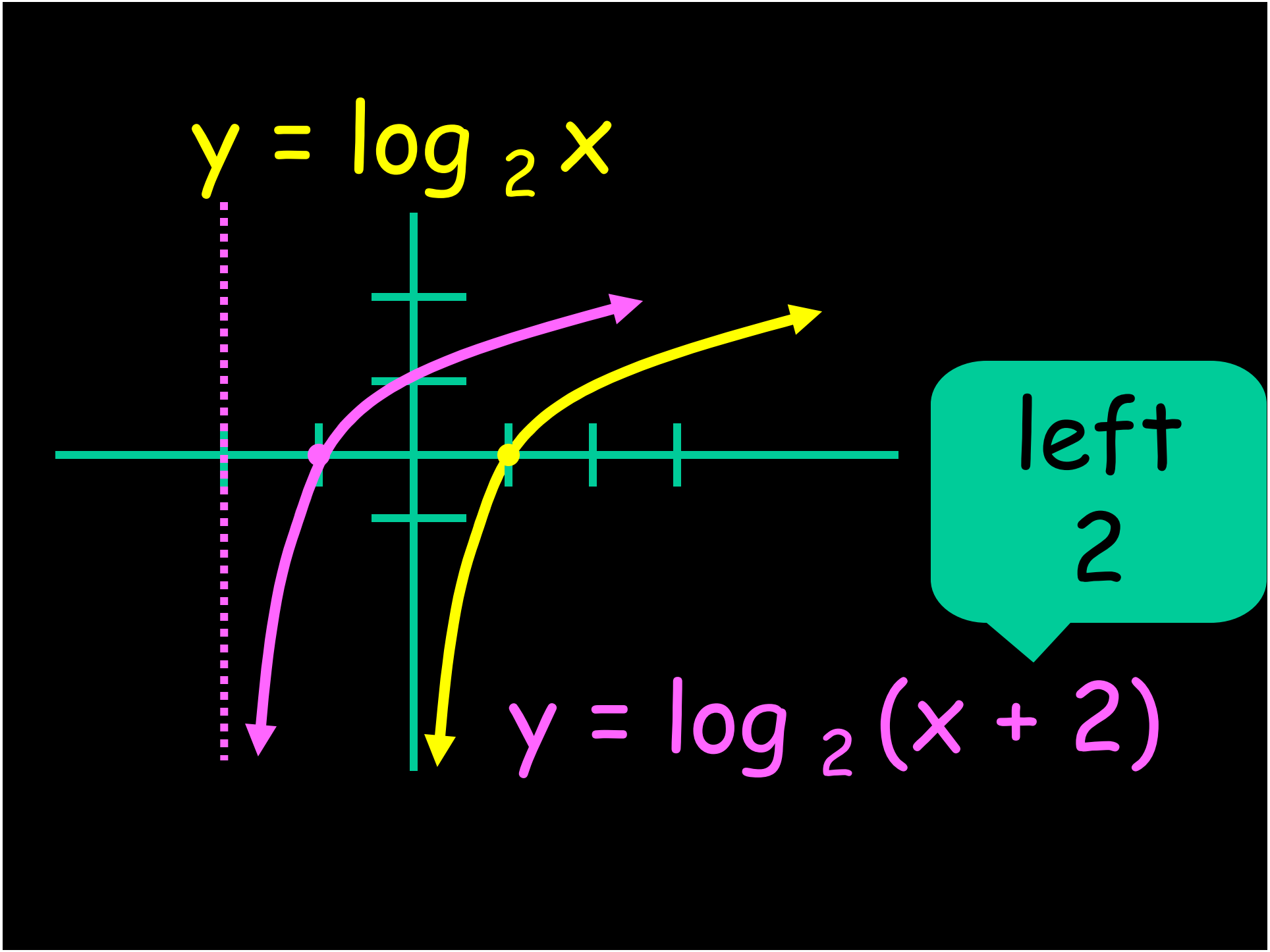
$y = x^2 + 3$

$y = x^2$

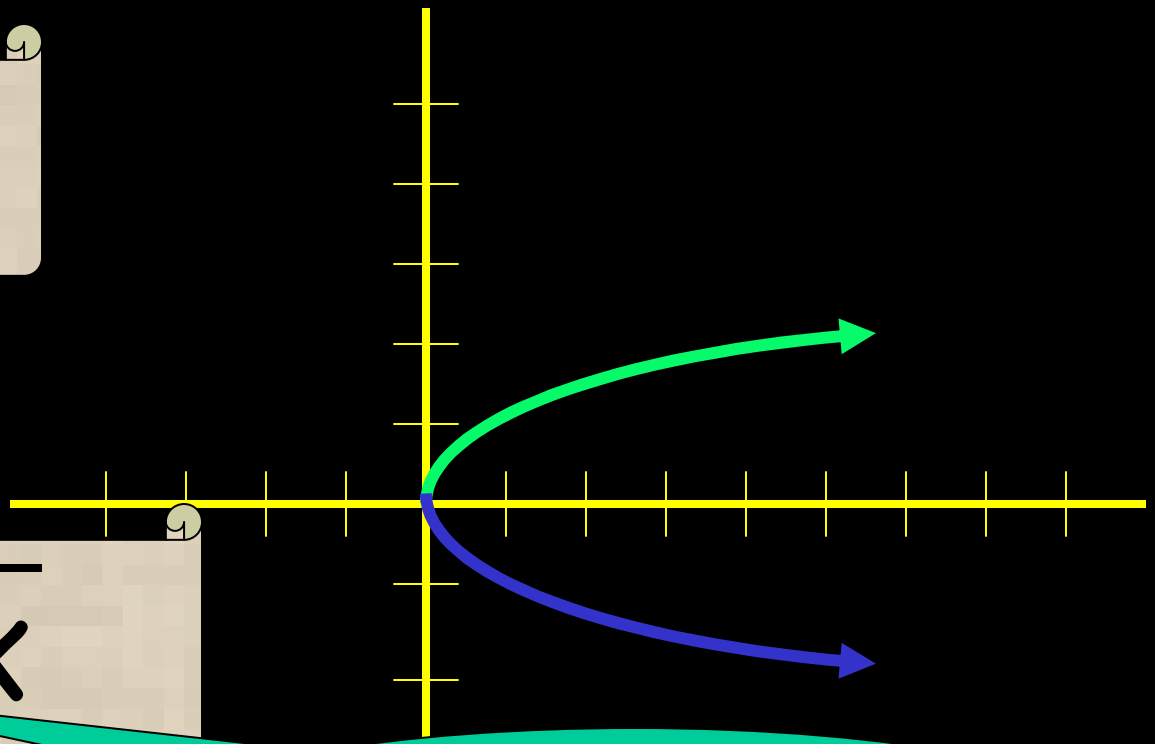
$y = x^2 - 2$



+ move left
- move right

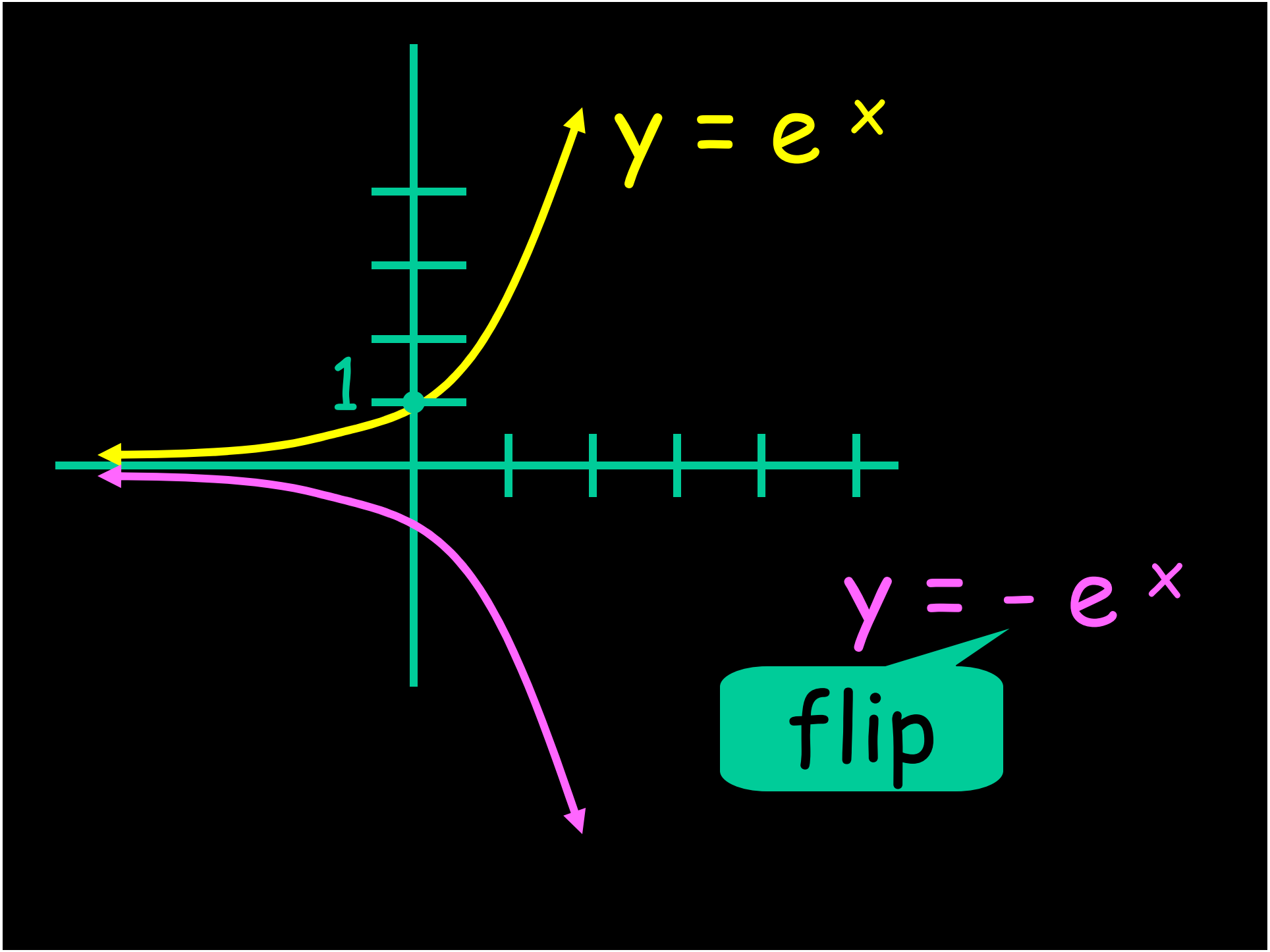

y = log 2 x
left
2
y = log 2 (x + 2)

$y = \sqrt{x}$

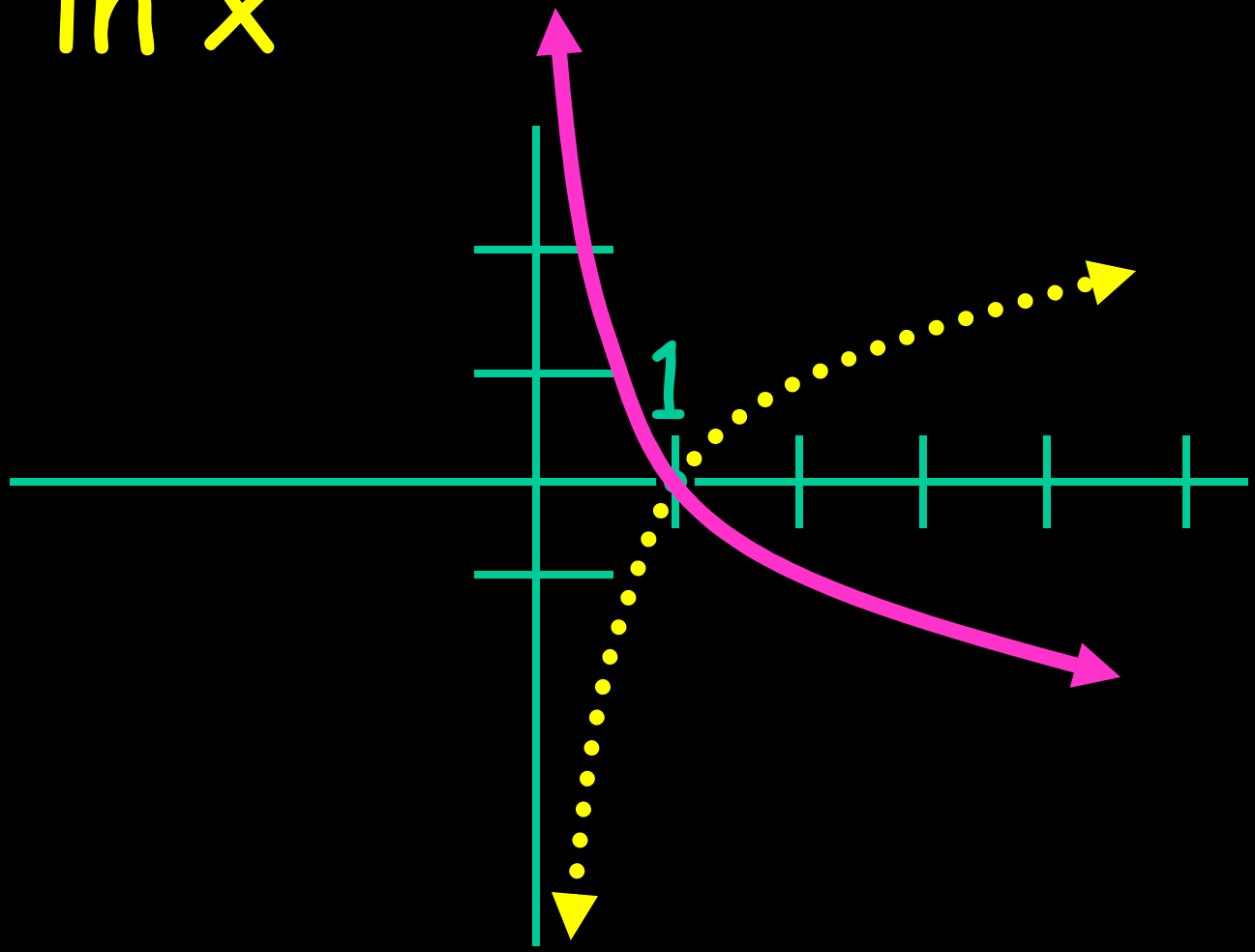


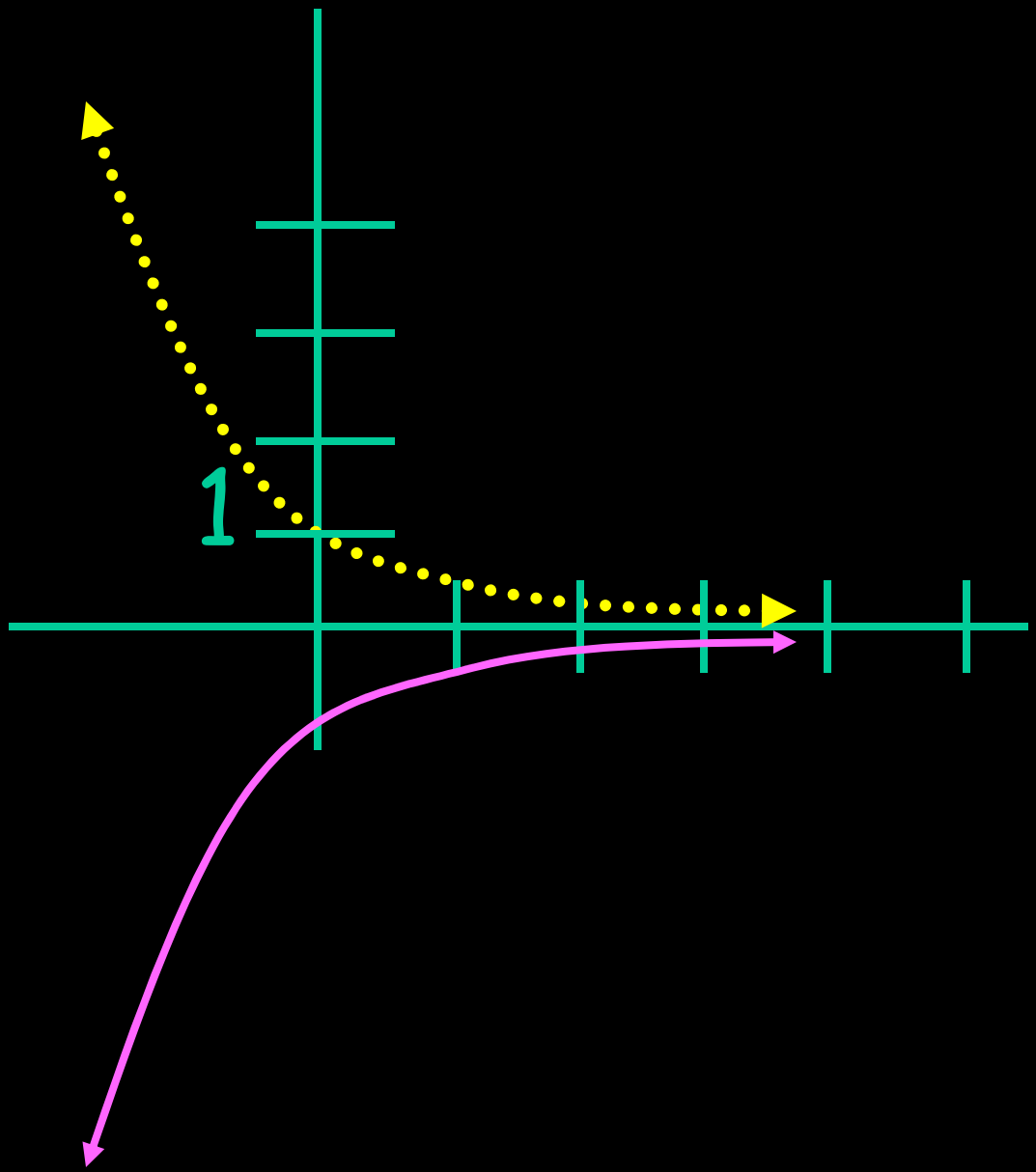
$y = -\sqrt{x}$

Flip about x


y = e x
1
y = - e x
flip

$y = -\ln x$





$$y = -e^{-x}$$

Given the graph of
$y = f(x)$,

To graph $y = f(x) \pm a$,

Move the graph of
$y = f(x)$ up/down a units

Given the graph of
$y = f(x)$,

To graph $y = f(x \pm a)$,

Move the graph of
$y = f(x)$ left/rt a units
+ is left, - is right!

Given the graph of
$y = f(x)$,

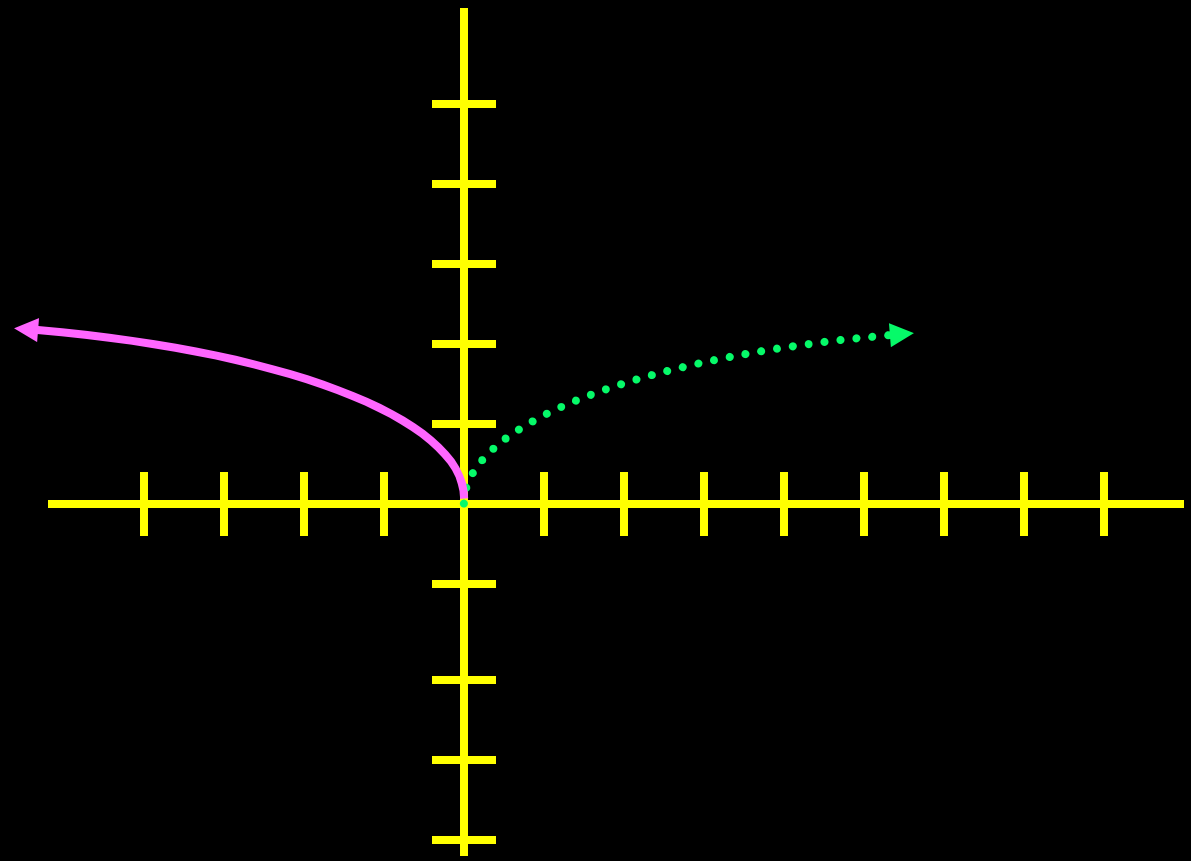
To graph $y = -f(x)$,

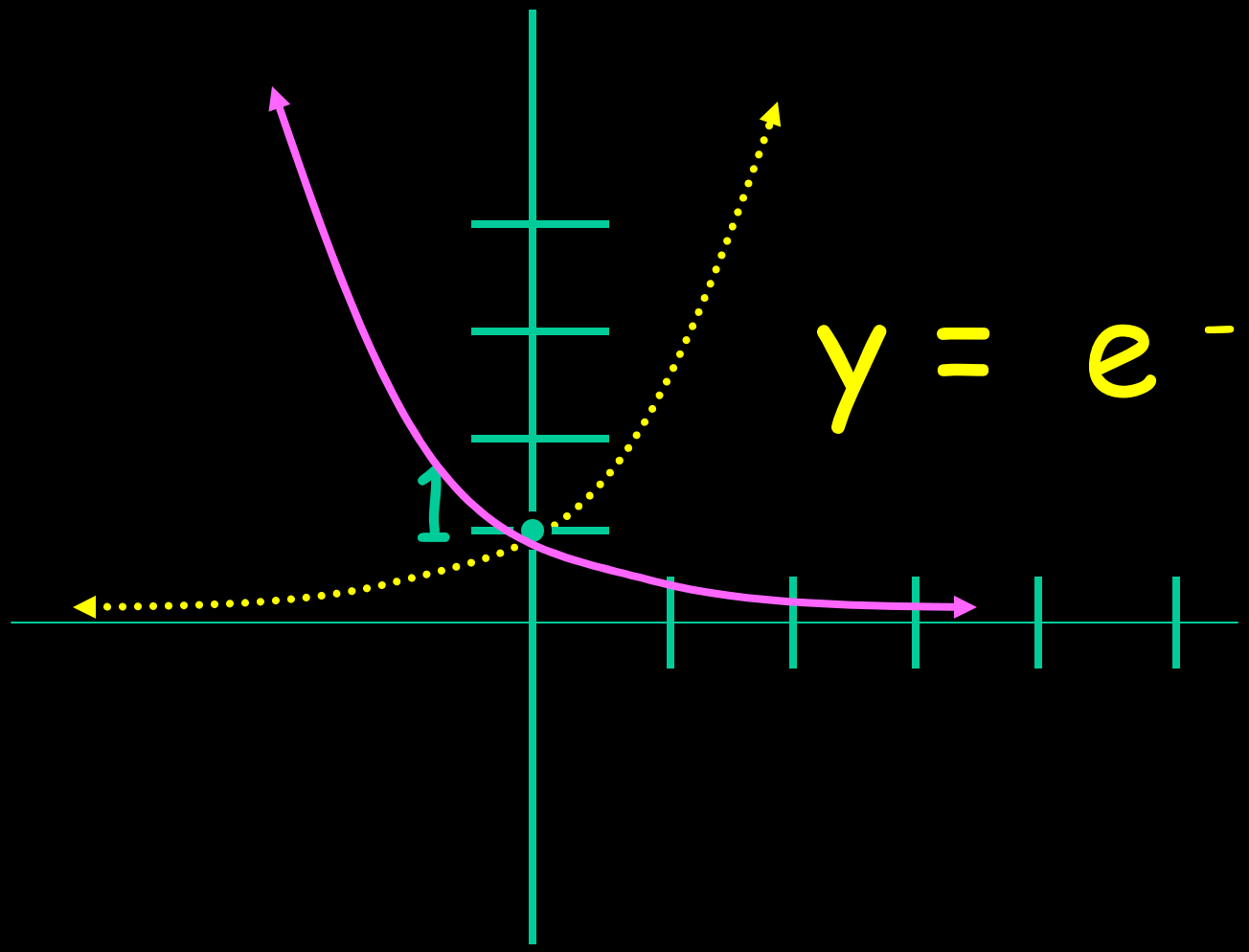
flip the graph of
$y = f(x)$ with respect to
the x-axis.

The graph of $y = -f(x)$ is flipped about the x-axis.

The graph of $y = f(-x)$ is flipped about the y-axis.

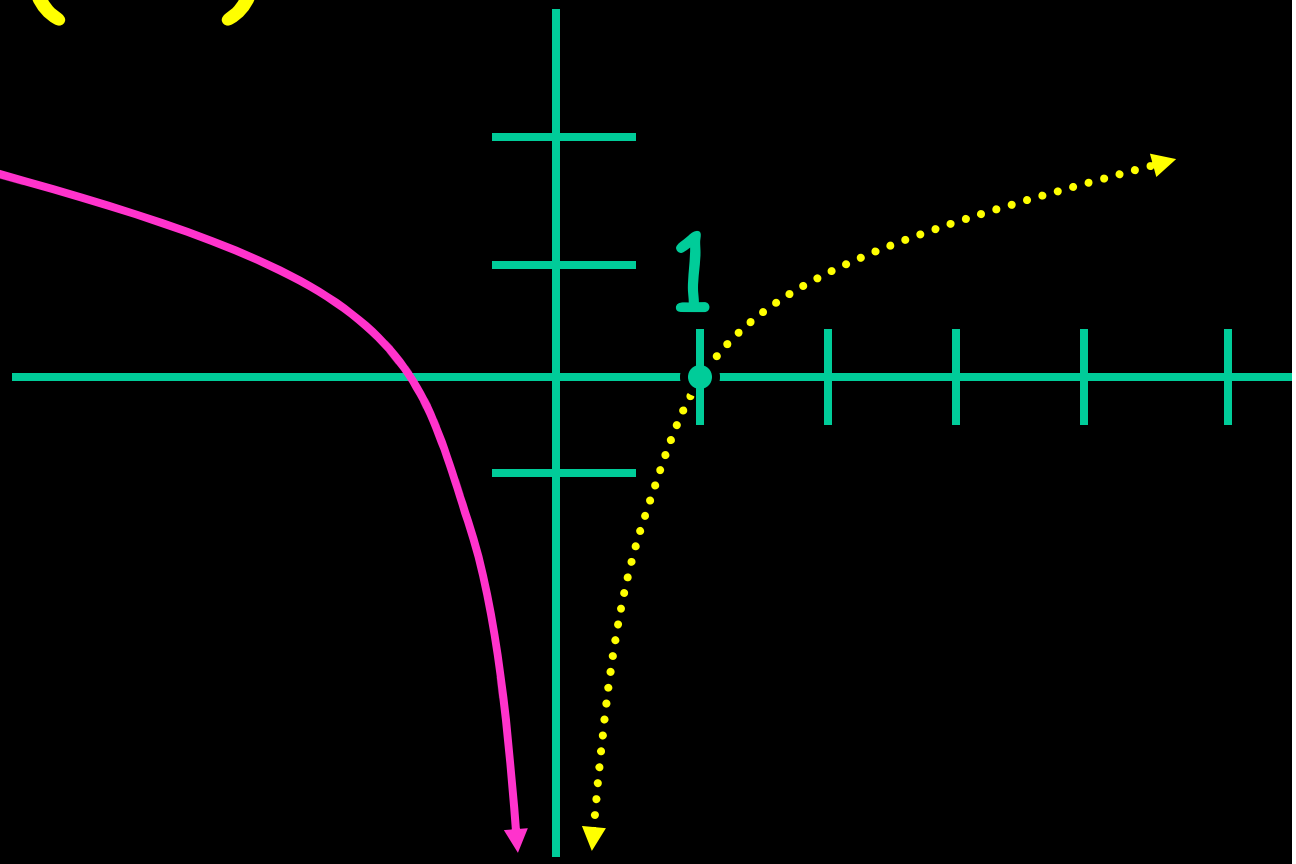
$$y = \sqrt{-x}$$





$y = e^{-x}$

$y = \ln(-x)$



Given the graph of $y = f(x)$,

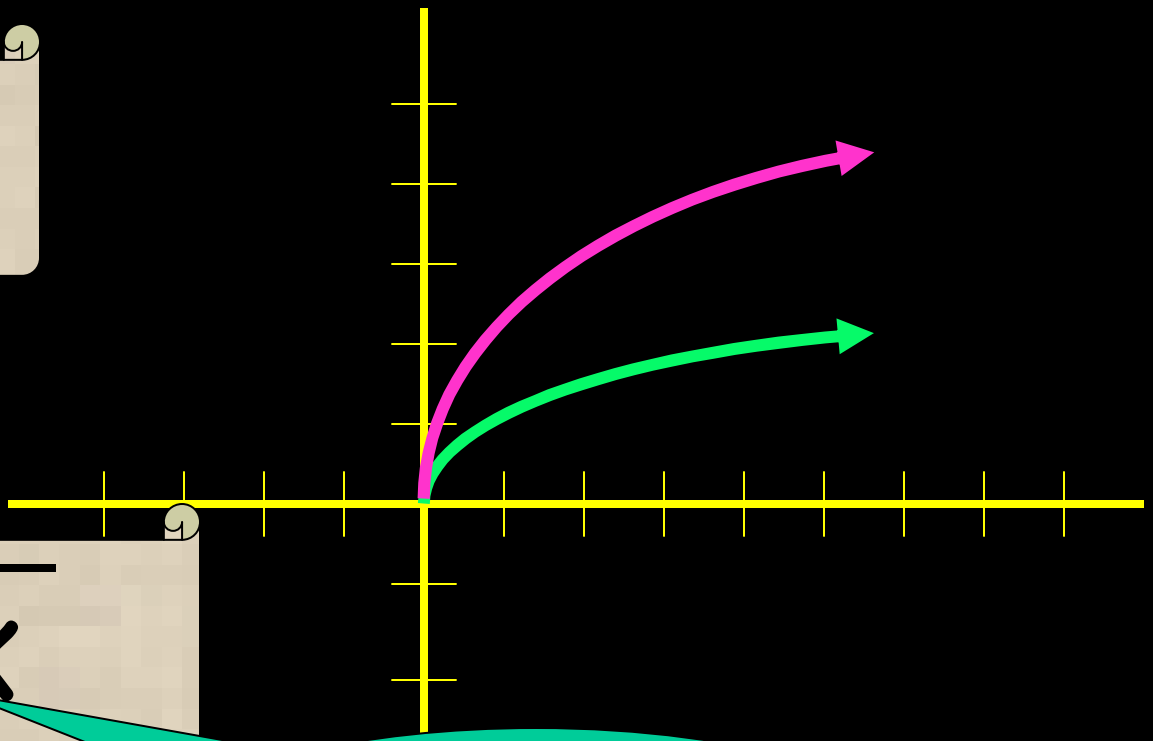
To graph $y = kf(x)$,

Multiply all the y values of $y = f(x)$ by k. Steeper if $k > 1$. Flatter if $k < 1$

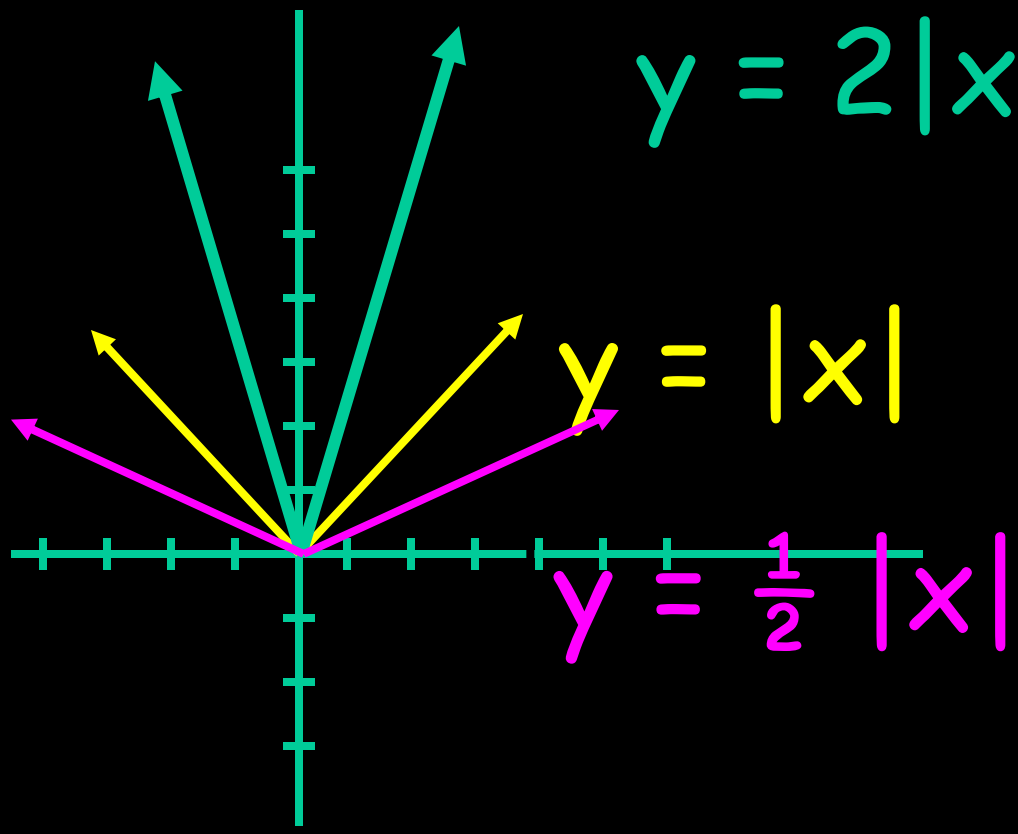
Square Root Graphs

$y = \sqrt{x}$

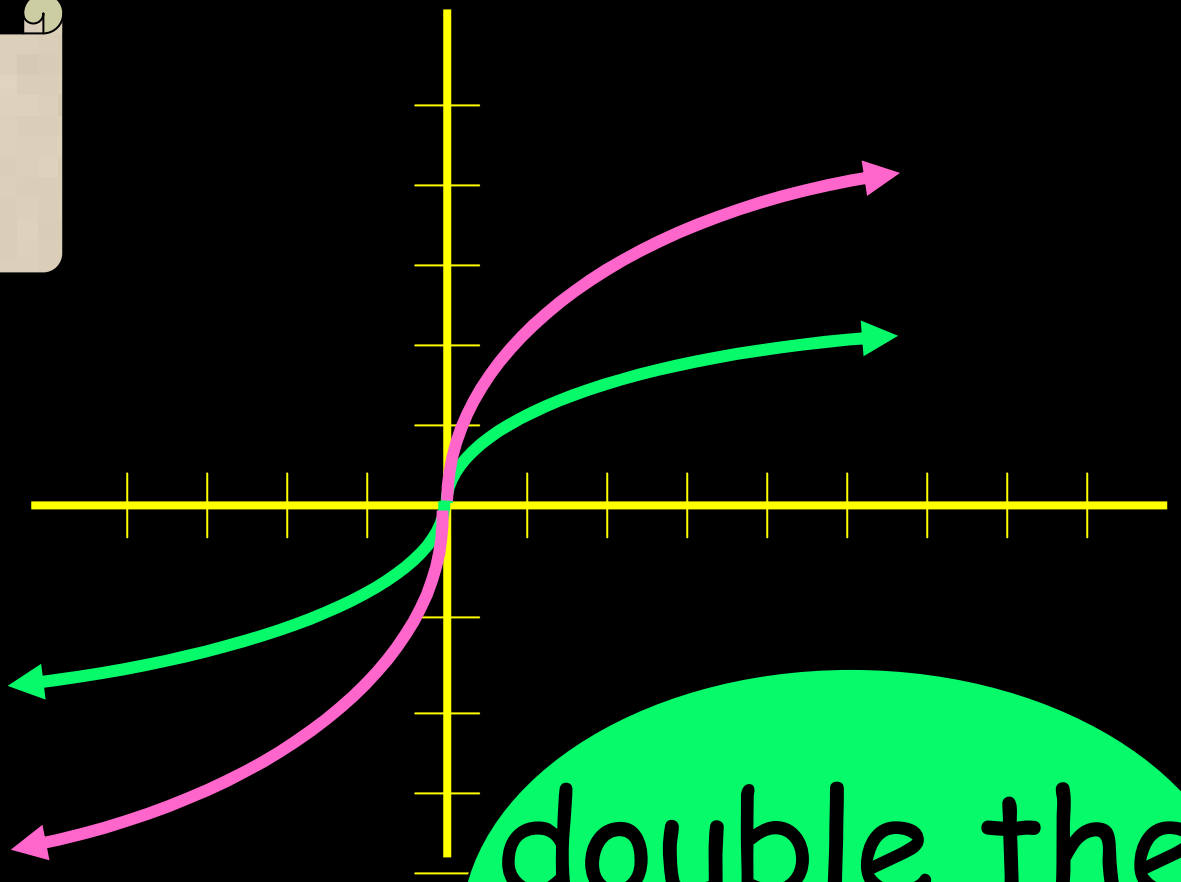
$y = 2\sqrt{x}$



steeper


y = 2|x|
y = |x|
y = ½ |x|

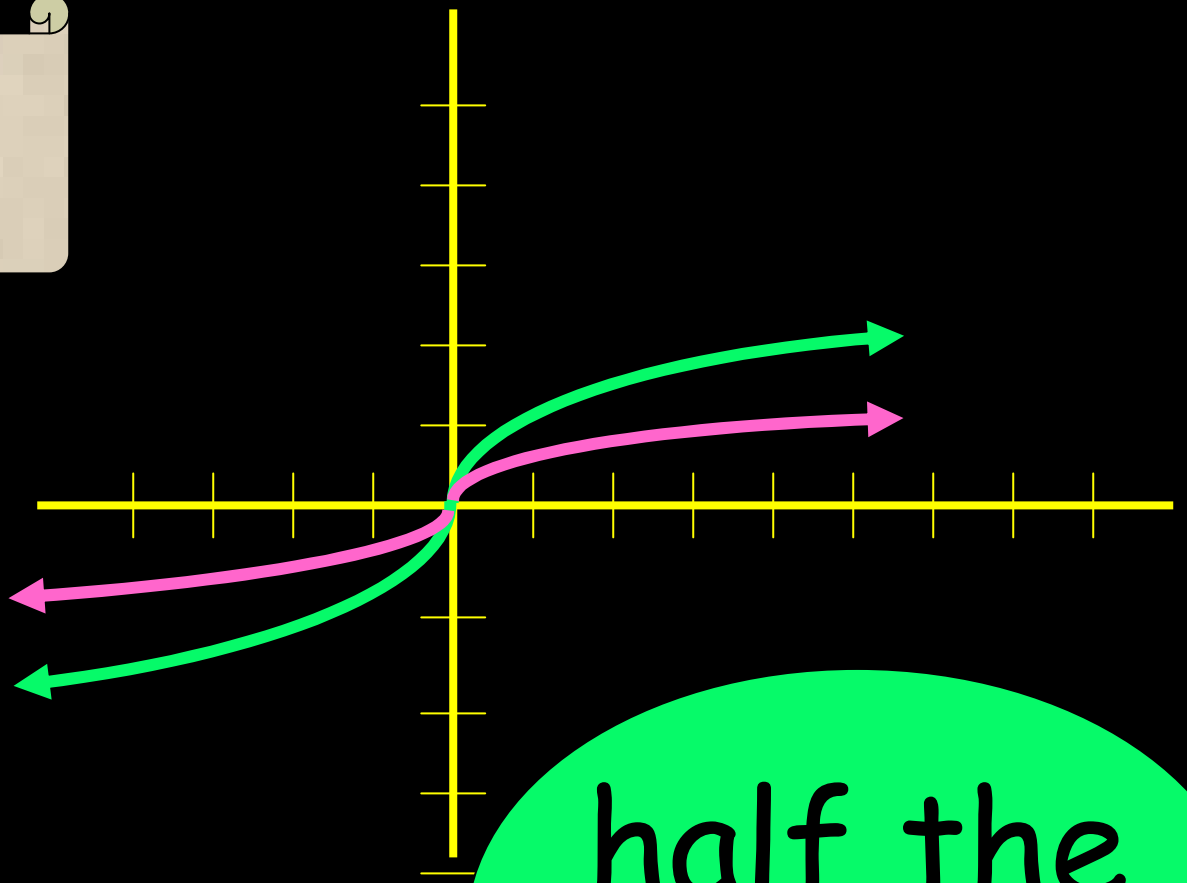
$y = \sqrt[3]{x}$



$y = 2\sqrt[3]{x}$

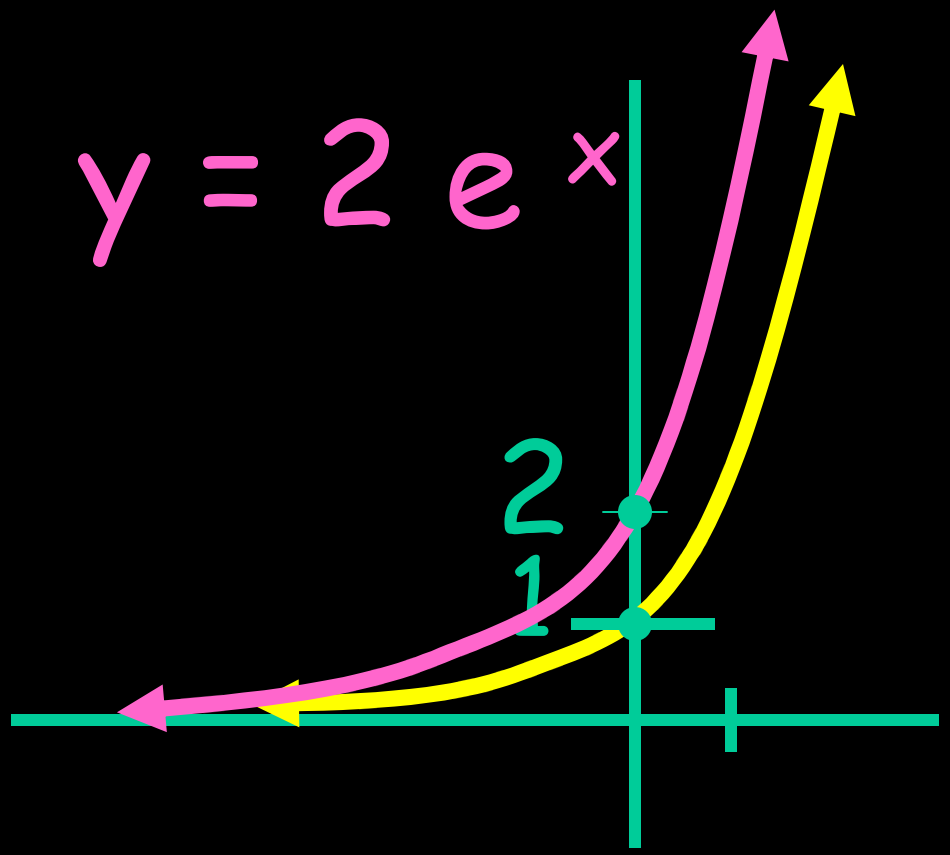
double the y-values

$y = \sqrt[3]{x}$



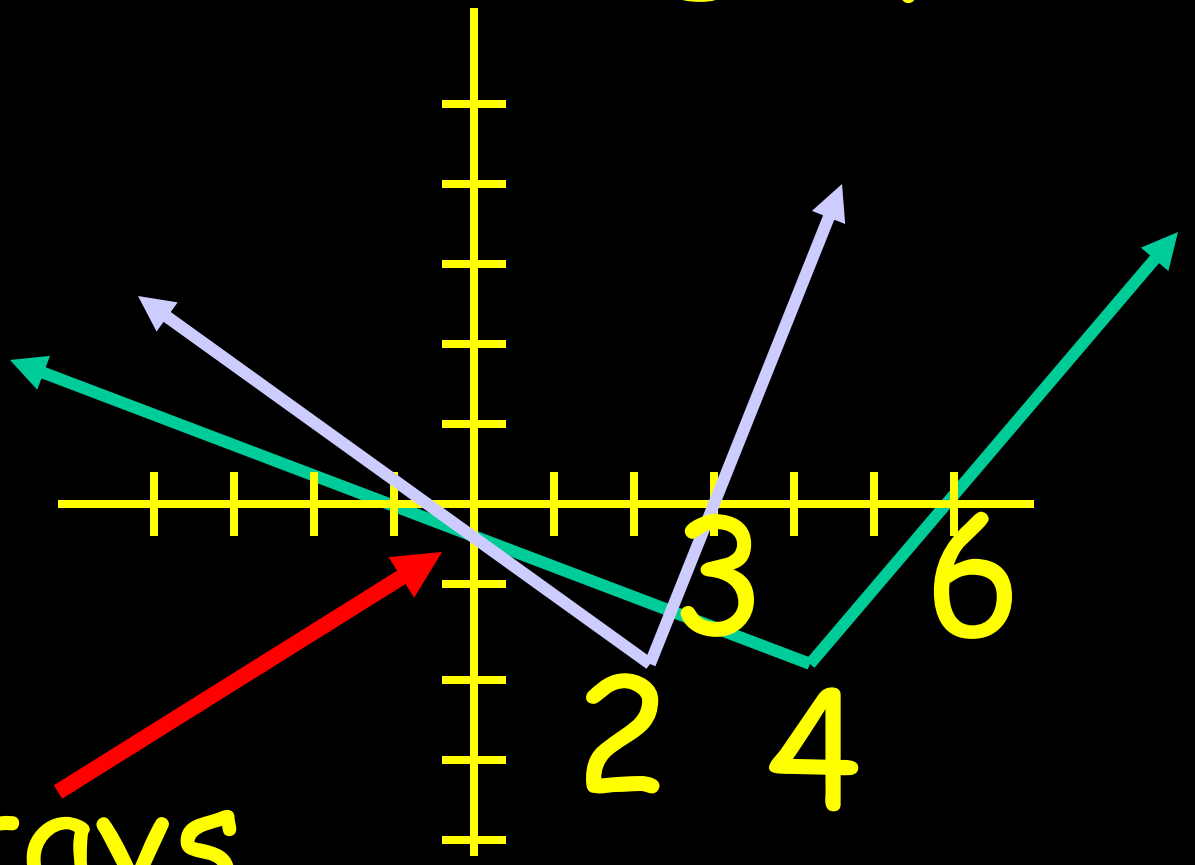
$y = \frac{1}{2}\sqrt[3]{x}$

half the y-values



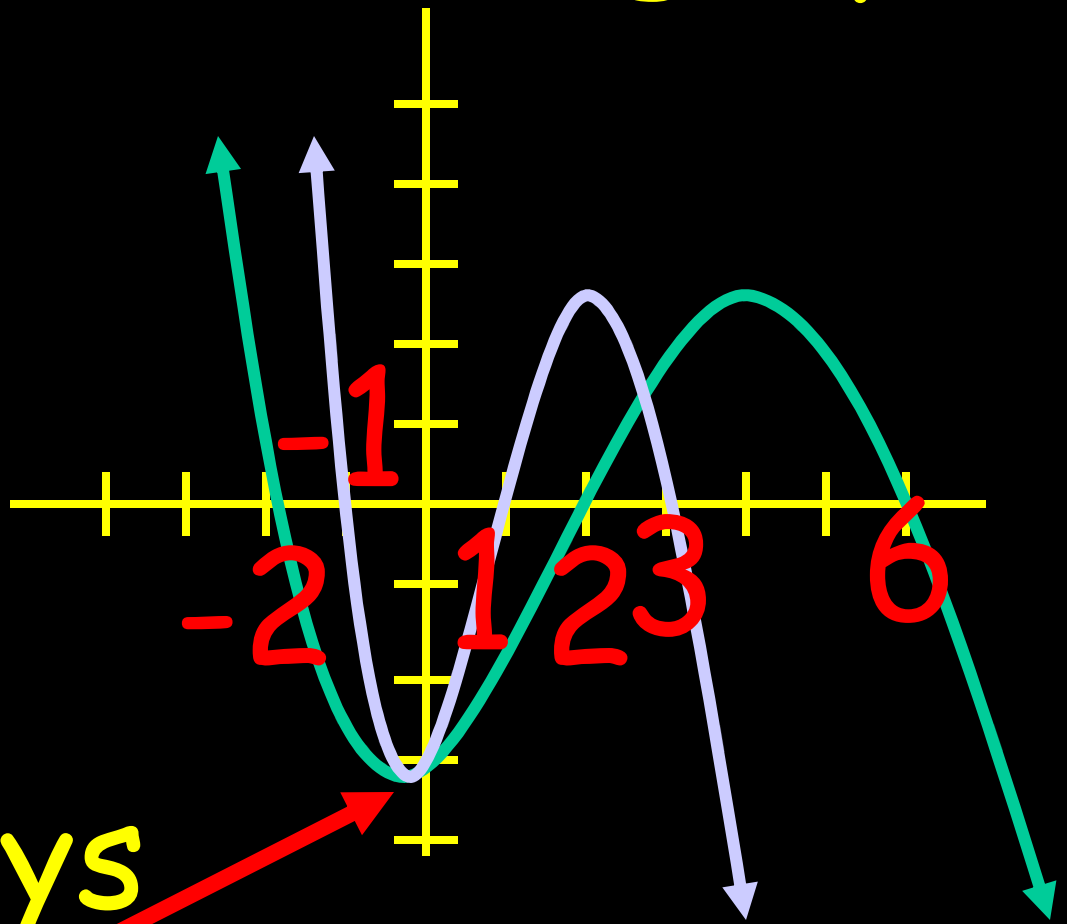
y = e x

Given f(x) graph f(2x)

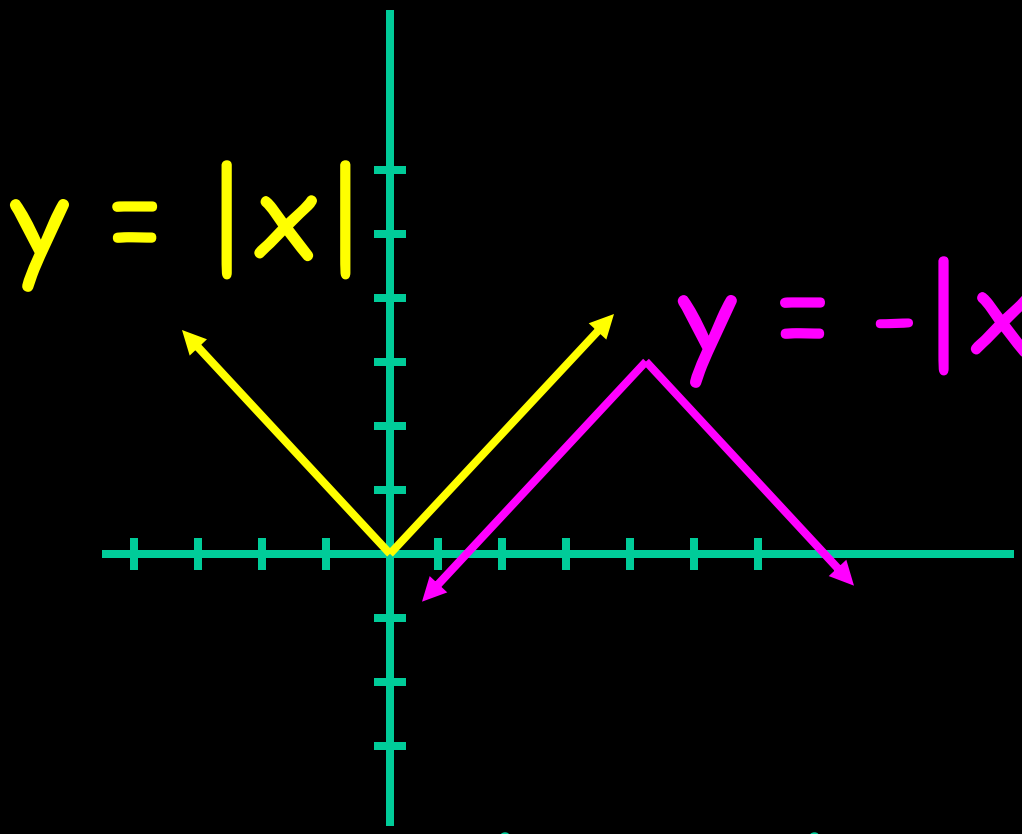


stays same

Given f(x) graph f(2x)



stays same

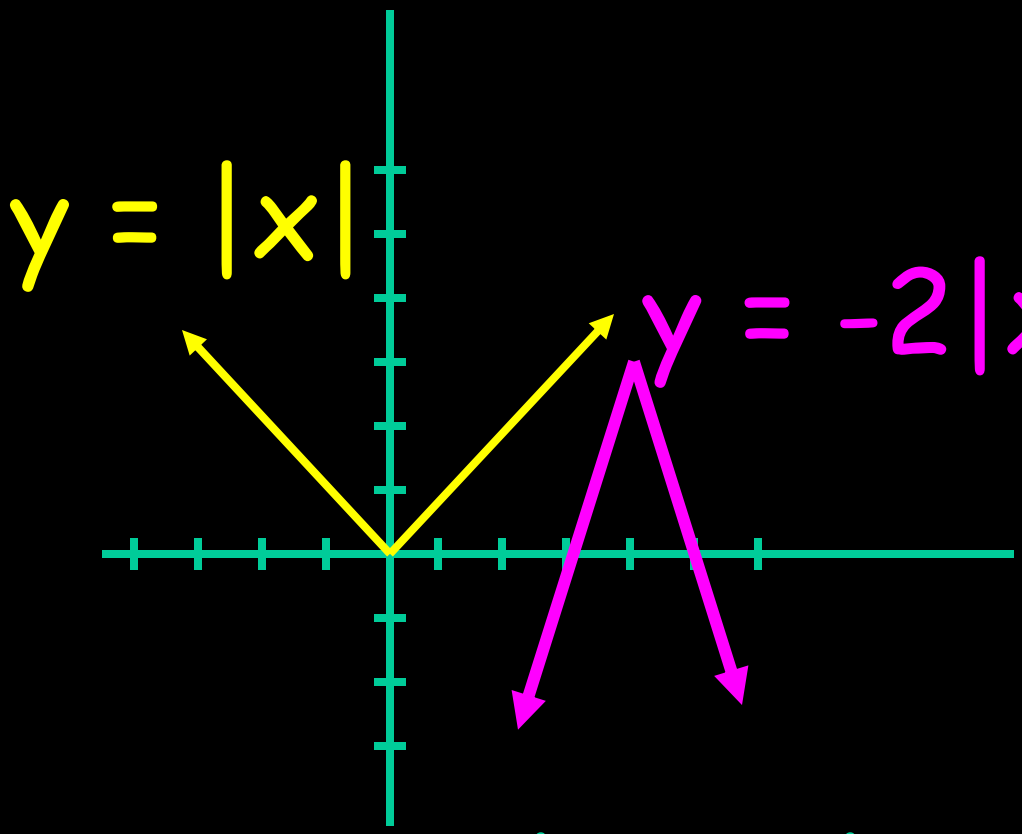


y = -|x - 4| + 3

flip

right 4

up 3



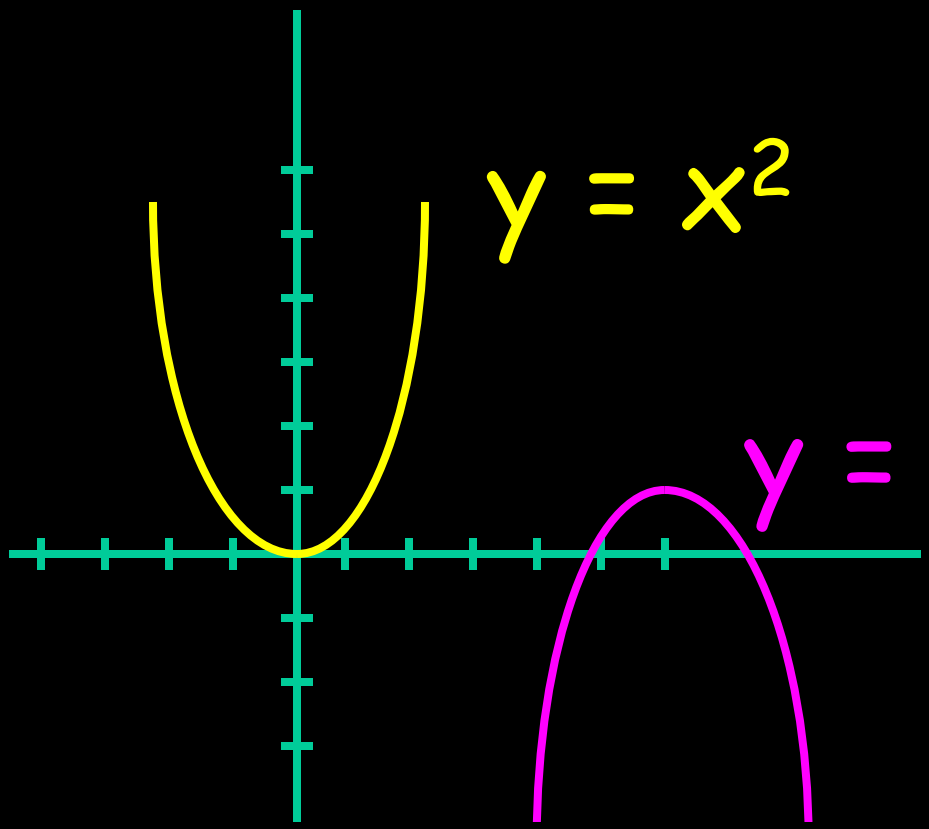
y = -2|x - 4| + 3

flip

steeper

right 4

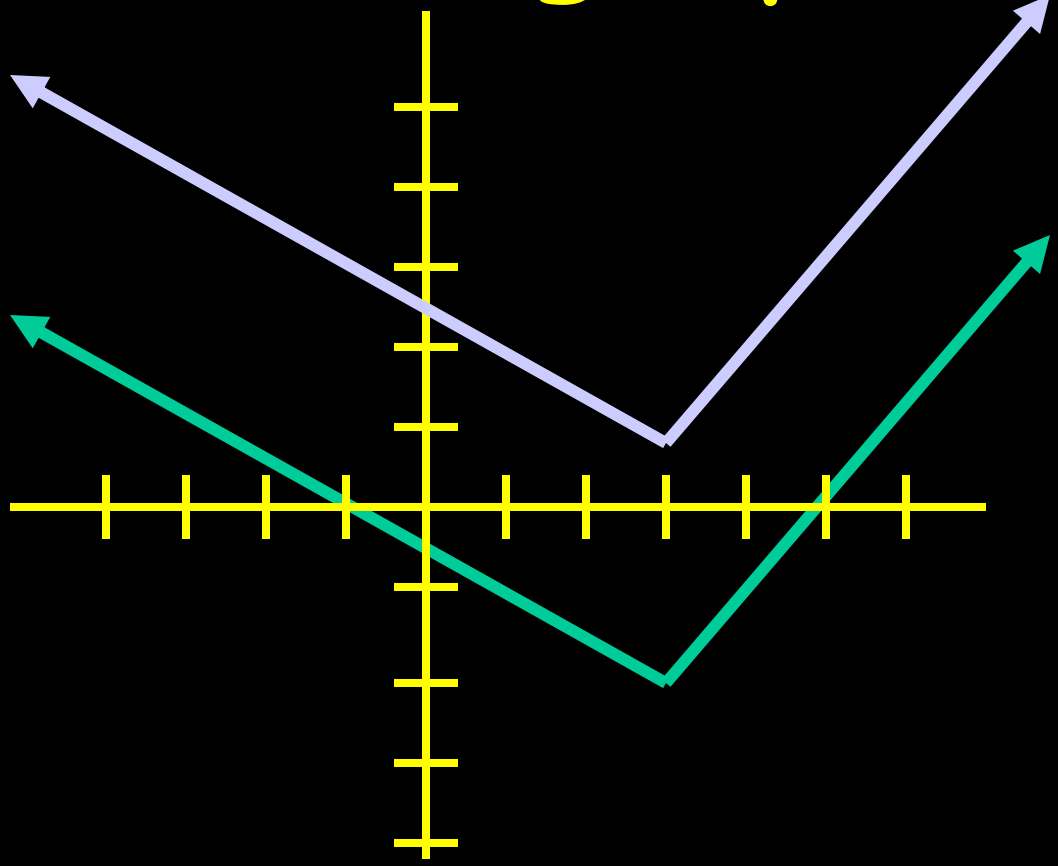
up 3



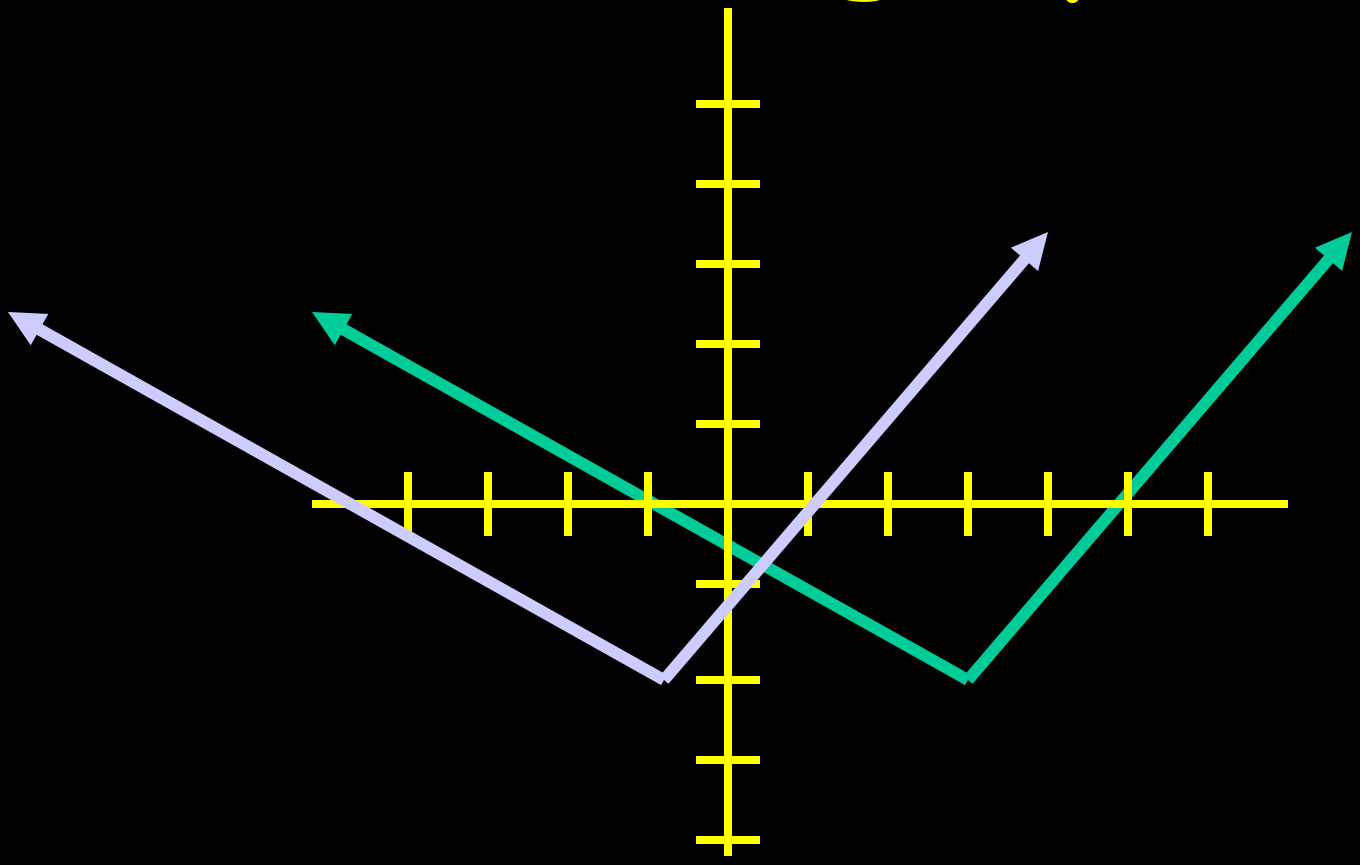
$y = -(x - 6)^2 + 1$

flip right 6 up 1

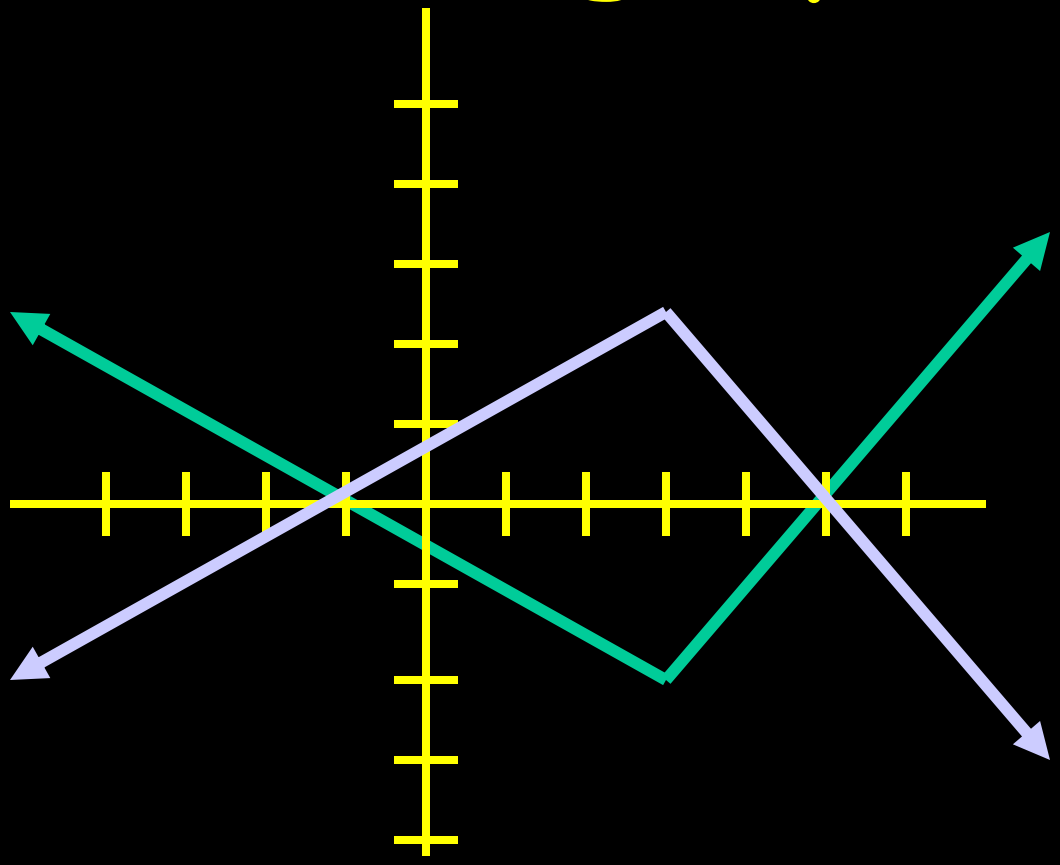
Given f(x), graph f(x) + k



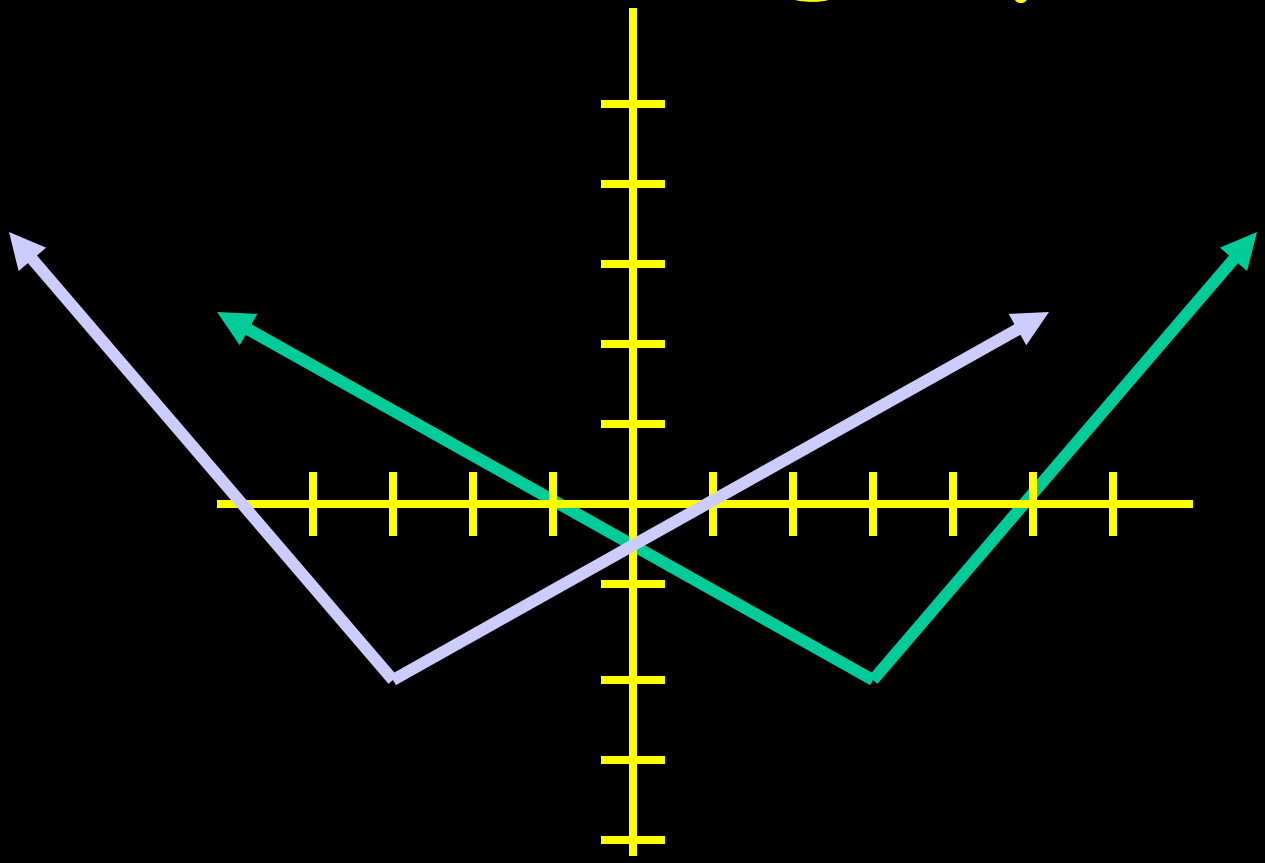
Given f(x), graph f(x + k)



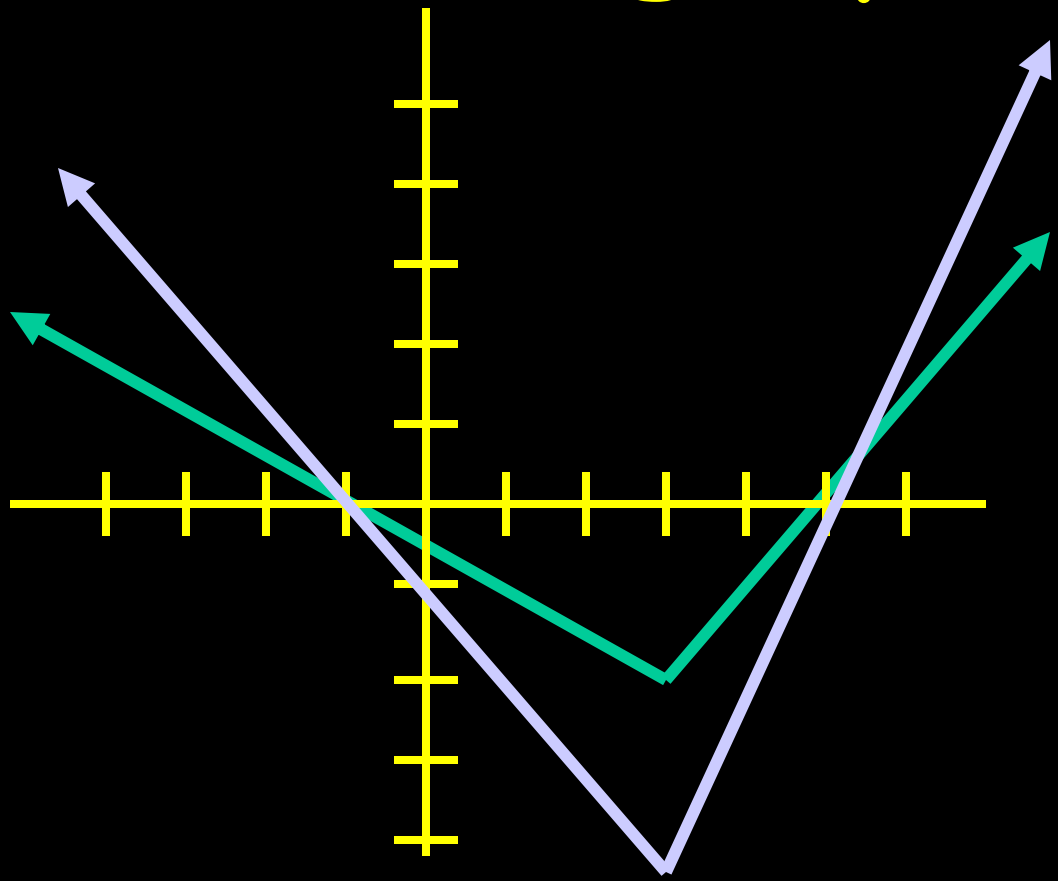
Given f(x), graph - f(x)



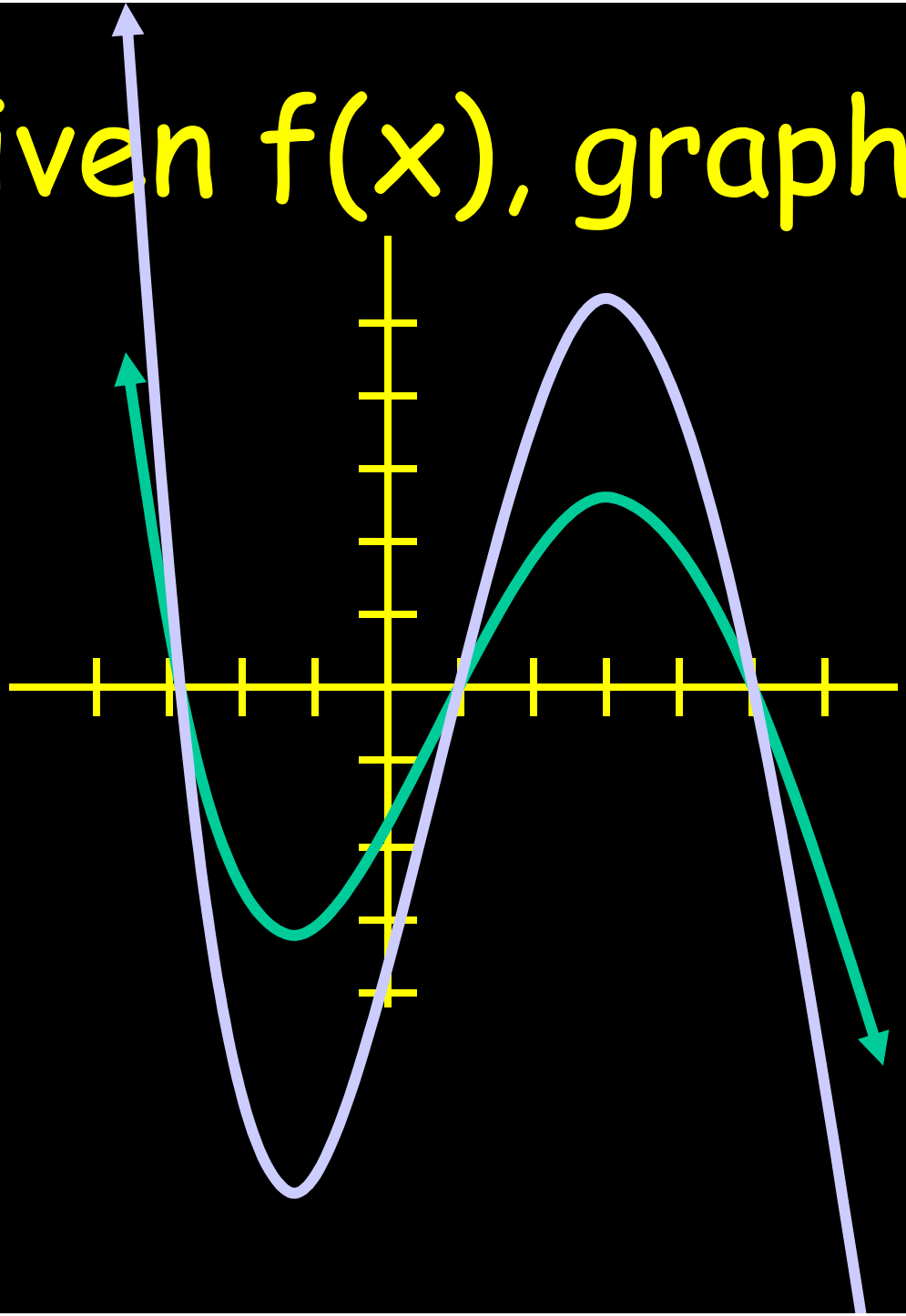
Given f(x), graph f(- x)



Given f(x), graph kf(x)



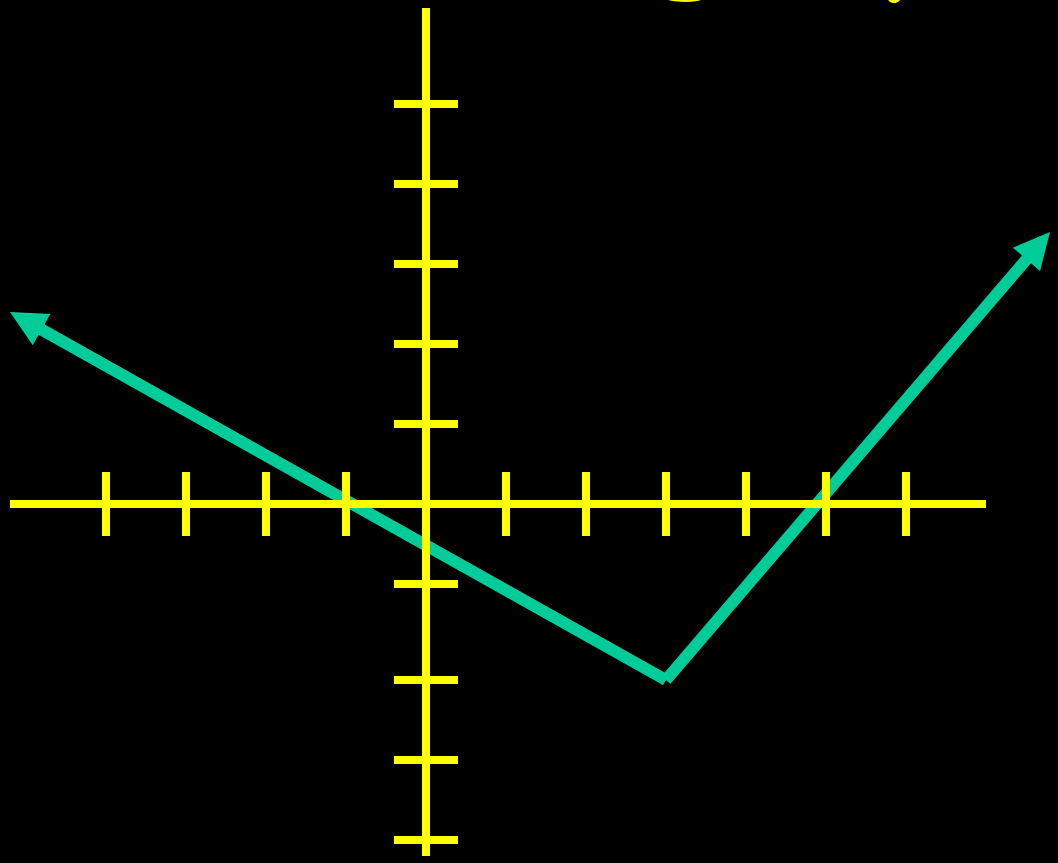
Given f(x), graph kf(x)



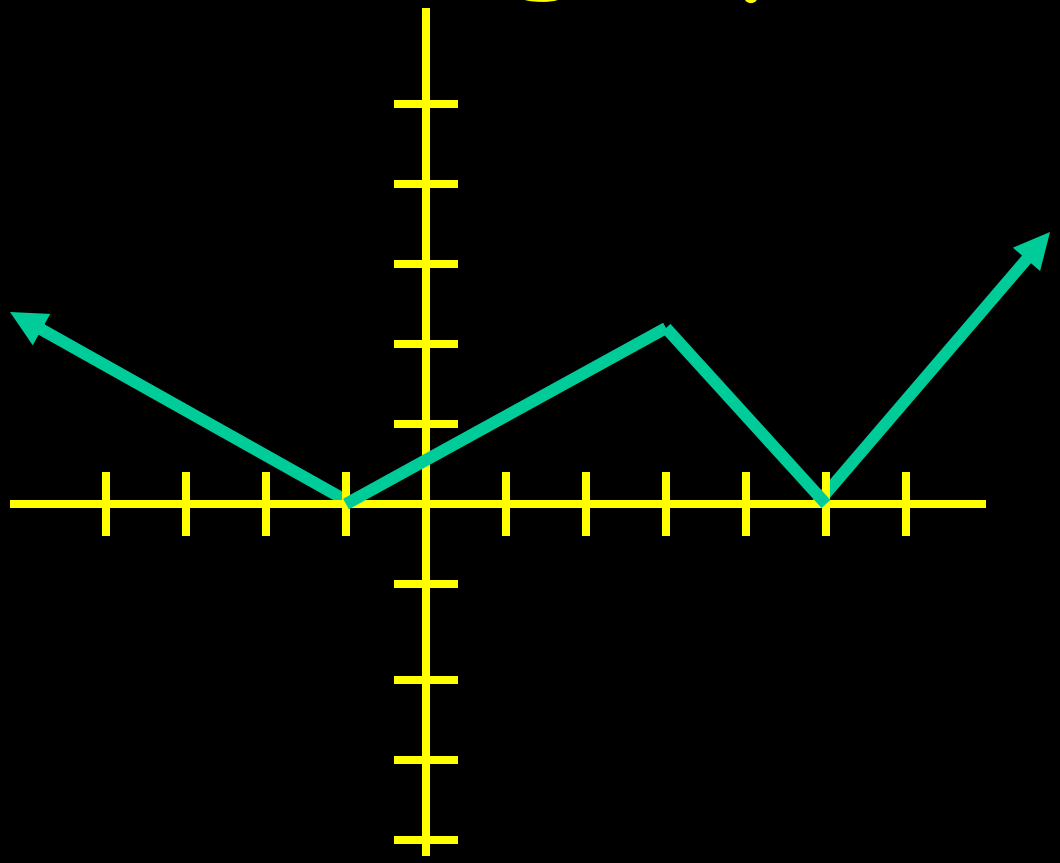
A look at absolute value

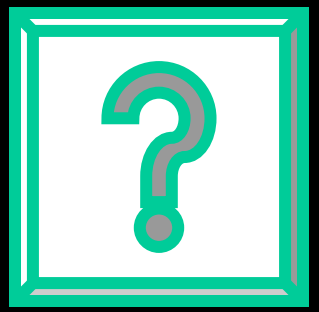
Given the graph of f(x), what does the graph of |f(x)| look like?

If this is the graph of f(x)

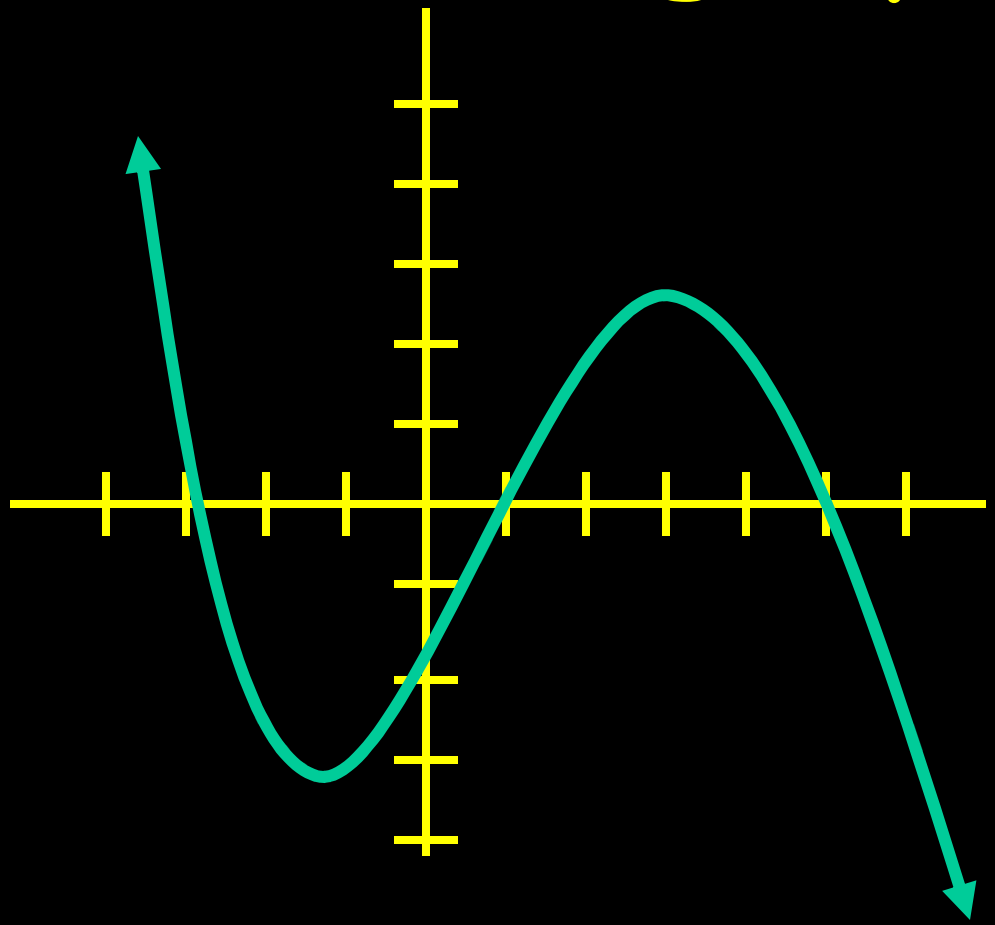


This is the graph of |f(x)|

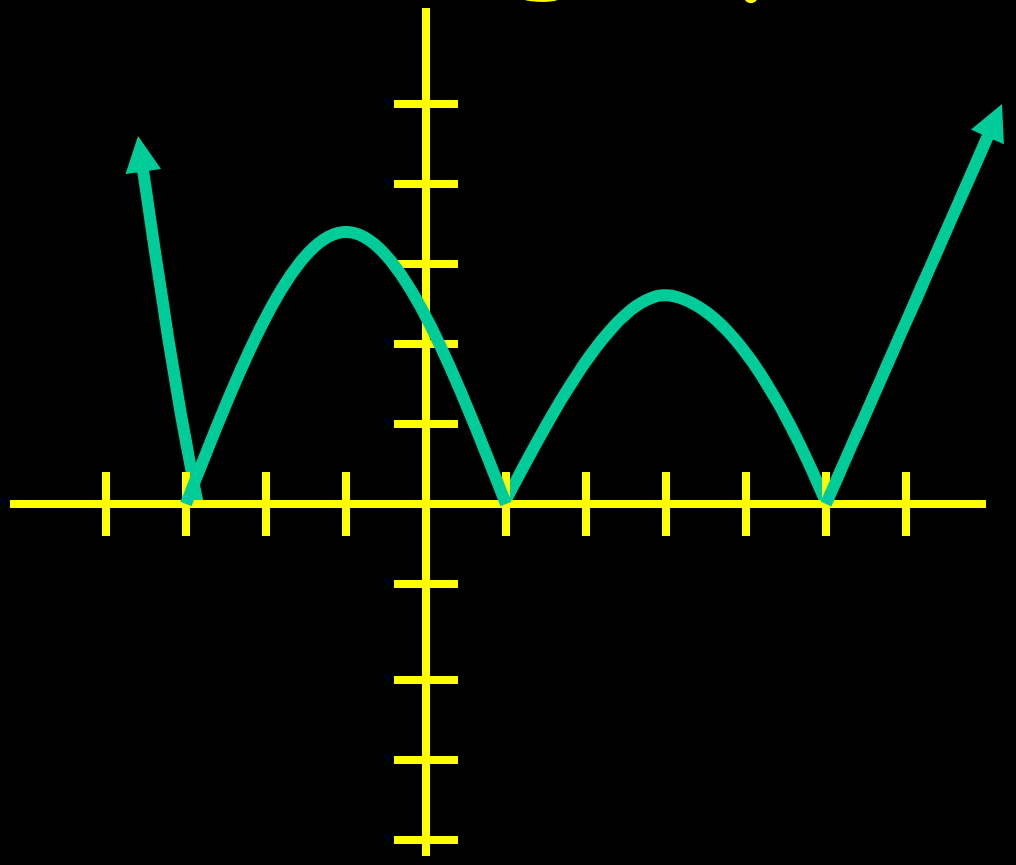




If this is the graph of f(x)



This is the graph of |f(x)|

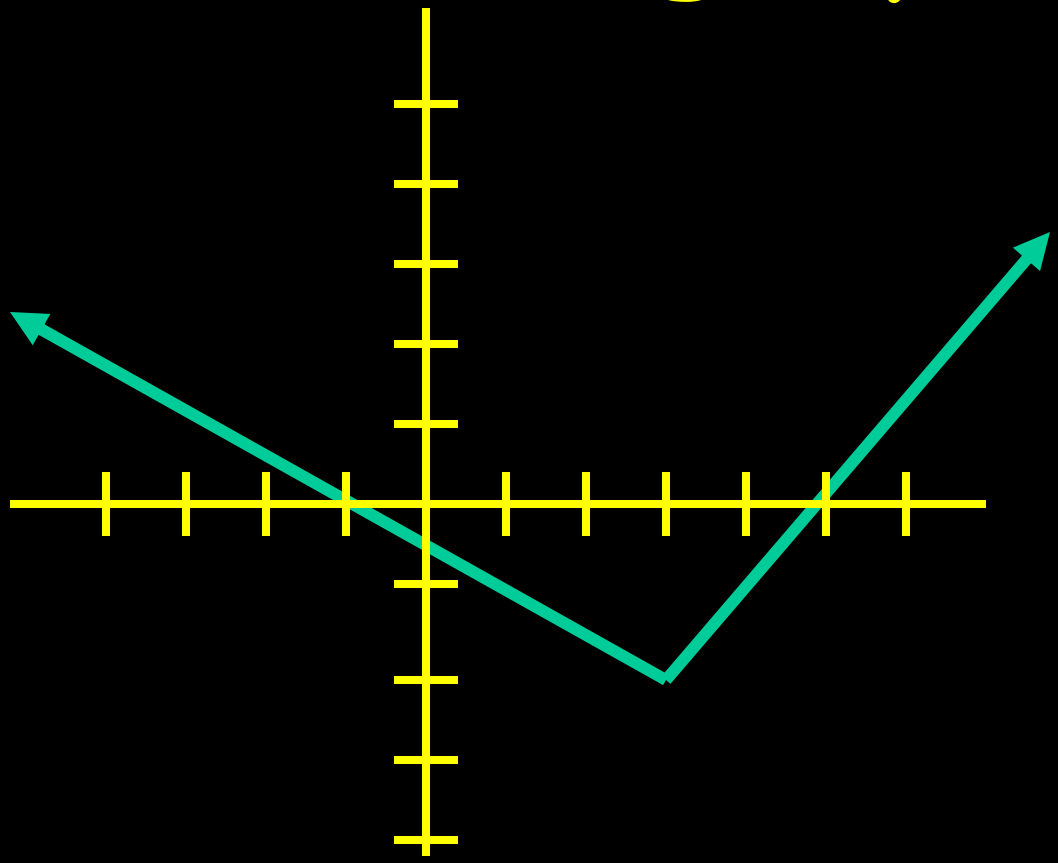


To graph $y = |f(x)|$, erase everything below the x-axis, and draw its mirror image above the x-axis

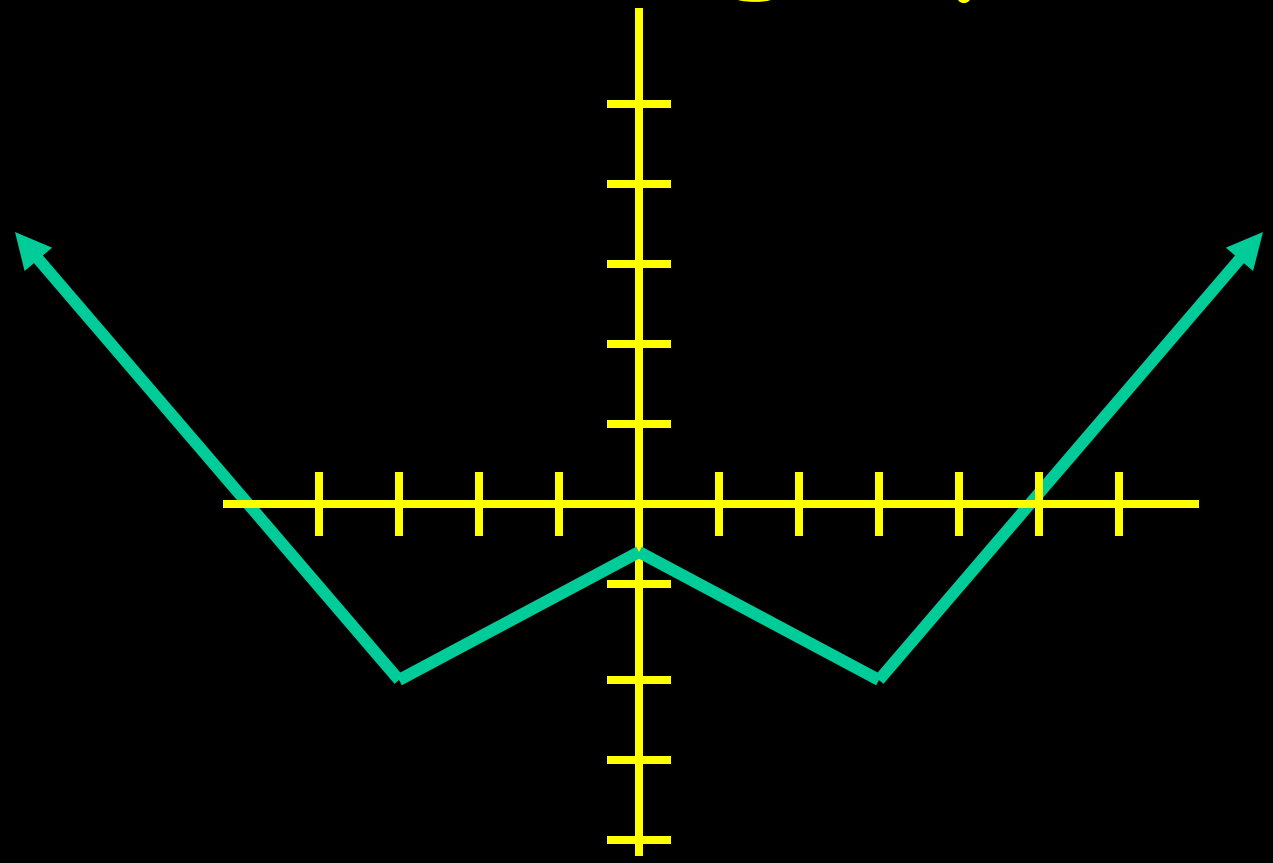
A look at absolute value

Given the graph of f(x), what does the graph of f(|x|) look like?

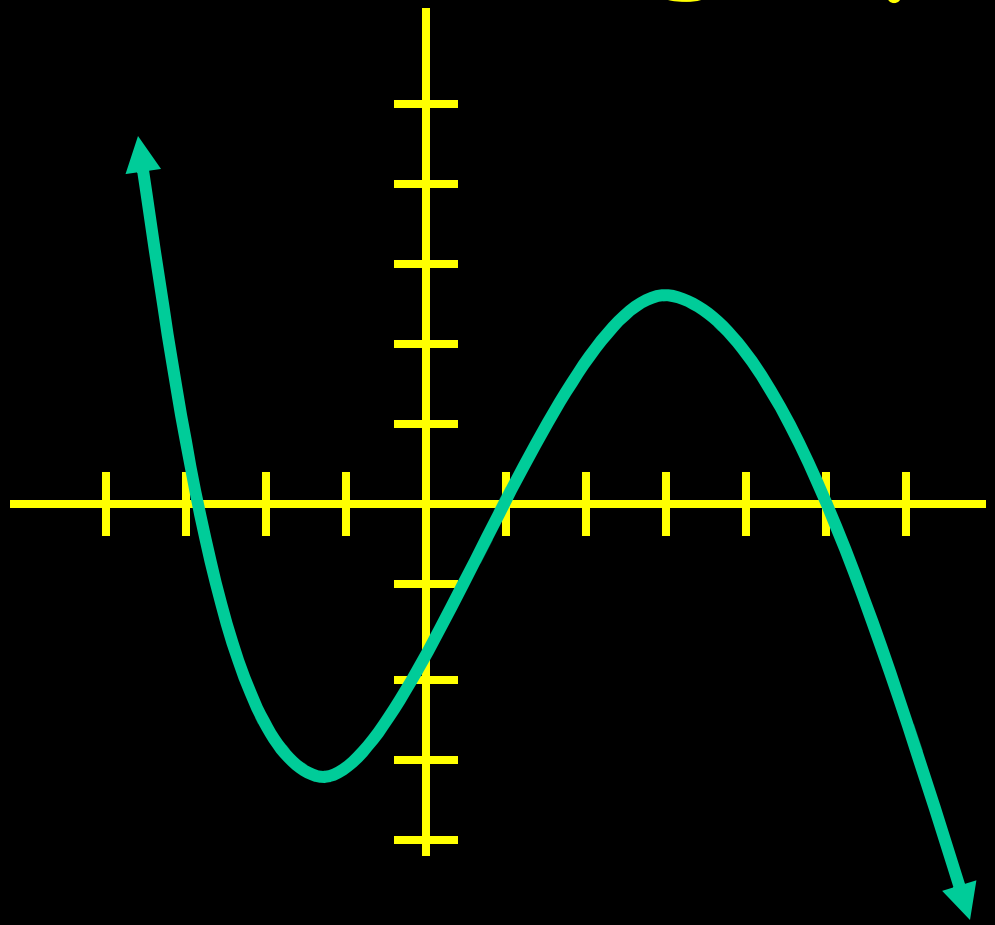
If this is the graph of f(x)



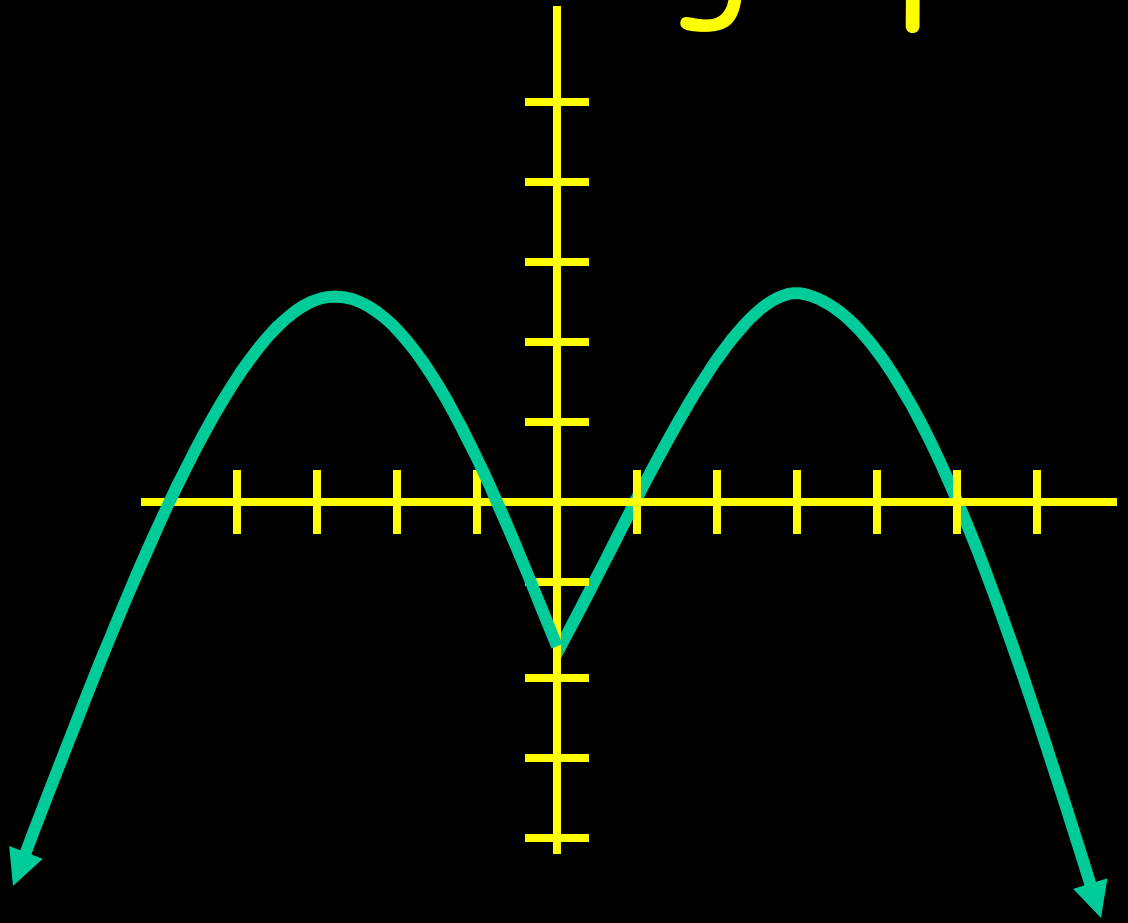
This is the graph of f(|x|)

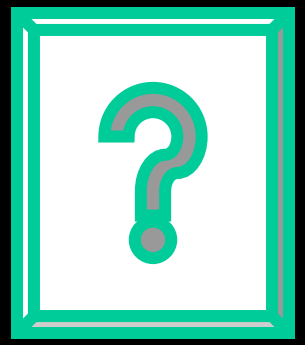


If this is the graph of f(x)



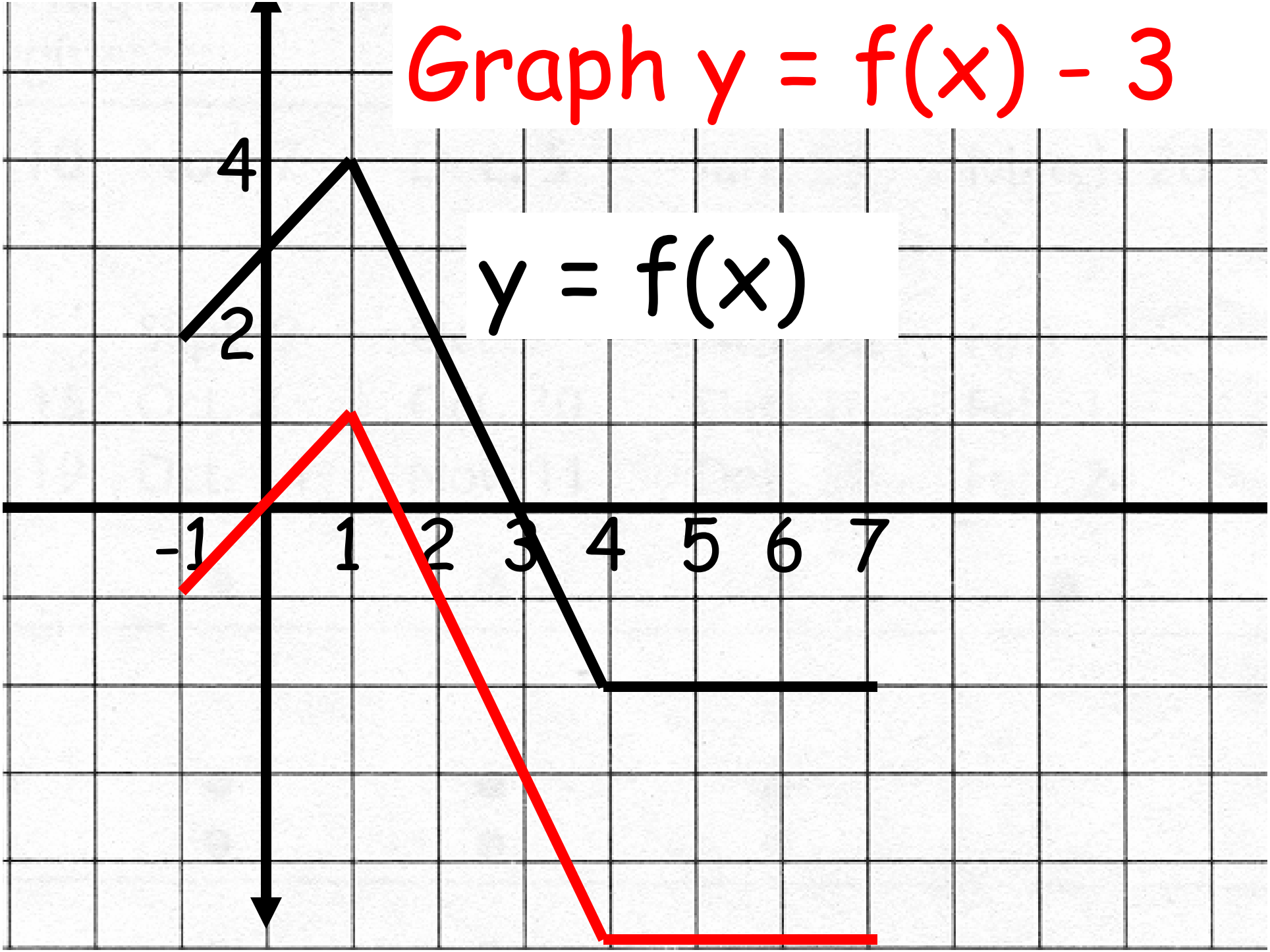
This is the graph of f(|x|)



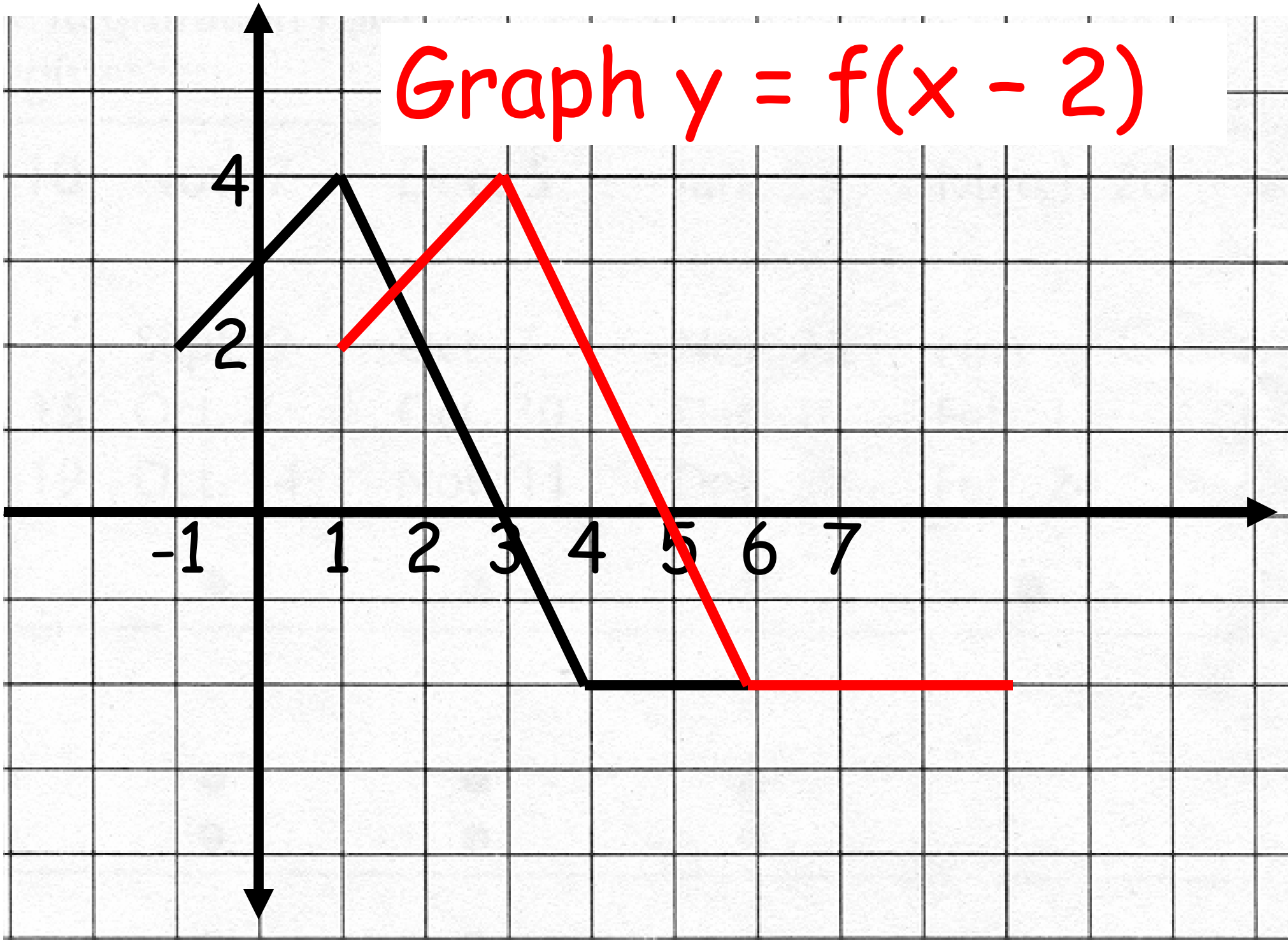


To graph $y = f(|x|)$, erase everything to the left of the y-axis, and replace it with a mirror of what is to the right of the y-axis

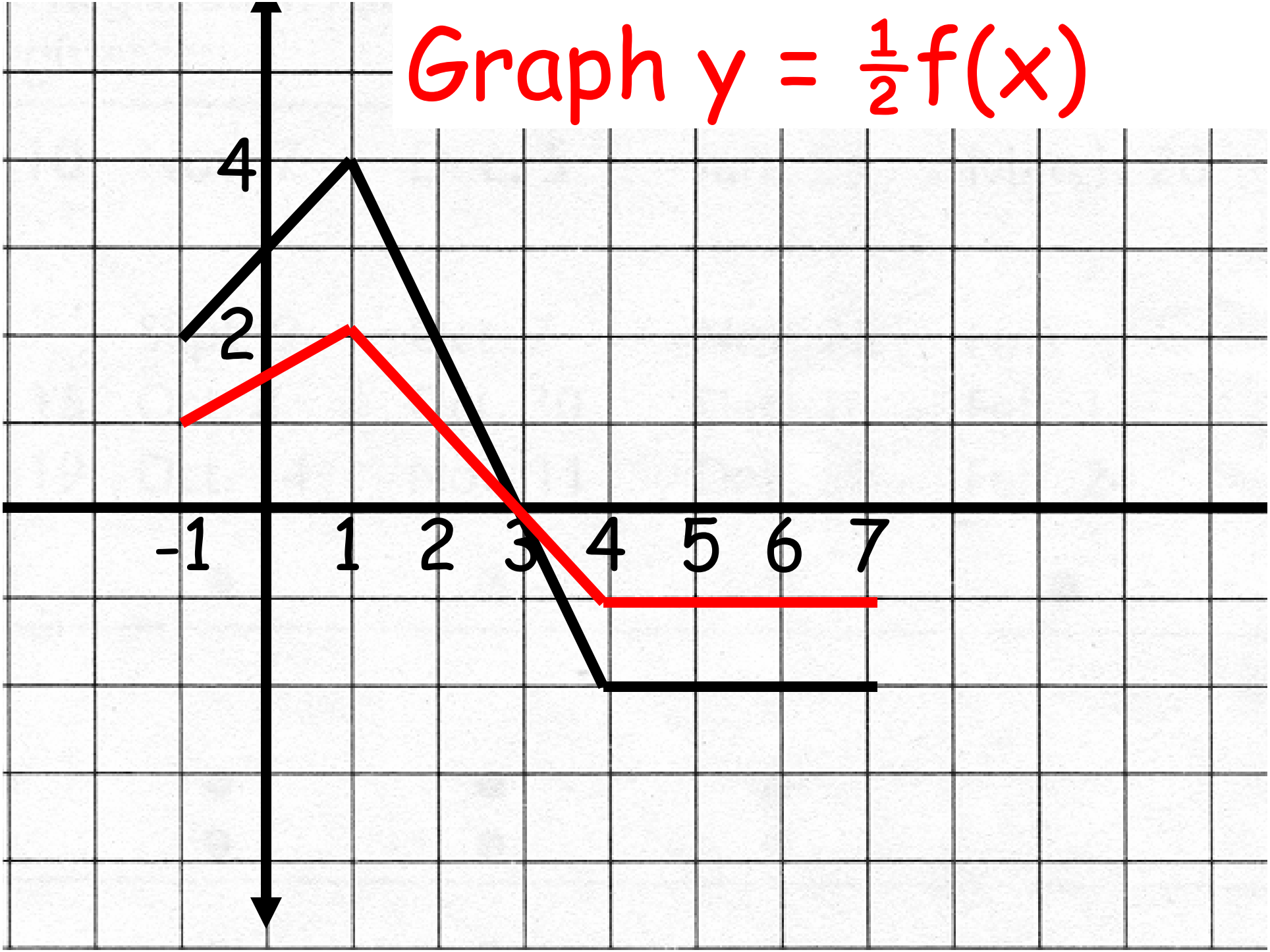
Graph y = f(x) - 3



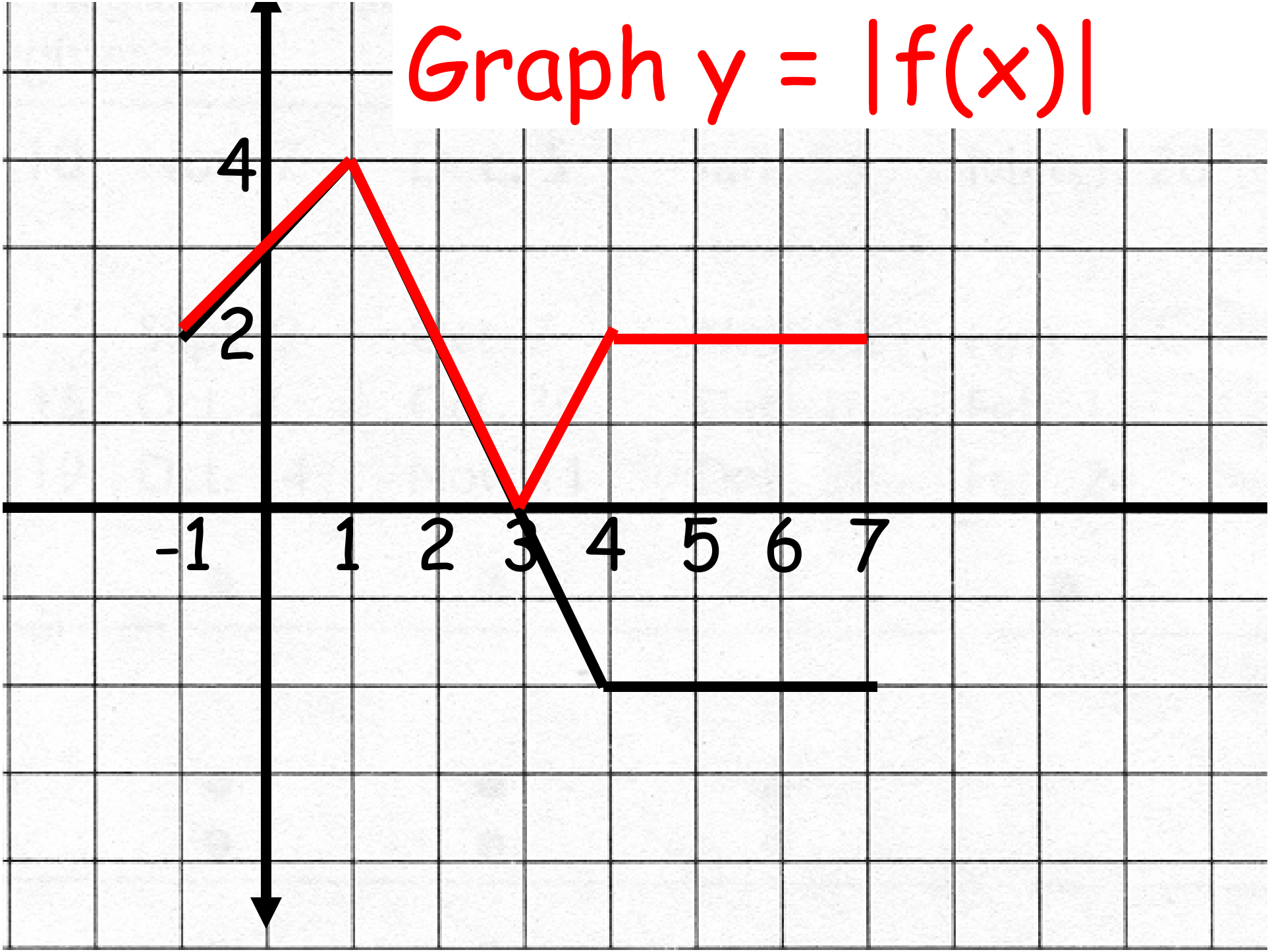
Graph y = f(x - 2)



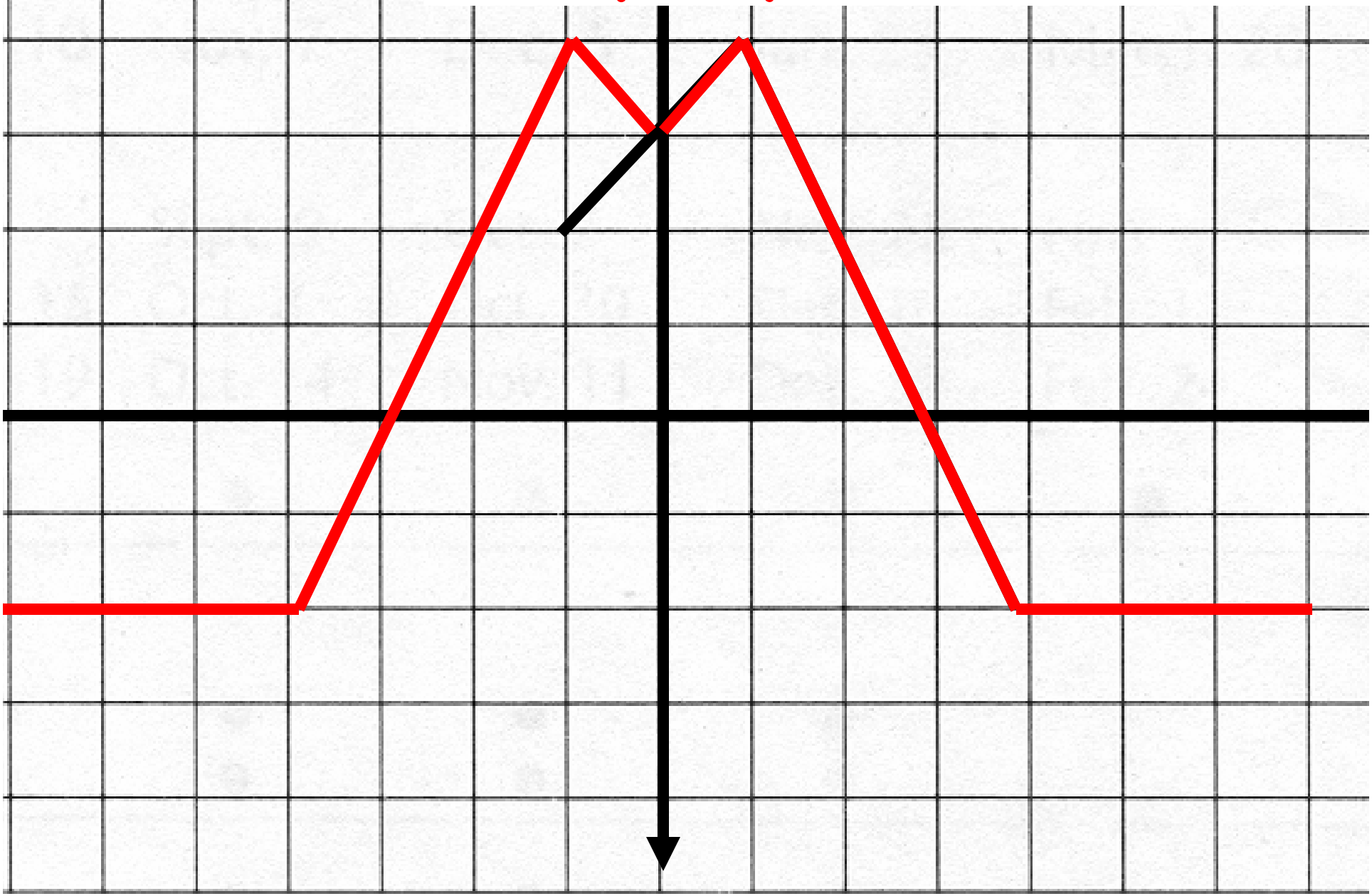
Graph $y = \frac{1}{2}f(x)$



Graph y = |f(x)|



Graph y = f(|x|)



Graph $y = x^2 - 4$

and $y = |x^2 - 4|$

Graph:

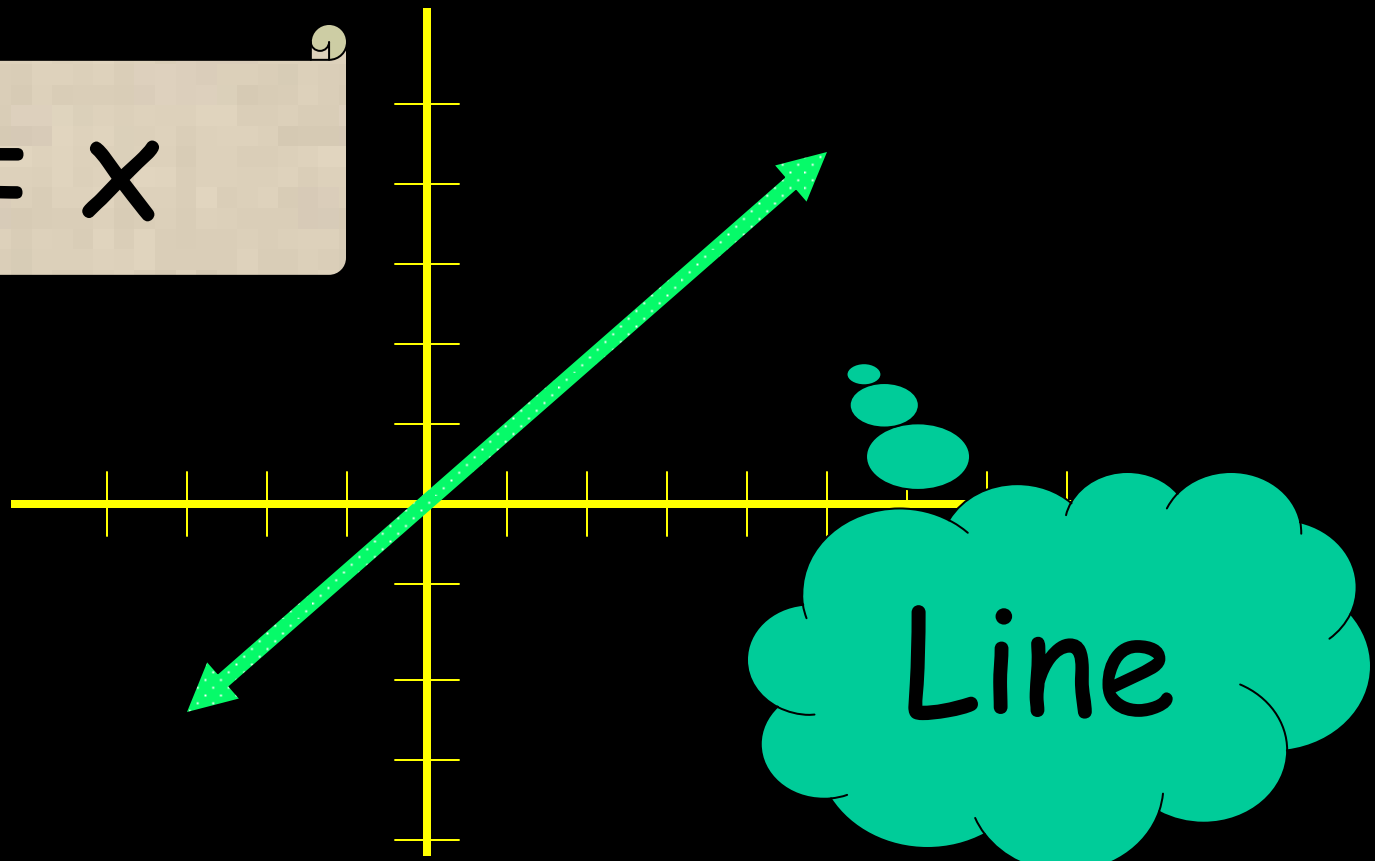
$y = \sin x$

$y = |\sin x|$

$y = \sin |x|$

Basic Graphs

$y = x$



sketch

$y = 5$

sketch

$x = -1.5$

sketch

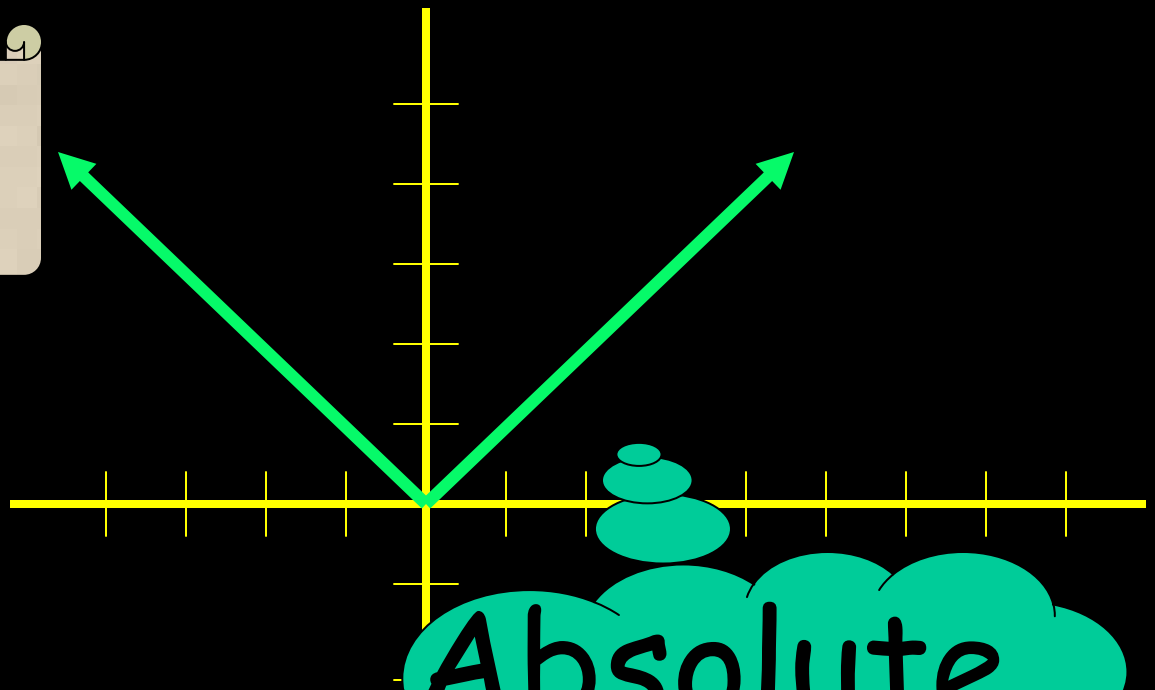
$$y = 5x - 3$$

sketch

$$y = 5 - \frac{1}{2}x$$

Basic Graphs

$y = |x|$



Absolute value

sketch

$$y = |x + 4| - 7$$

sketch

$$y = 7 - |x + 4|$$

sketch

$$y = |x| - 7$$

sketch

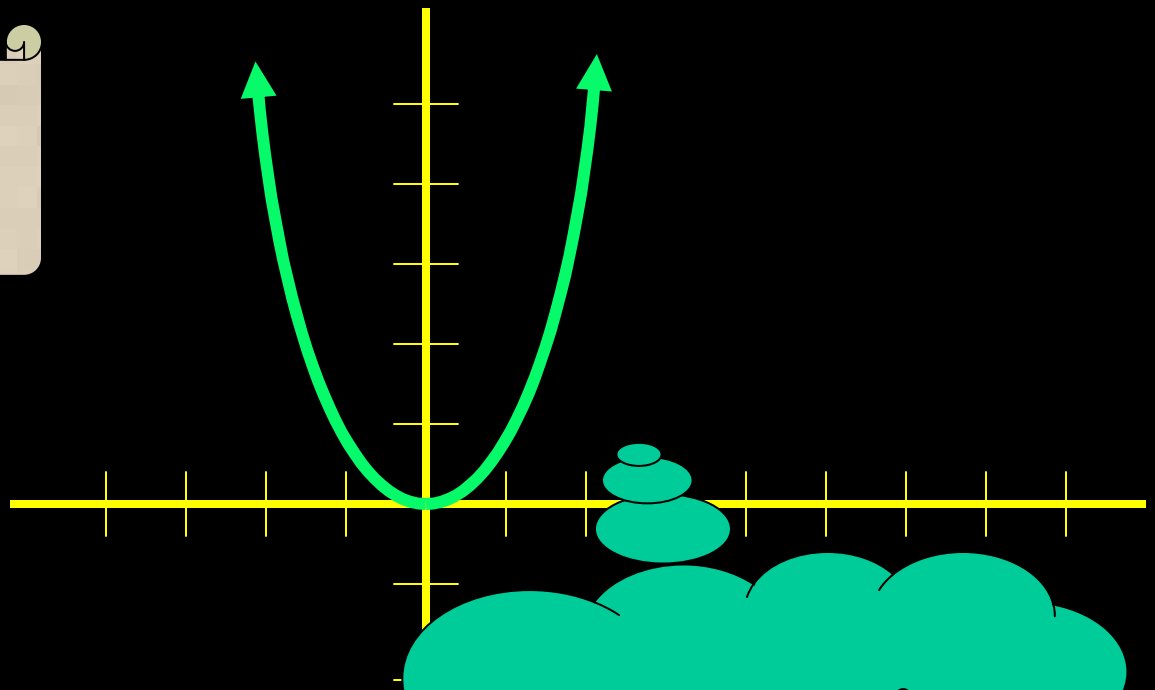
$$y = 6 - |x + 8|$$

sketch

$$y = 6 - 3|x + 8|$$

Basic Graphs

$y = x^2$



parabola

sketch

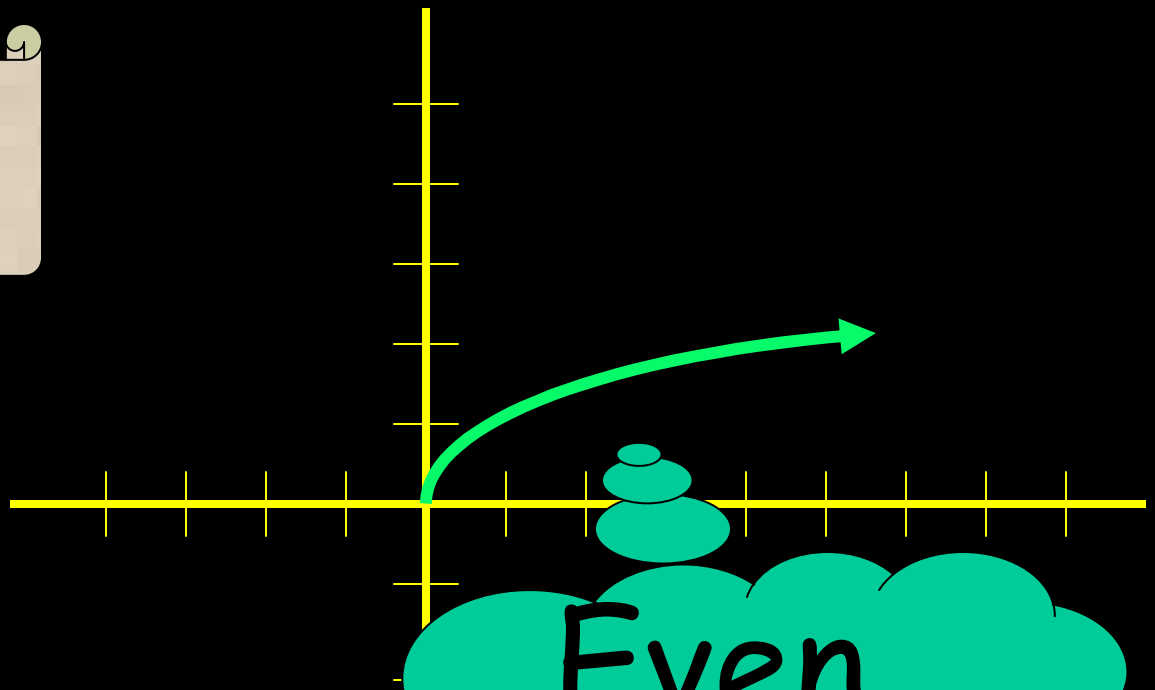
$$y = 9 - x^2$$

sketch

$$y = x^2 + 1$$

Basic Graphs

$$y = \sqrt{x}$$



Even root

sketch

$$y = \sqrt{x + 3}$$

sketch

$$y = \sqrt{x - 3} + 5$$

sketch

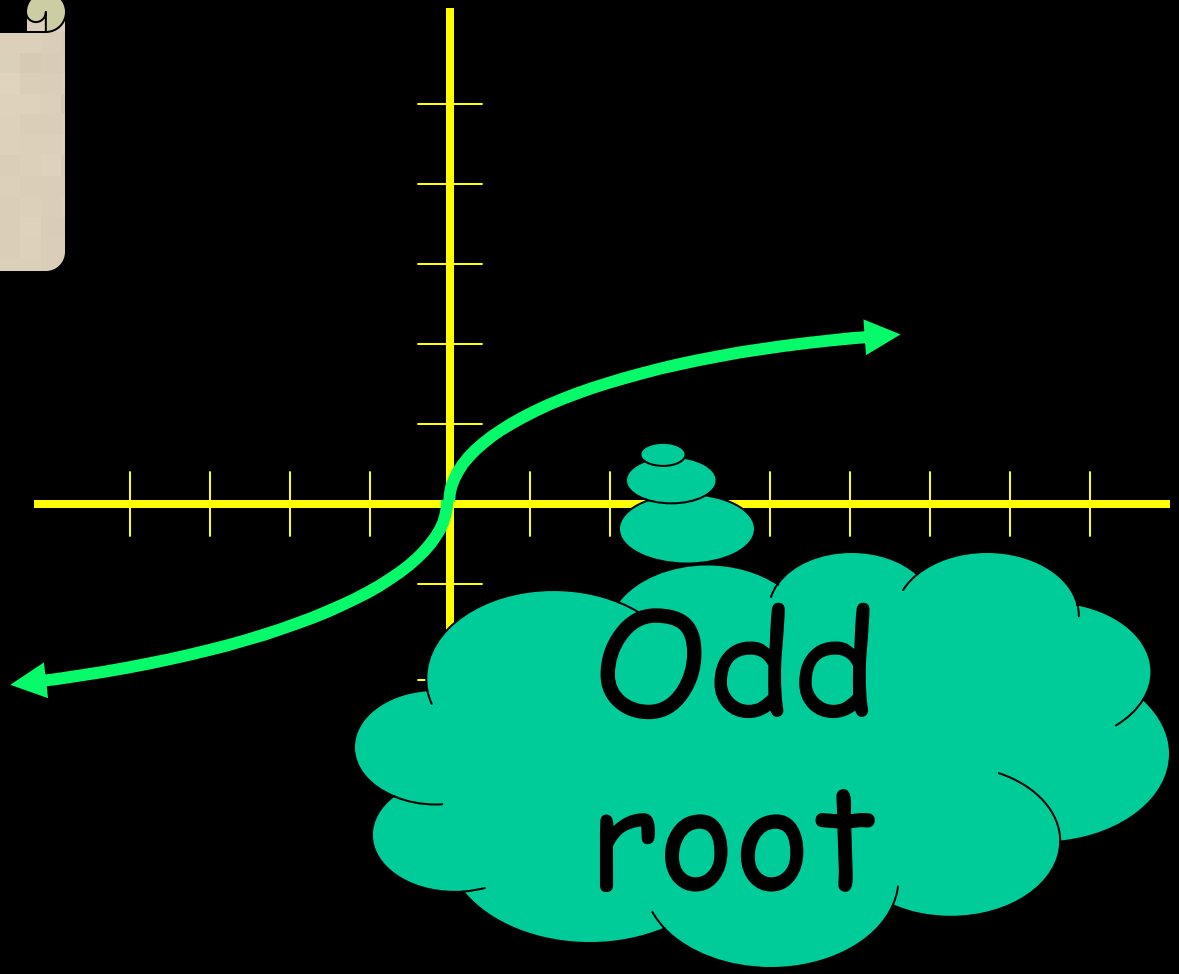
$$y = \frac{1}{2}\sqrt{x - 3} + 5$$

sketch

$$y = 1 - \sqrt{x + 3}$$

Basic Graphs

$$y = \sqrt[3]{x}$$

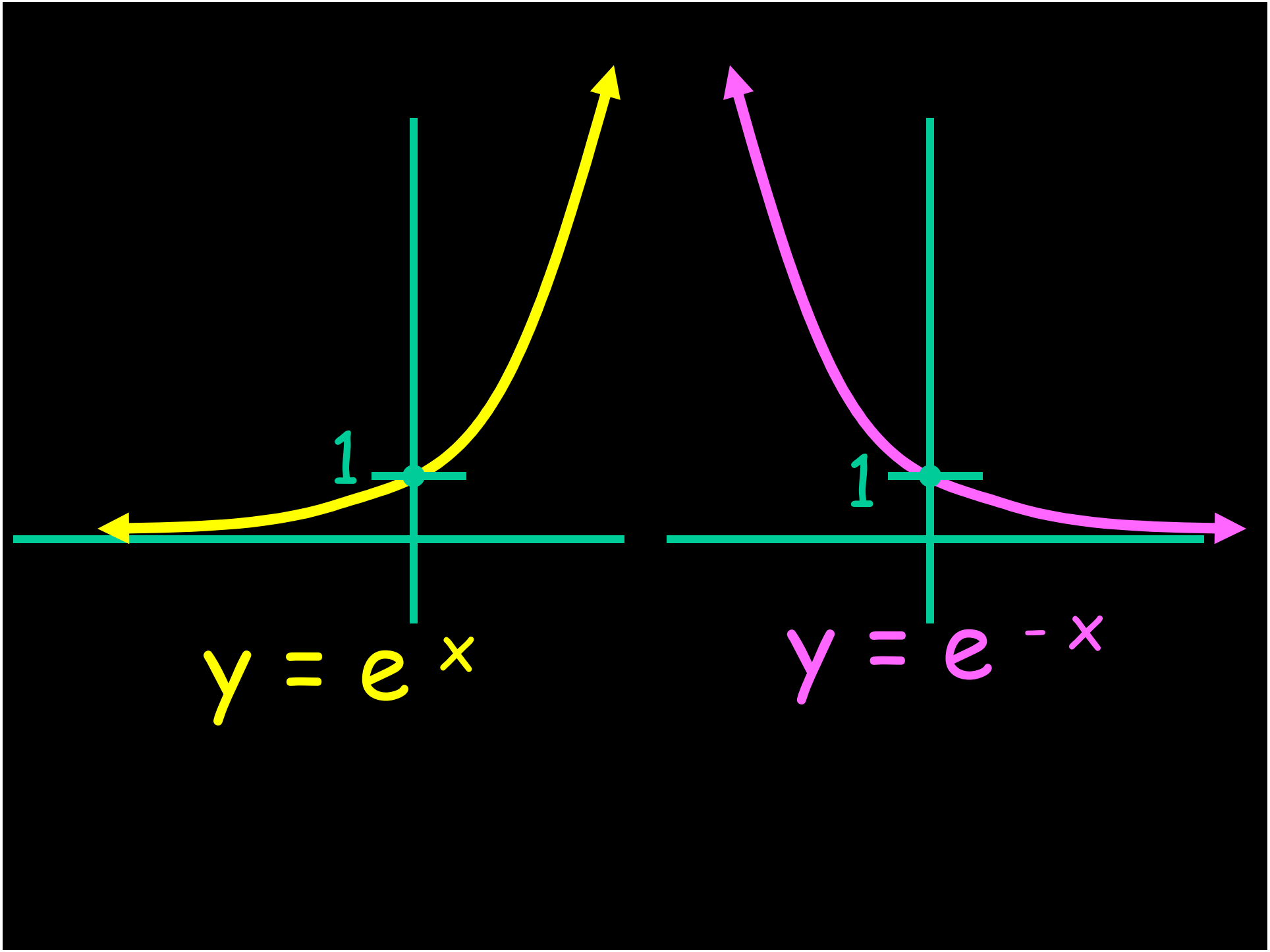


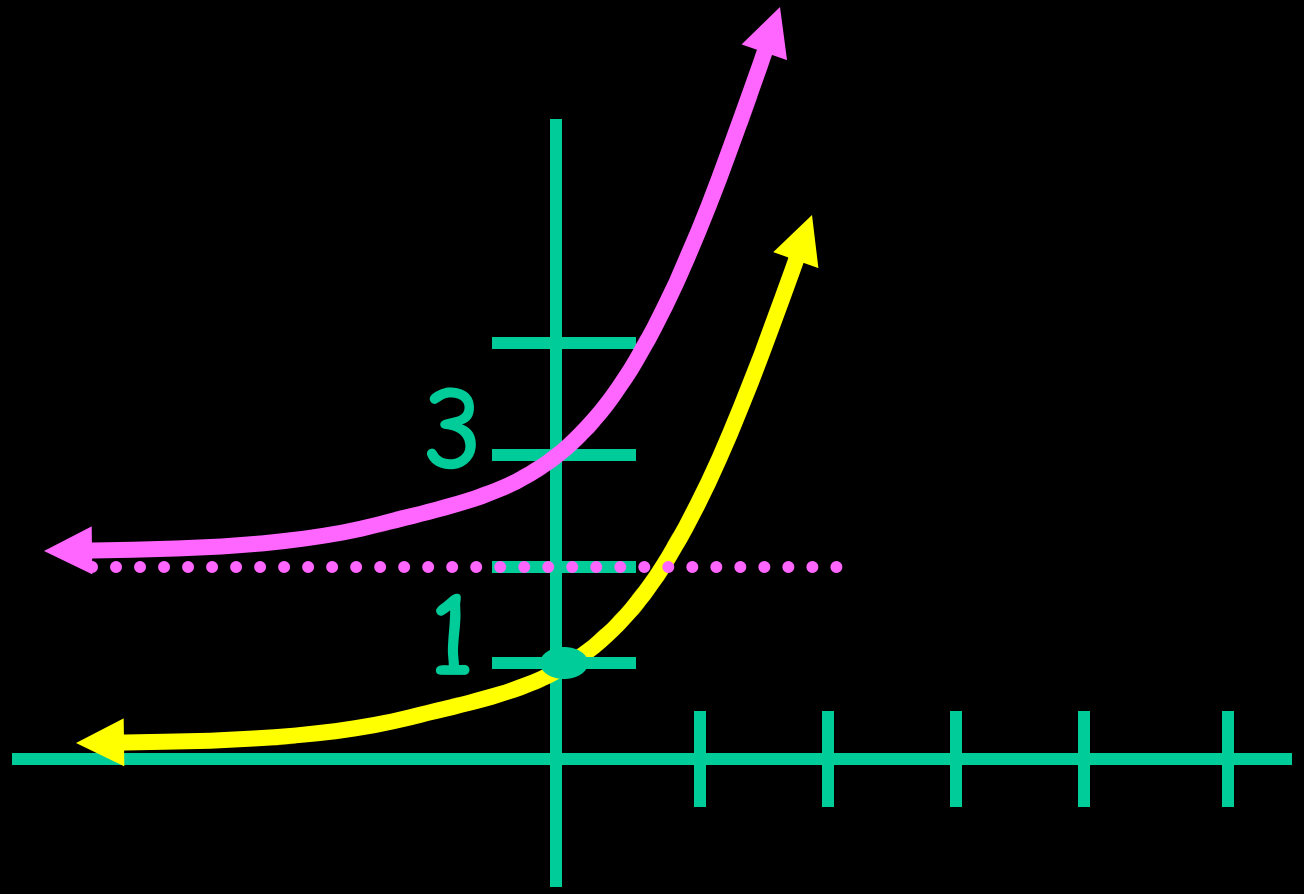
sketch

$$y = \sqrt[3]{x + 3}$$

sketch

$$y = 1 - \sqrt[3]{x + 3}$$

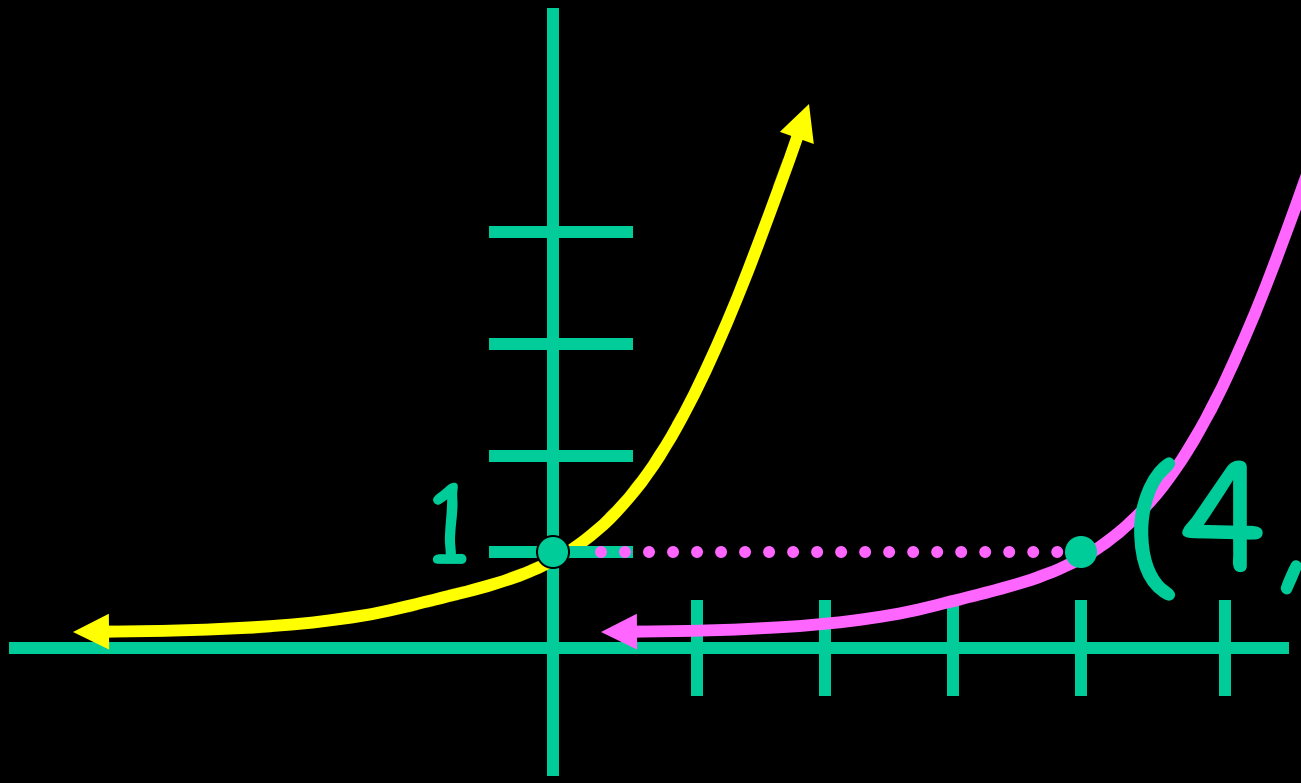

1
1
y = e x
y = e - x



Up 2

$y = e^x$

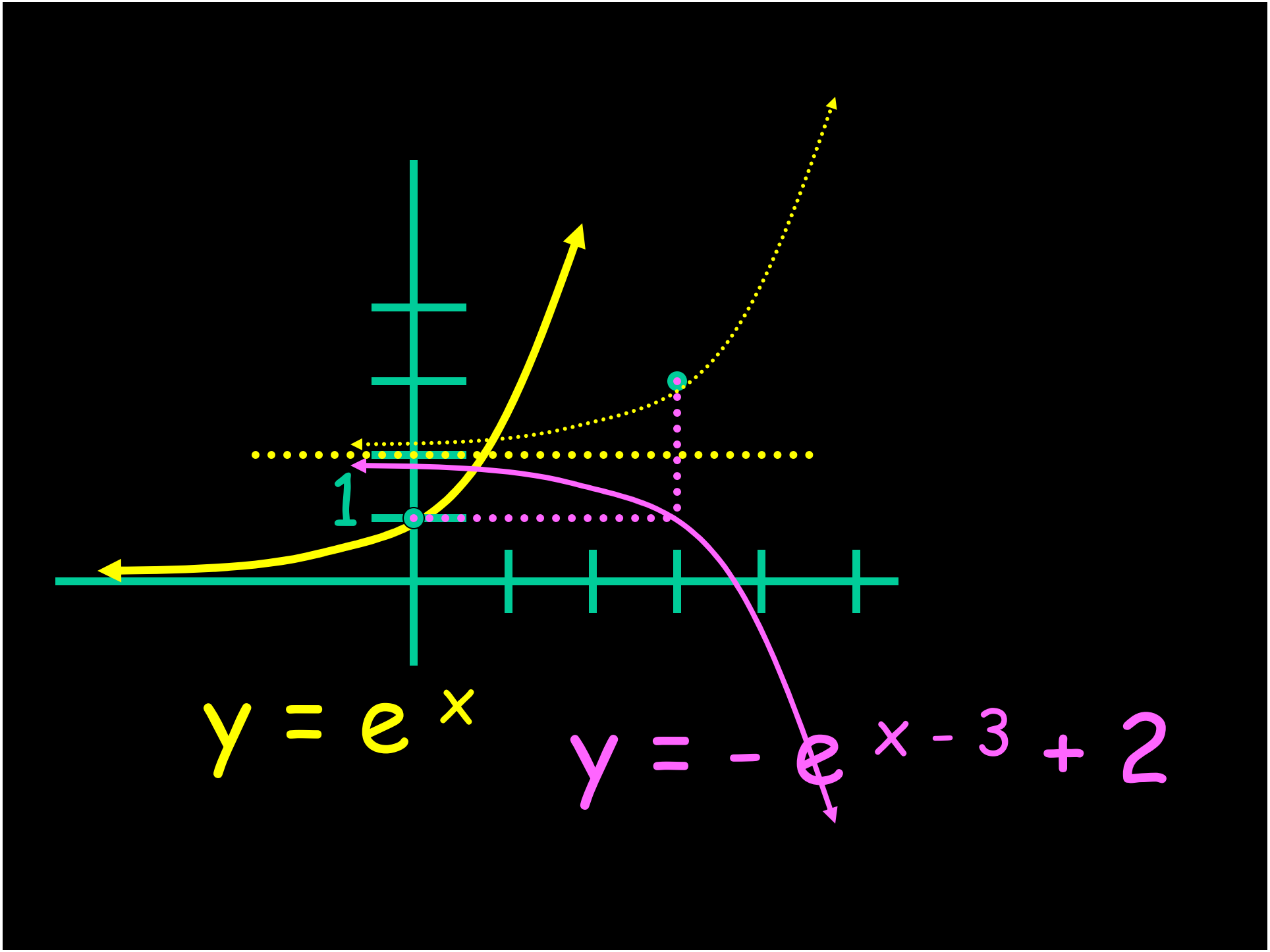
$y = e^x + 2$

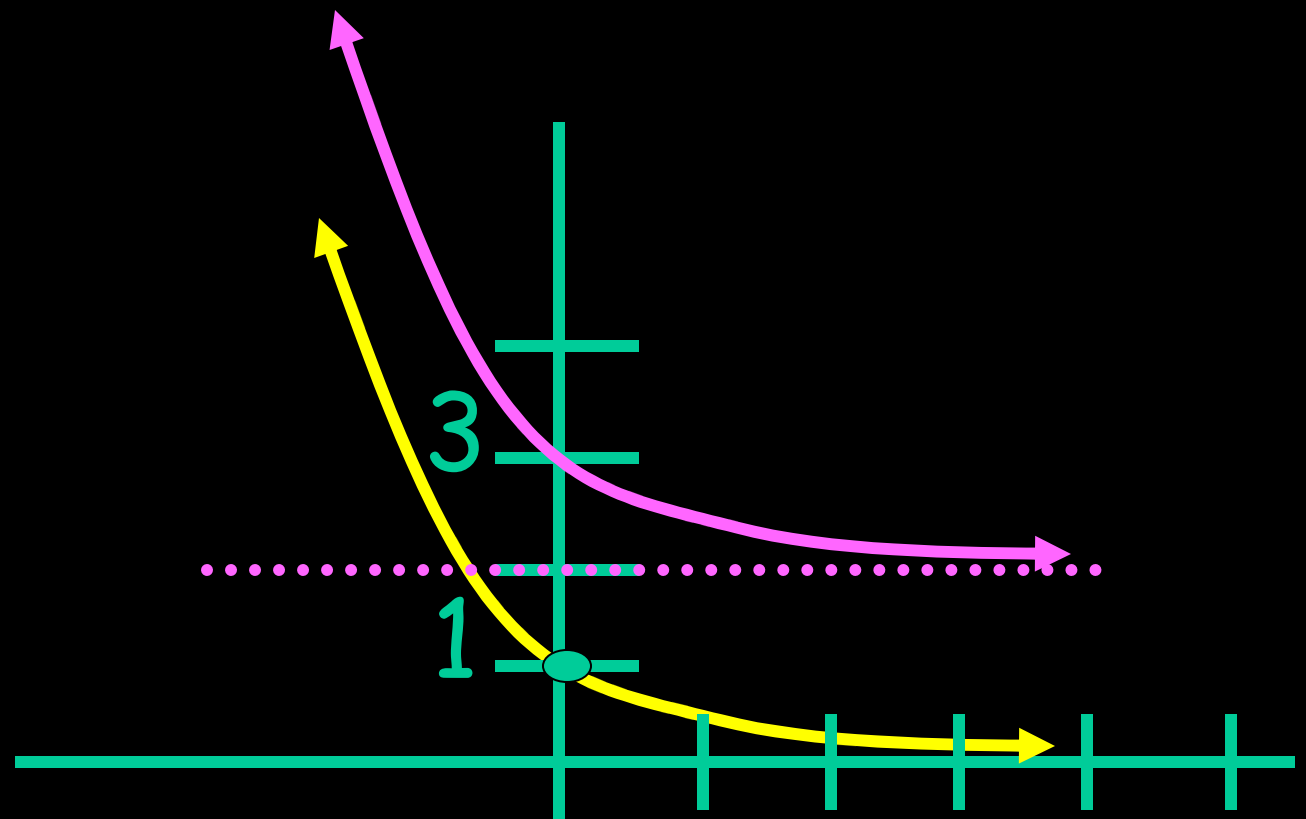


$y = e^x$

$y = e^{x-4}$

Right 4

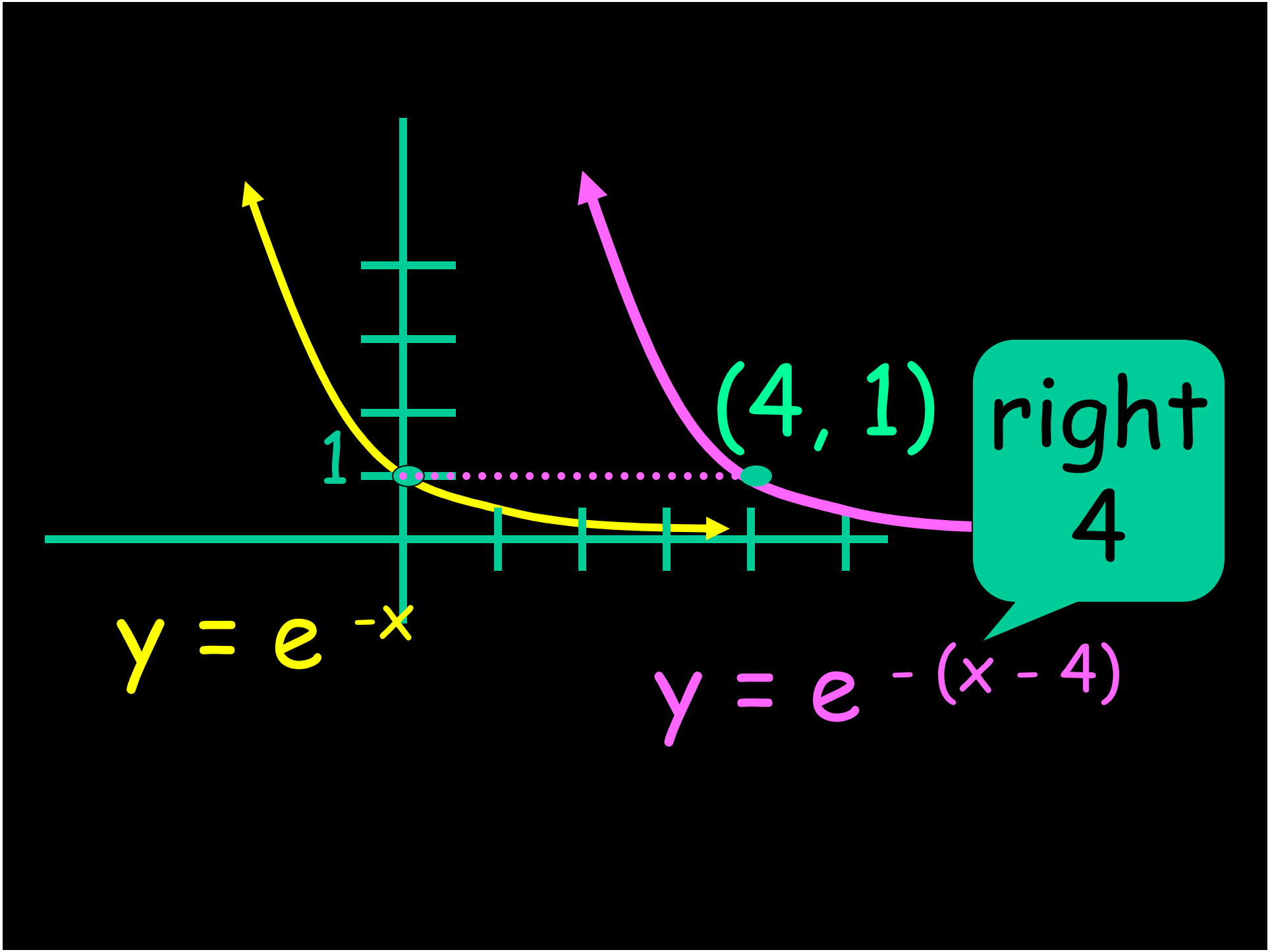

1
y = e x
y = - e x - 3 + 2

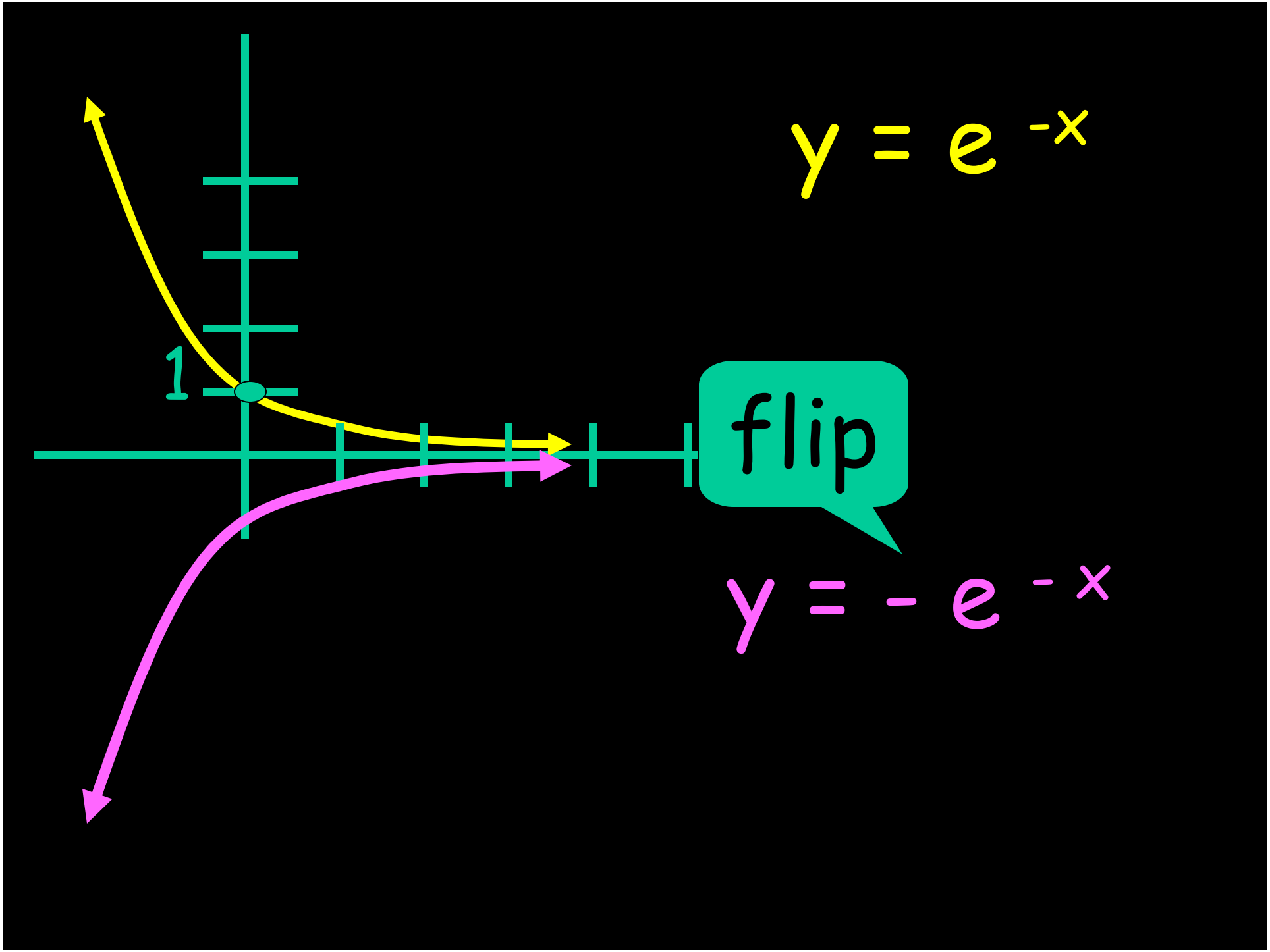


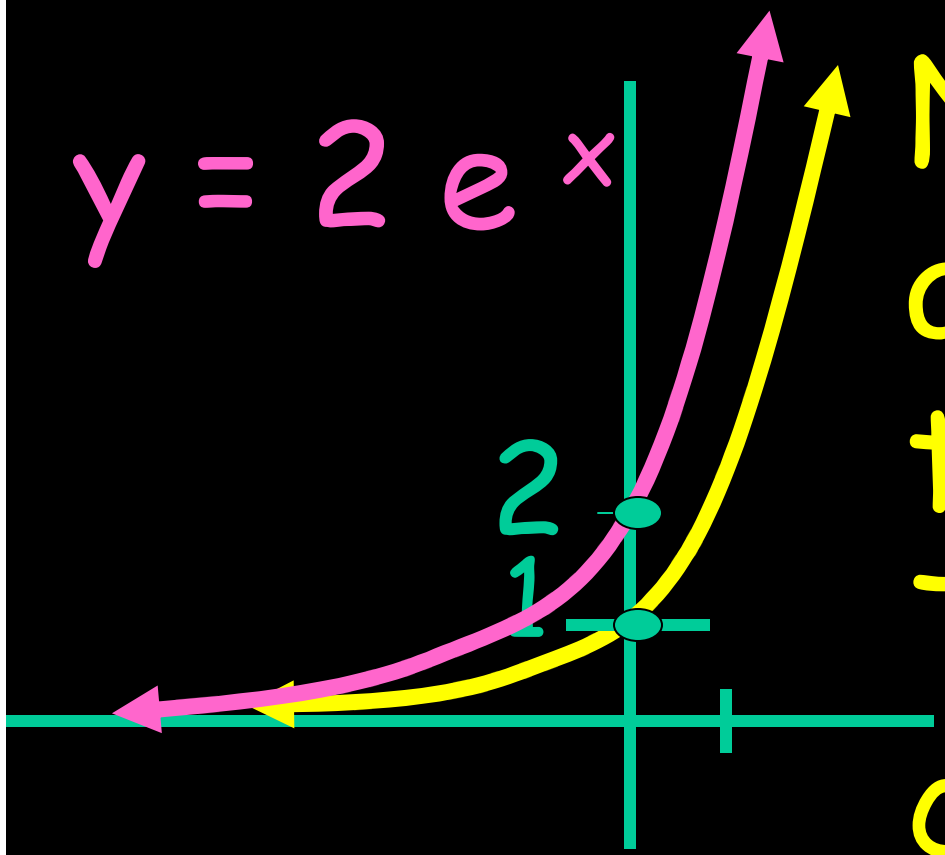
$y = e^{-x}$

Up 2

$y = e^{-x} + 2$

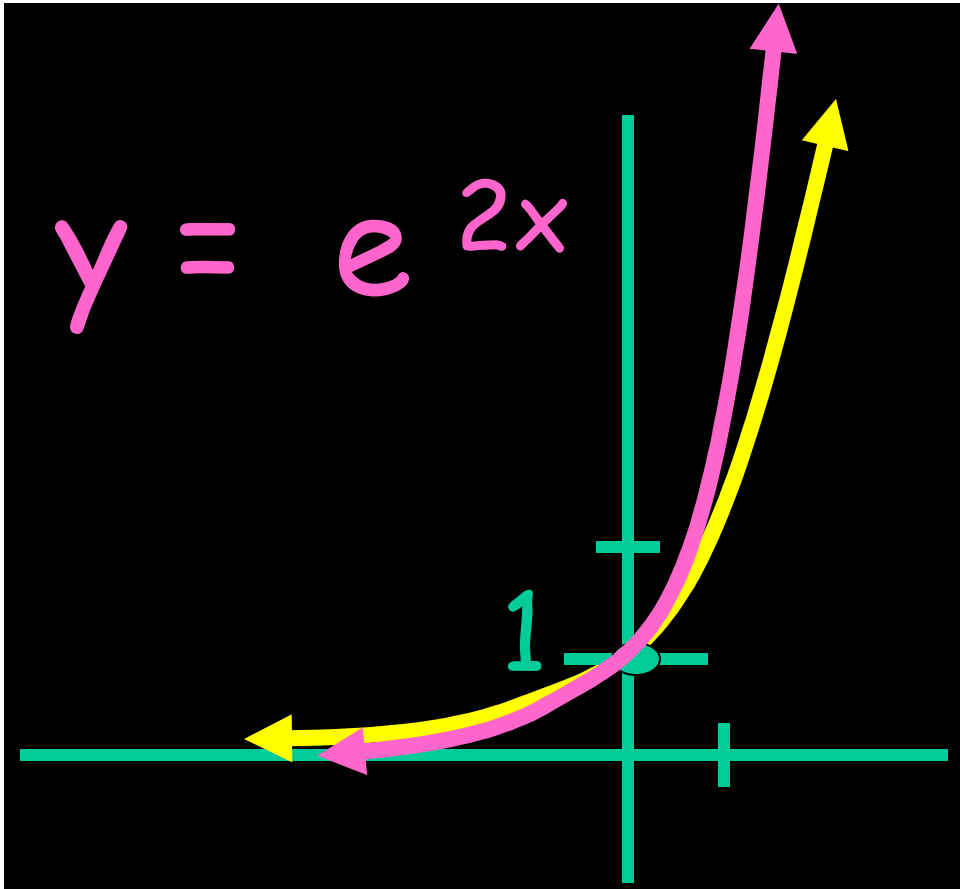

(4, 1)
right
4
1
y = e -x
y = e - (x - 4)


y = e -x
1
flip
y = - e - x



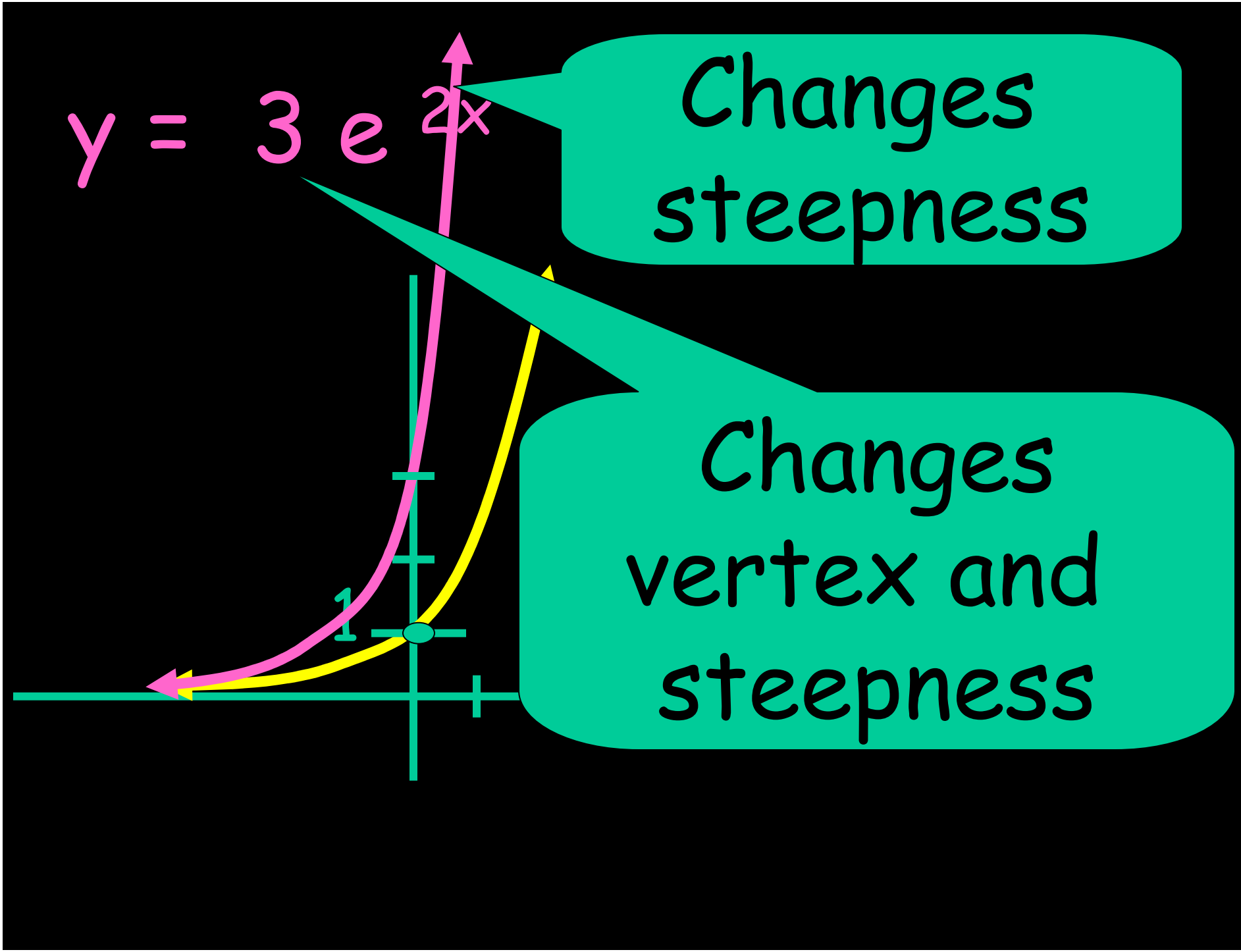
$y = e^x$

Note: the asymptote is the same. The vertex changed. The graph is steeper.



$y = e^{x}$

Note: the asymptote and vertex are the same. The graph is steeper.


y = 3 e 2x
Changes steepness
Changes vertex and steepness
1

sketch

$$y = e^x$$

sketch

$$y = e^x + 2$$

sketch

$$y = e^{x-3} + 1$$

sketch

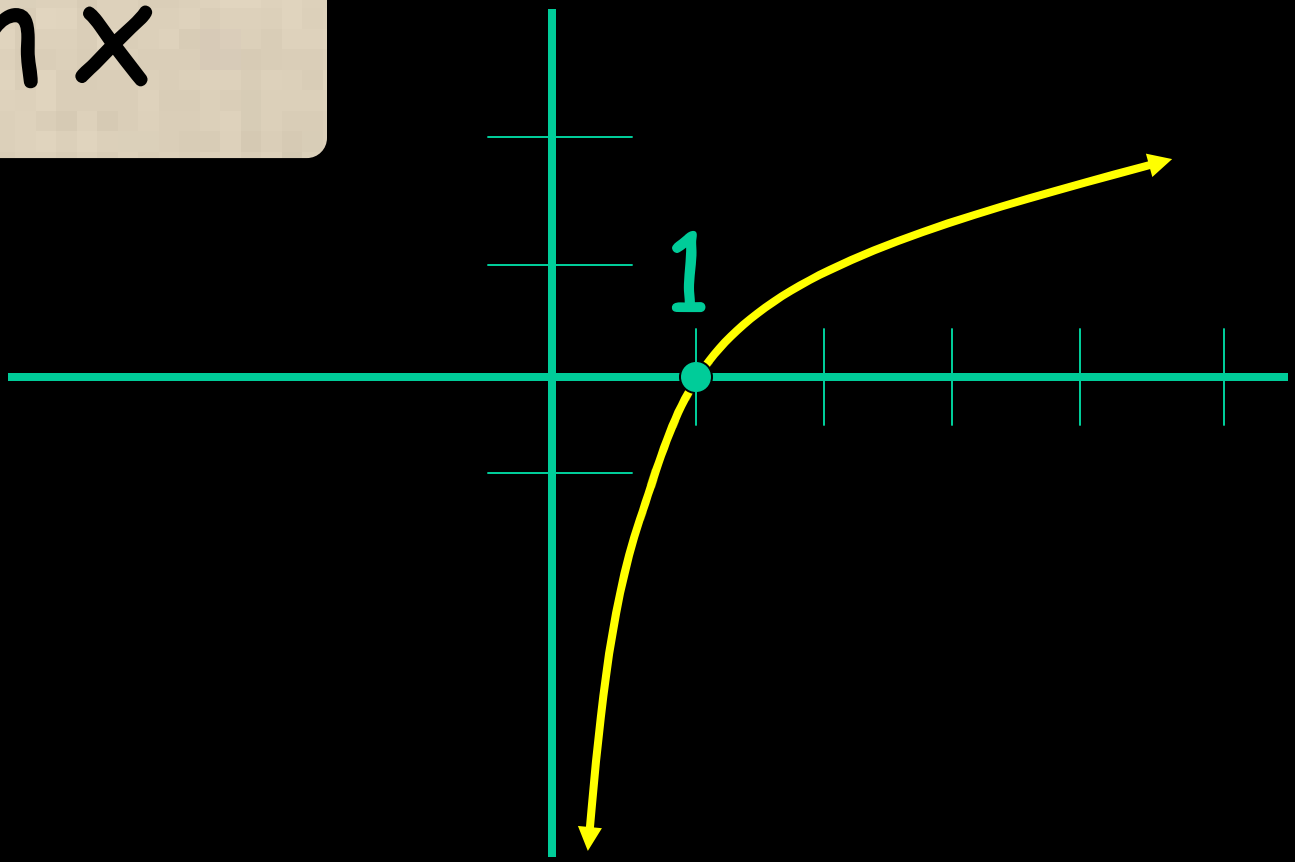
$$y = 4 + e^{-x}$$

sketch

$$y = 4 + e^{-(4+x)}$$

Basic Graphs

$y = \ln x$



sketch

$$y = \ln x - 4$$

sketch

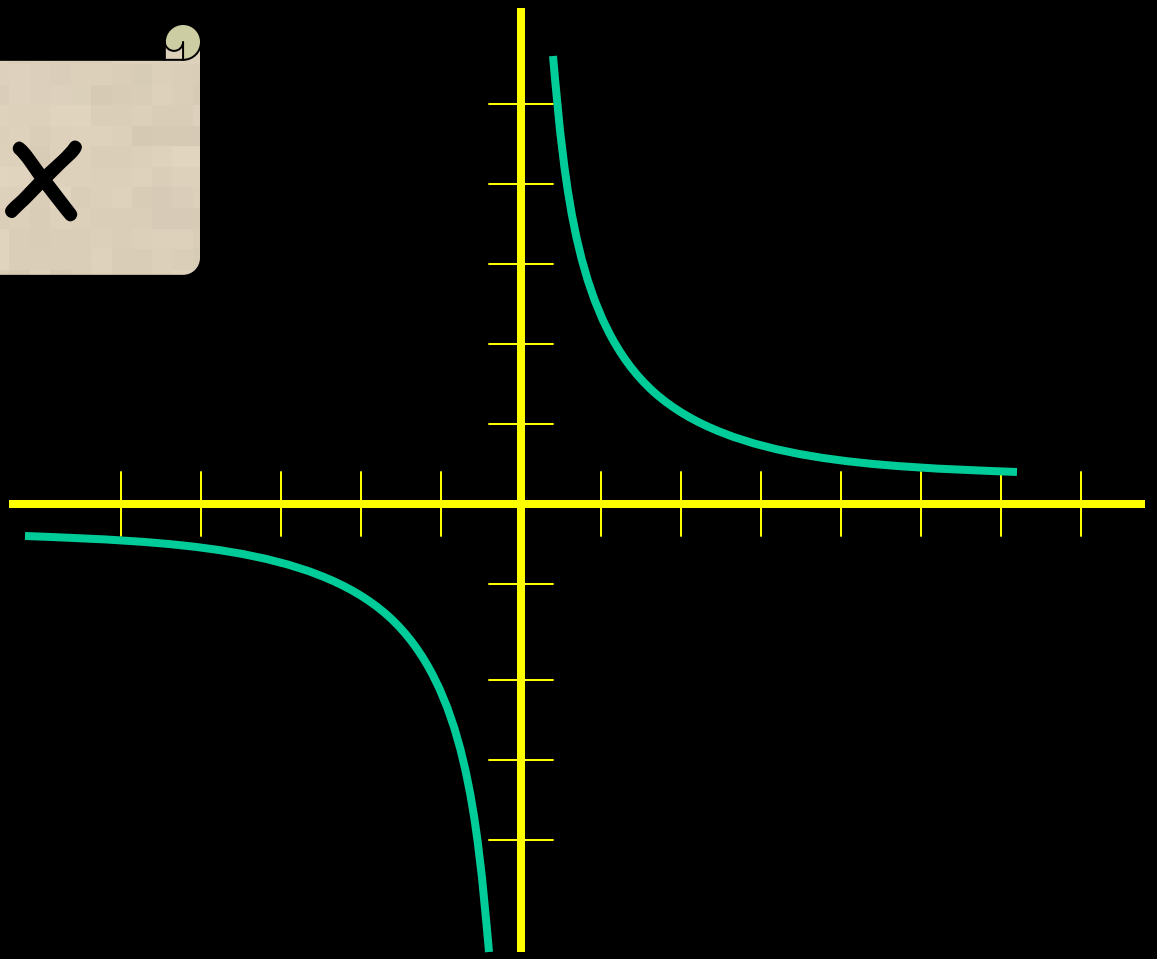
$$y = \ln(x - 4)$$

sketch

$$y = \ln(x + 1) - 3$$

Basic Graphs

$y = 1/x$



sketch

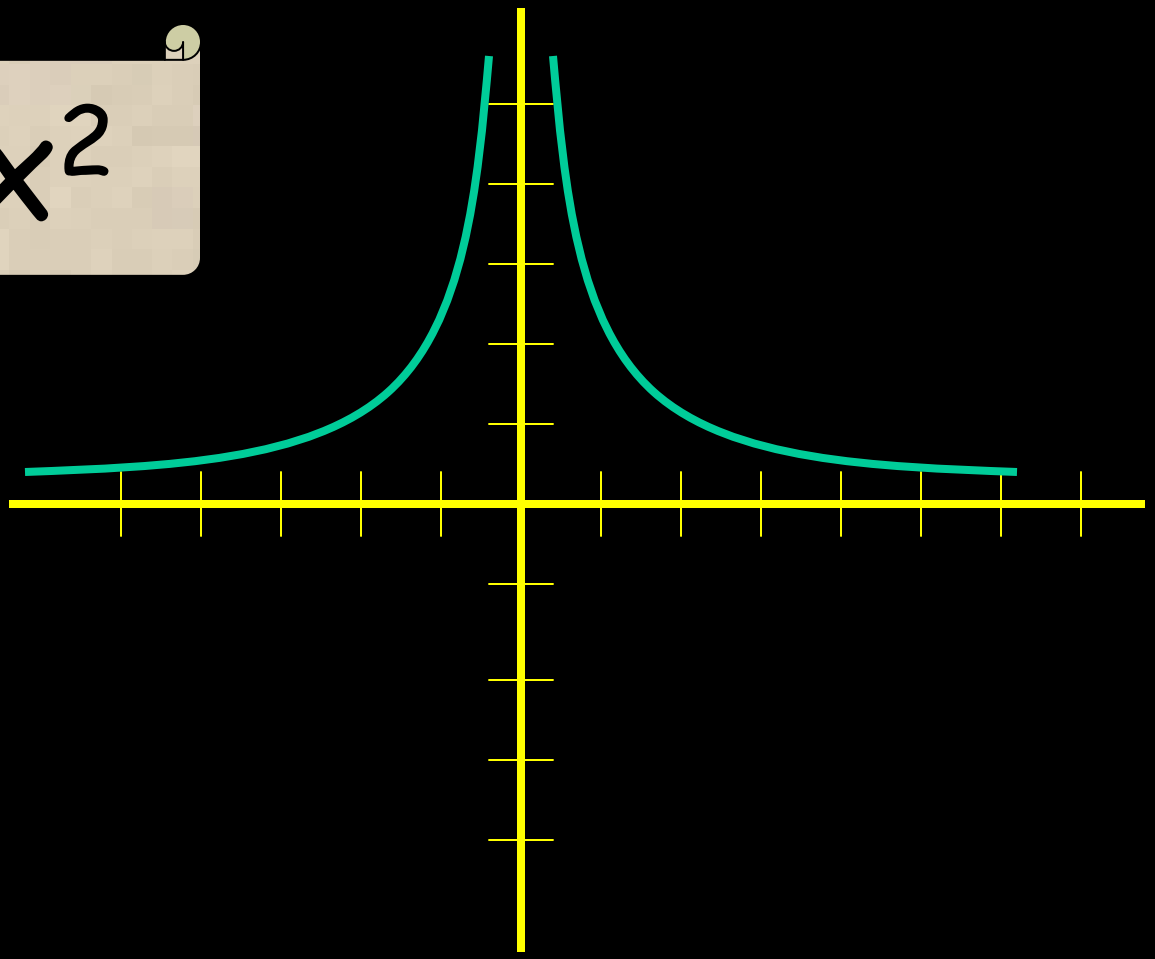
$$y = \frac{2}{x}$$

sketch

$$y = 3 + \frac{1}{(x+2)}$$

Basic Graphs

$y = 1/x^2$



sketch

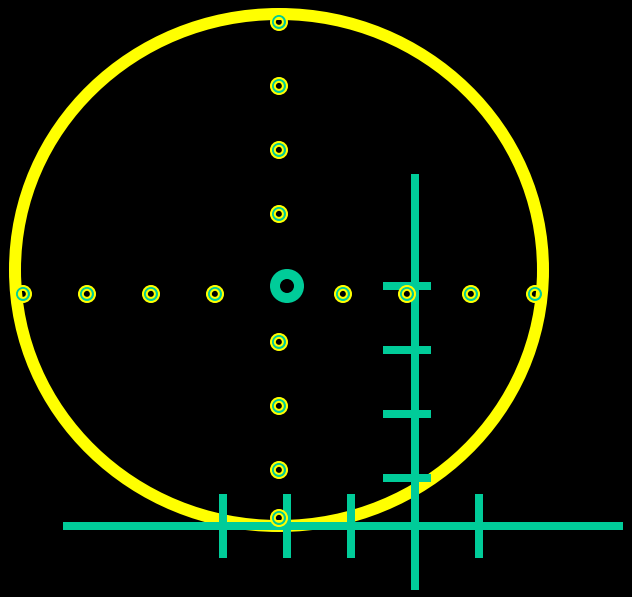
$$y = \frac{-2}{x^2}$$

sketch

$$y = 1 / (x+ 2)^2$$

Graph:

$$(x + 2)^2 + (y - 4)^2 = 16$$

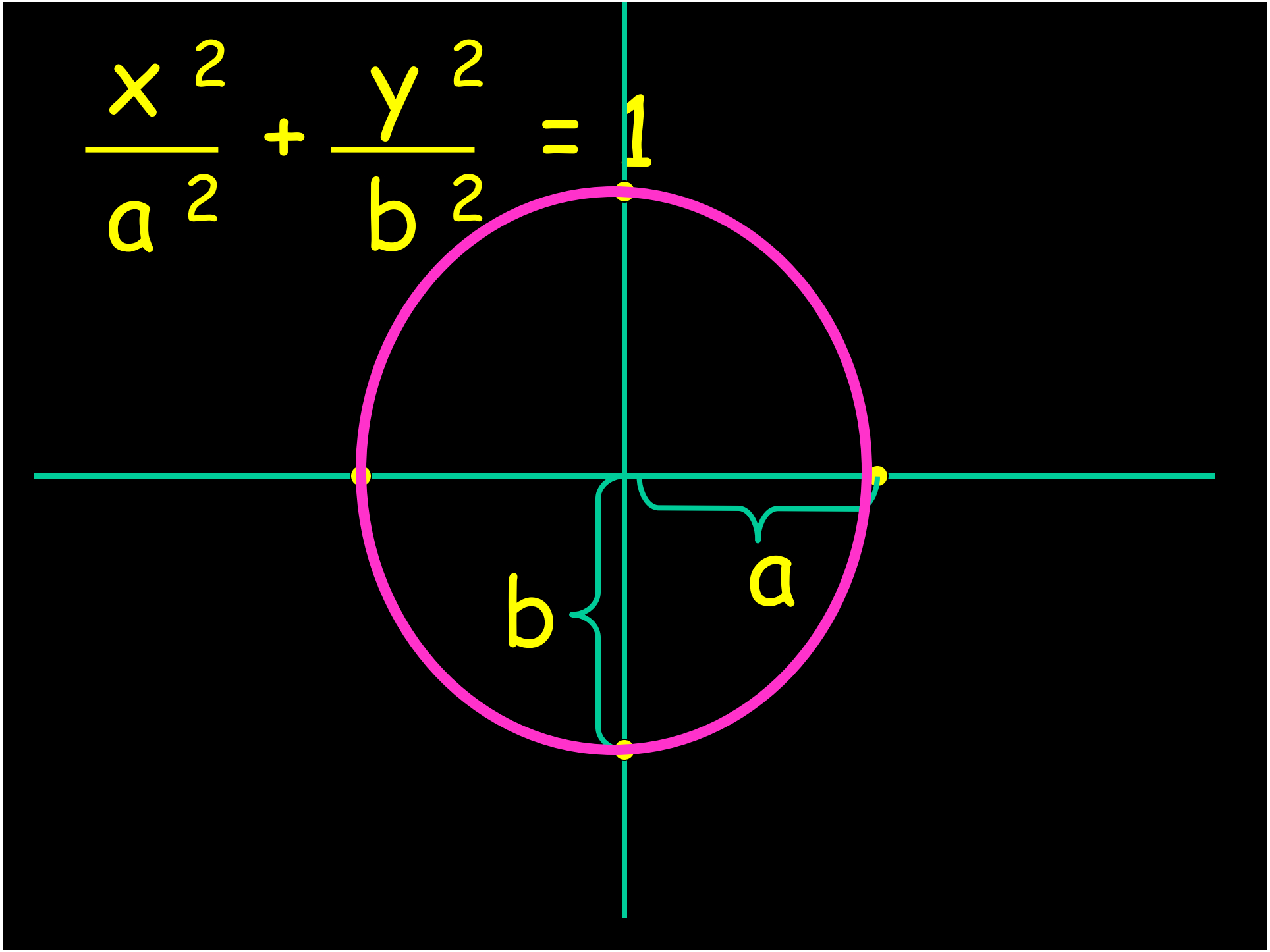


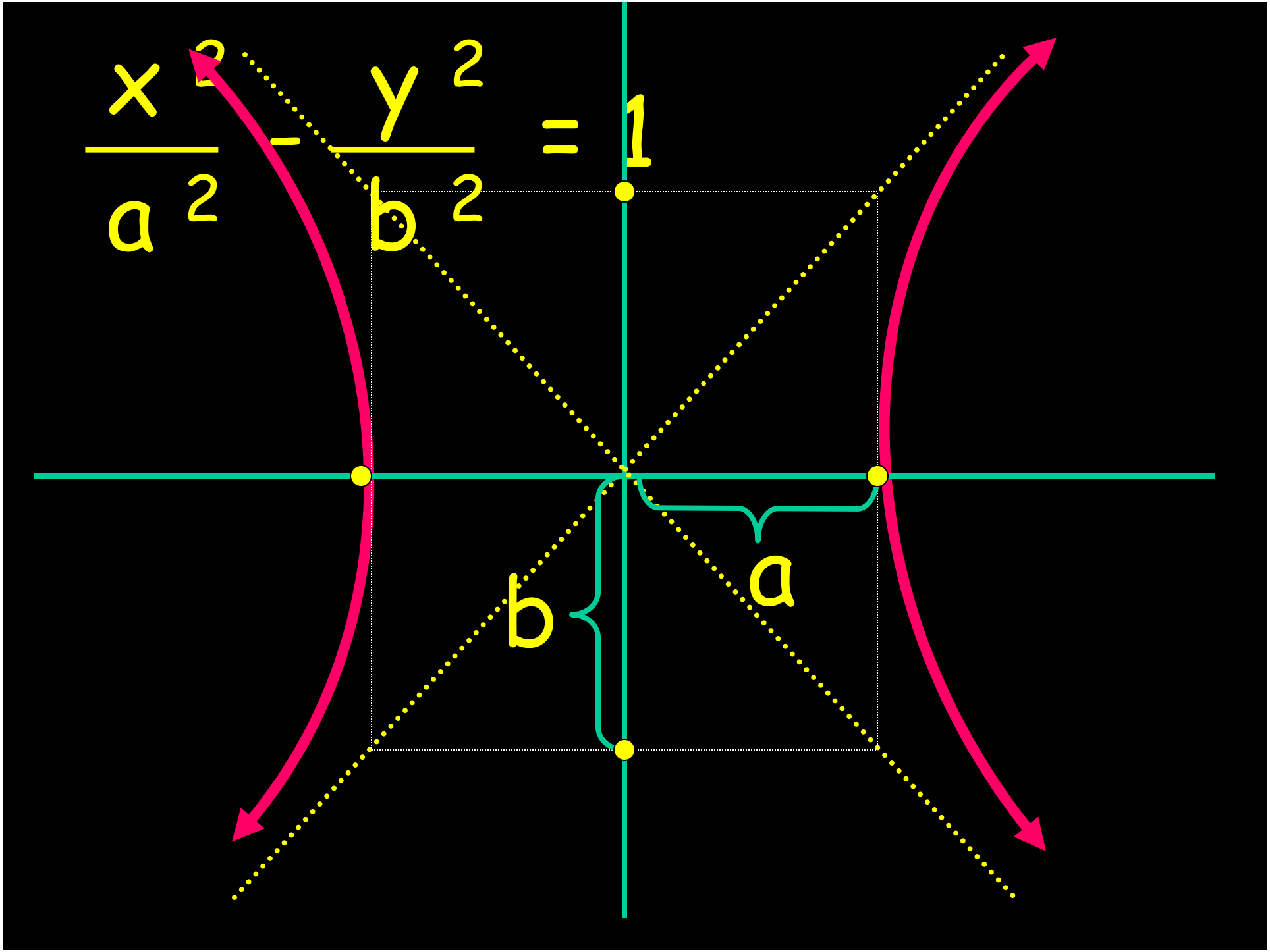
Center

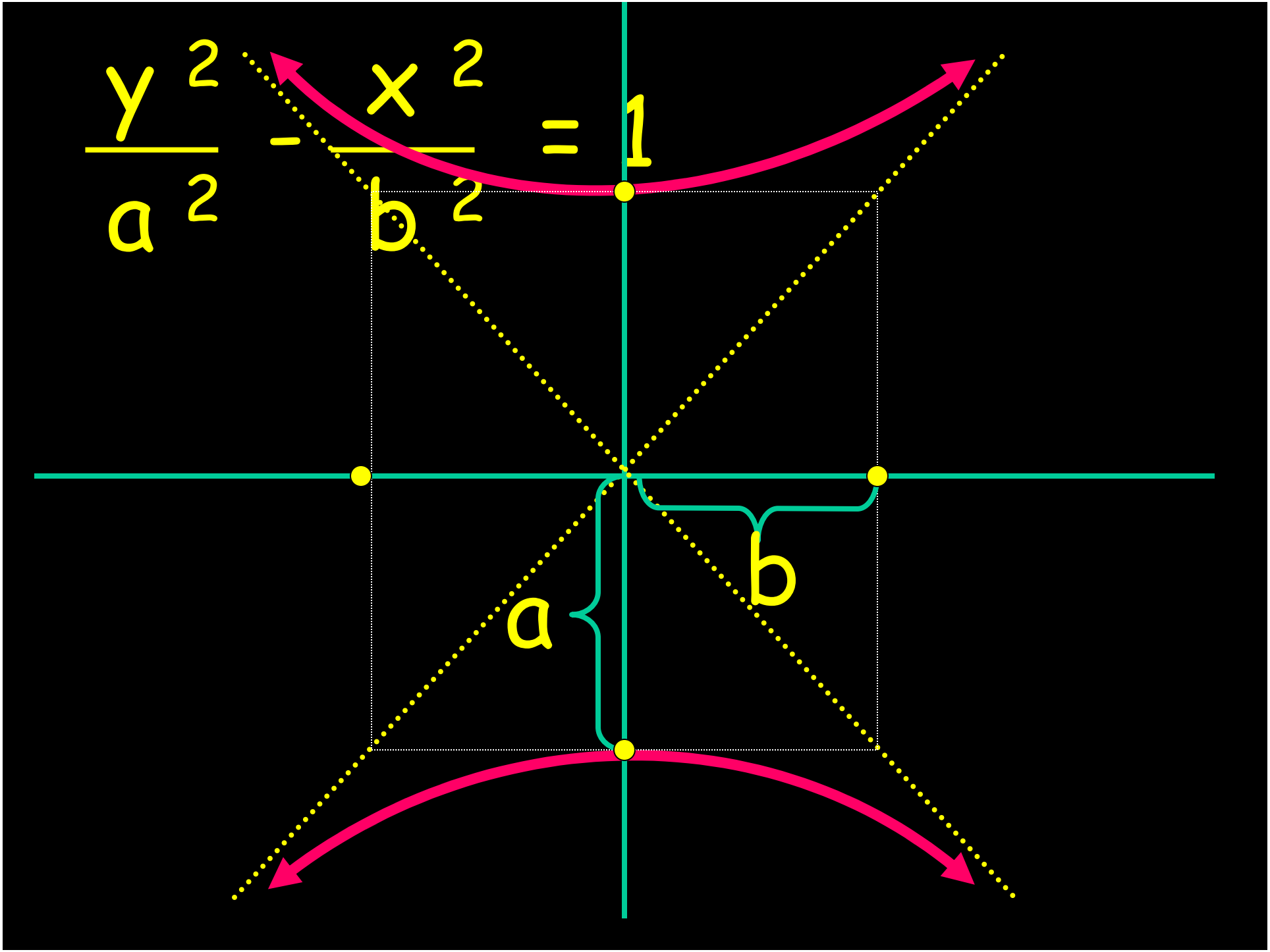
(-2, 4)

r = 4

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$




x 2 / a 2 − y 2 / b 2 = 1
b
a


y 2 / a 2 − x 2 / b 2 = 1
a
b

sketch

$$x^2 + y^2 = 9$$

sketch

$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$

sketch

$$4x^2 + 9y^2 = 36$$

sketch

$$\frac{(y+2)^2}{25} - \frac{x^2}{9} = 1$$

sketch

$$(x - 1)^2 + y^2 = 9$$

sketch

$$4x^2 - 9y^2 = 36$$

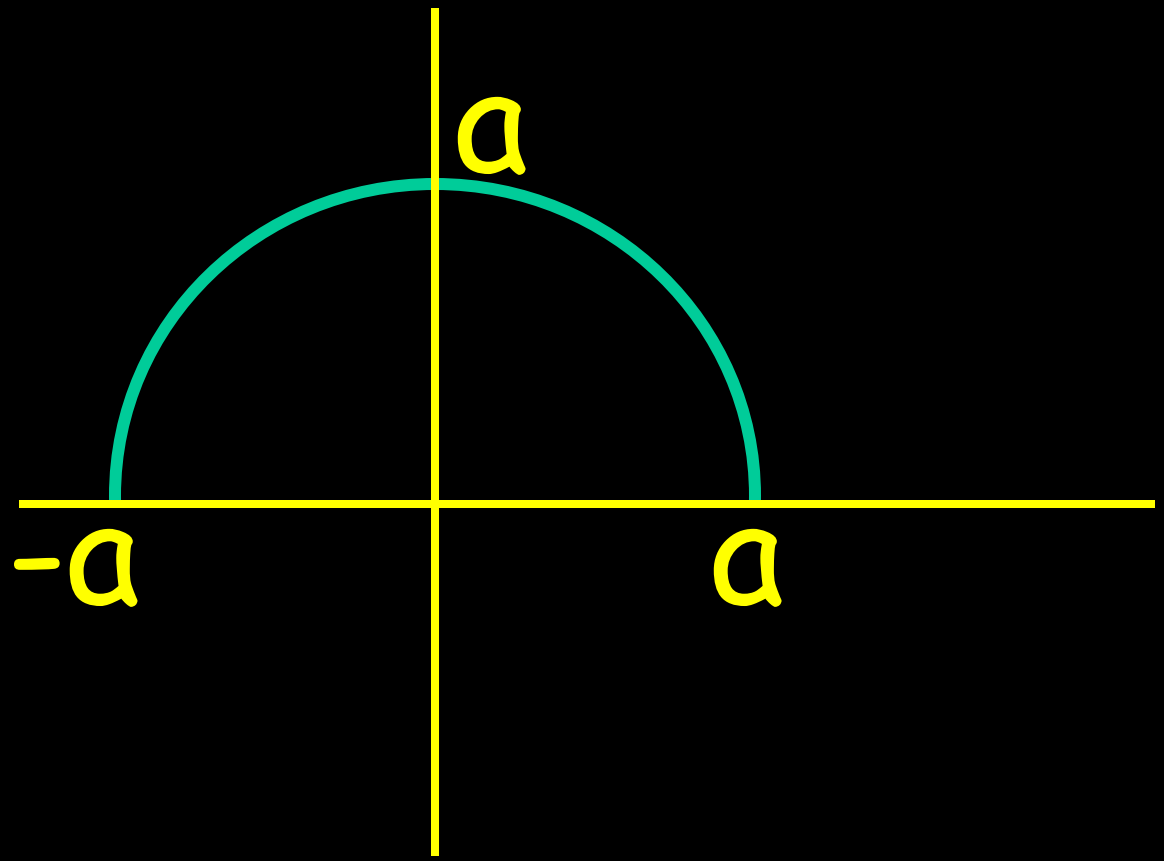
sketch

$$(x - 1)^2 + (y + 4)^2 = 9$$

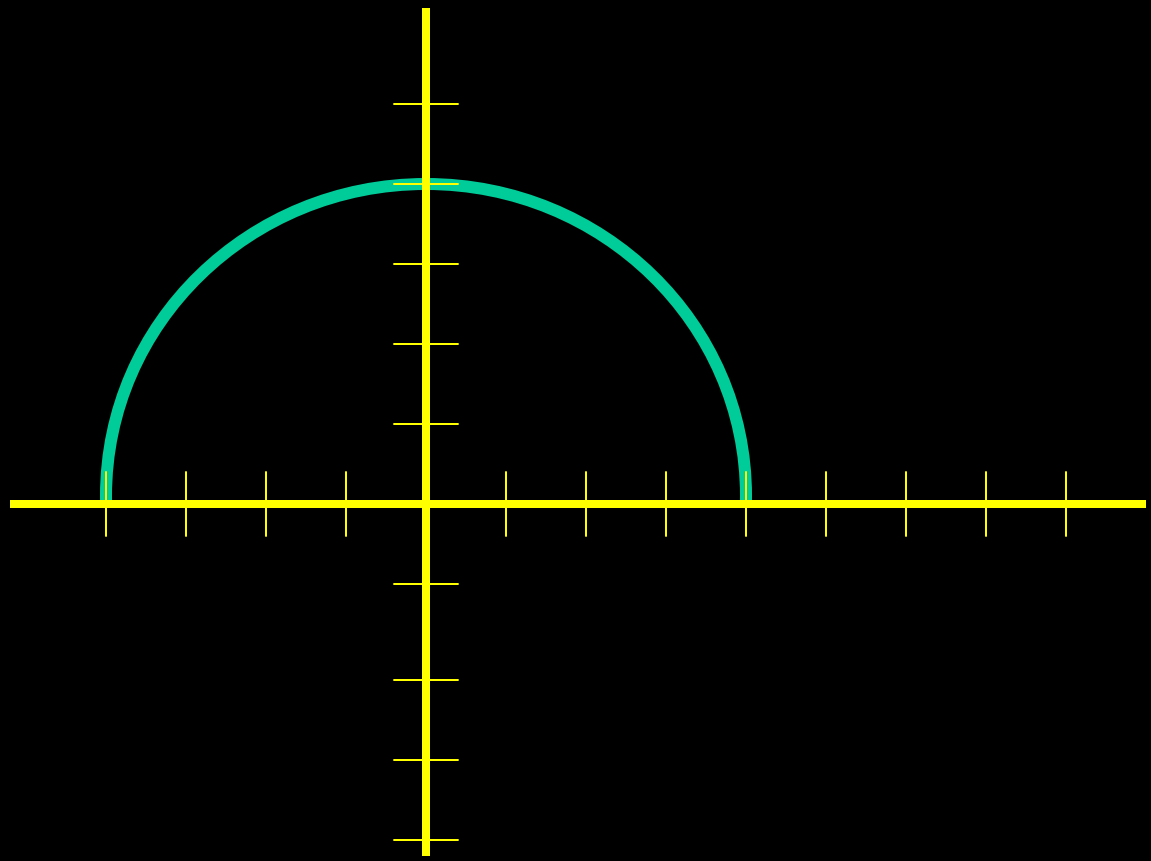
sketch

$$\frac{(x+3)^2}{16} - \frac{y^2}{9} = 1$$

$$y = \sqrt{a^2 - x^2}$$



$$y = \sqrt{16 - x^2}$$



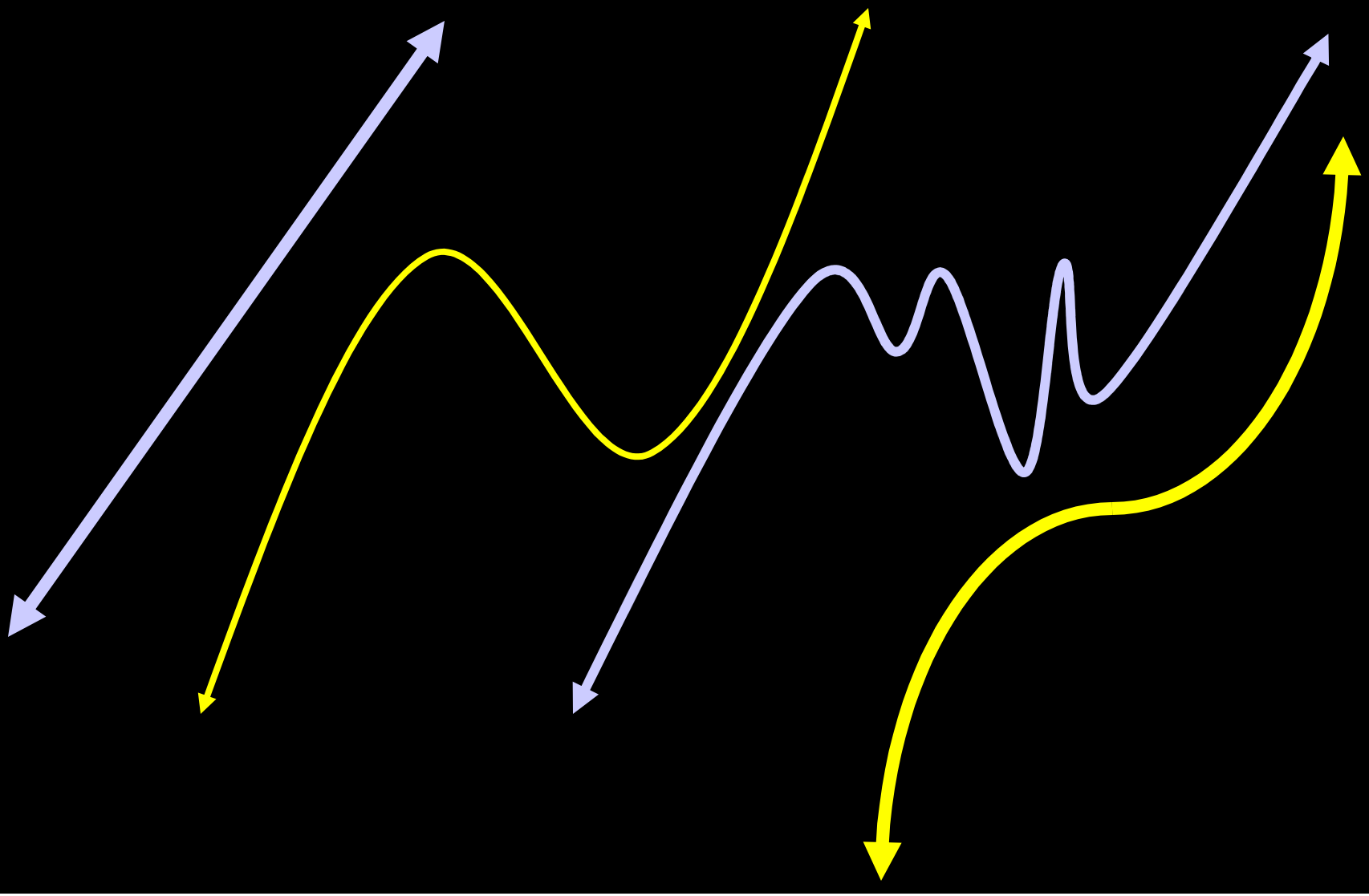
sketch

$$y = \sqrt{25 - x^2}$$

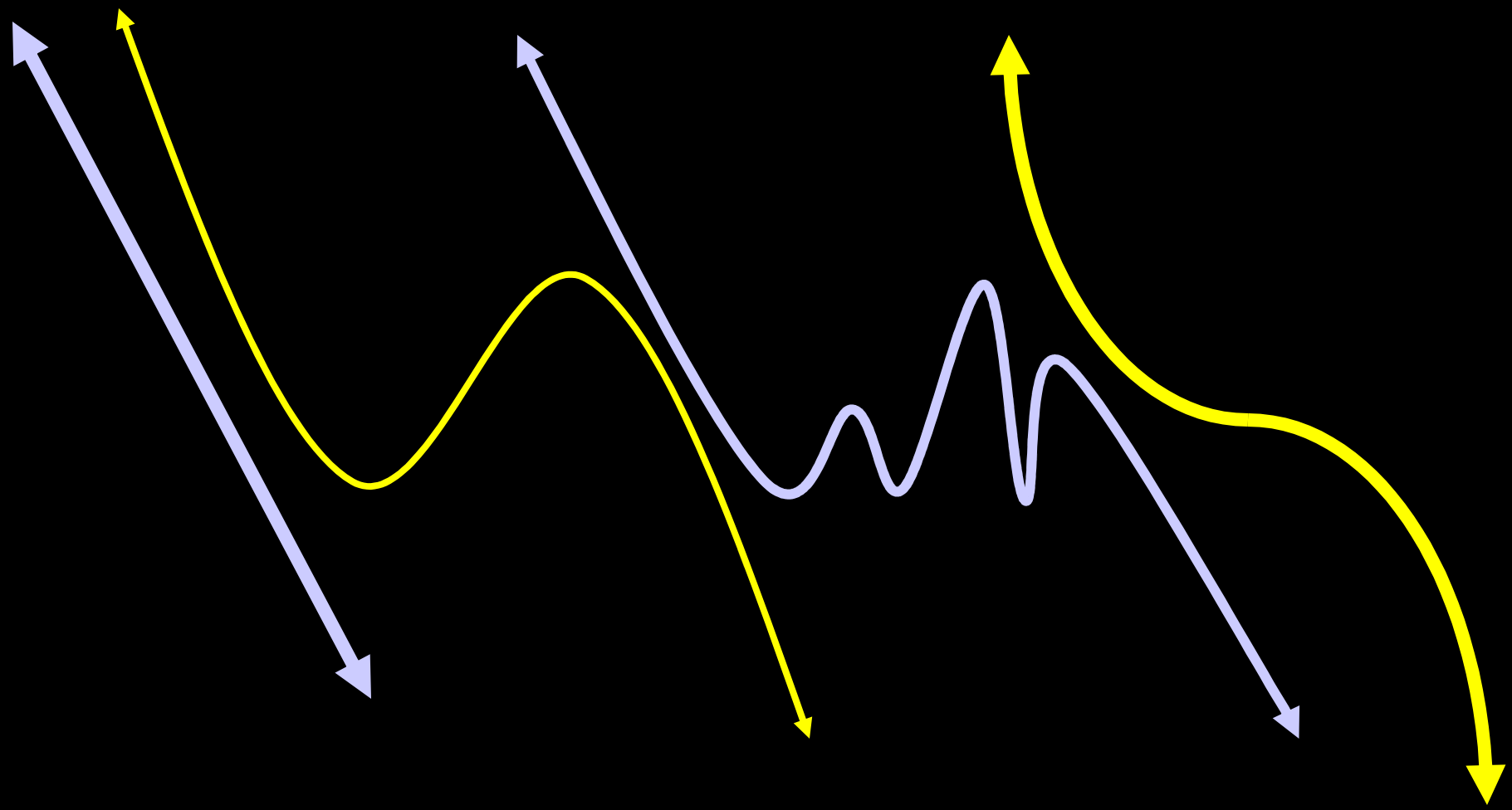
sketch

$$y = \sqrt{7 - x^2}$$

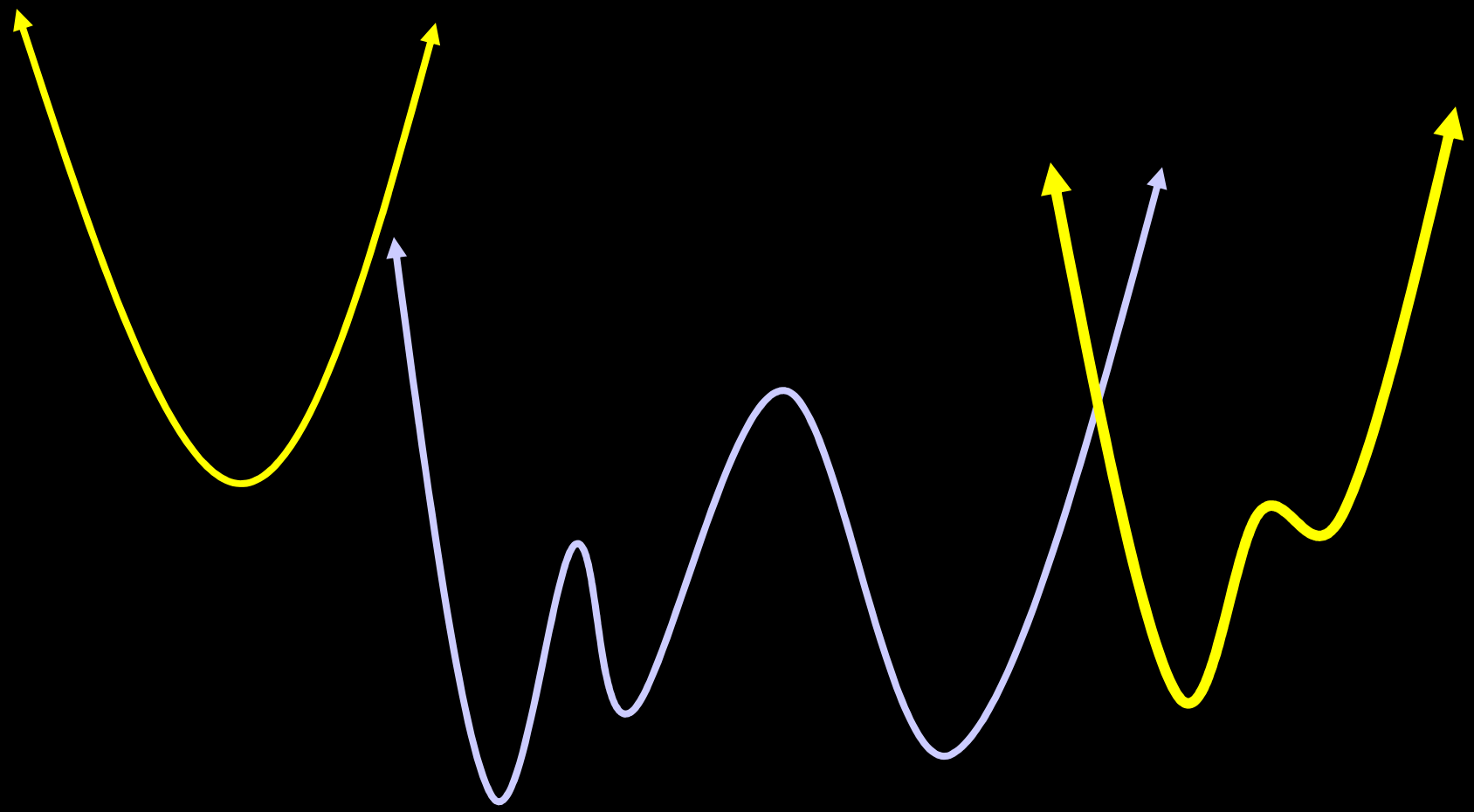
ODD power, POS coefficient



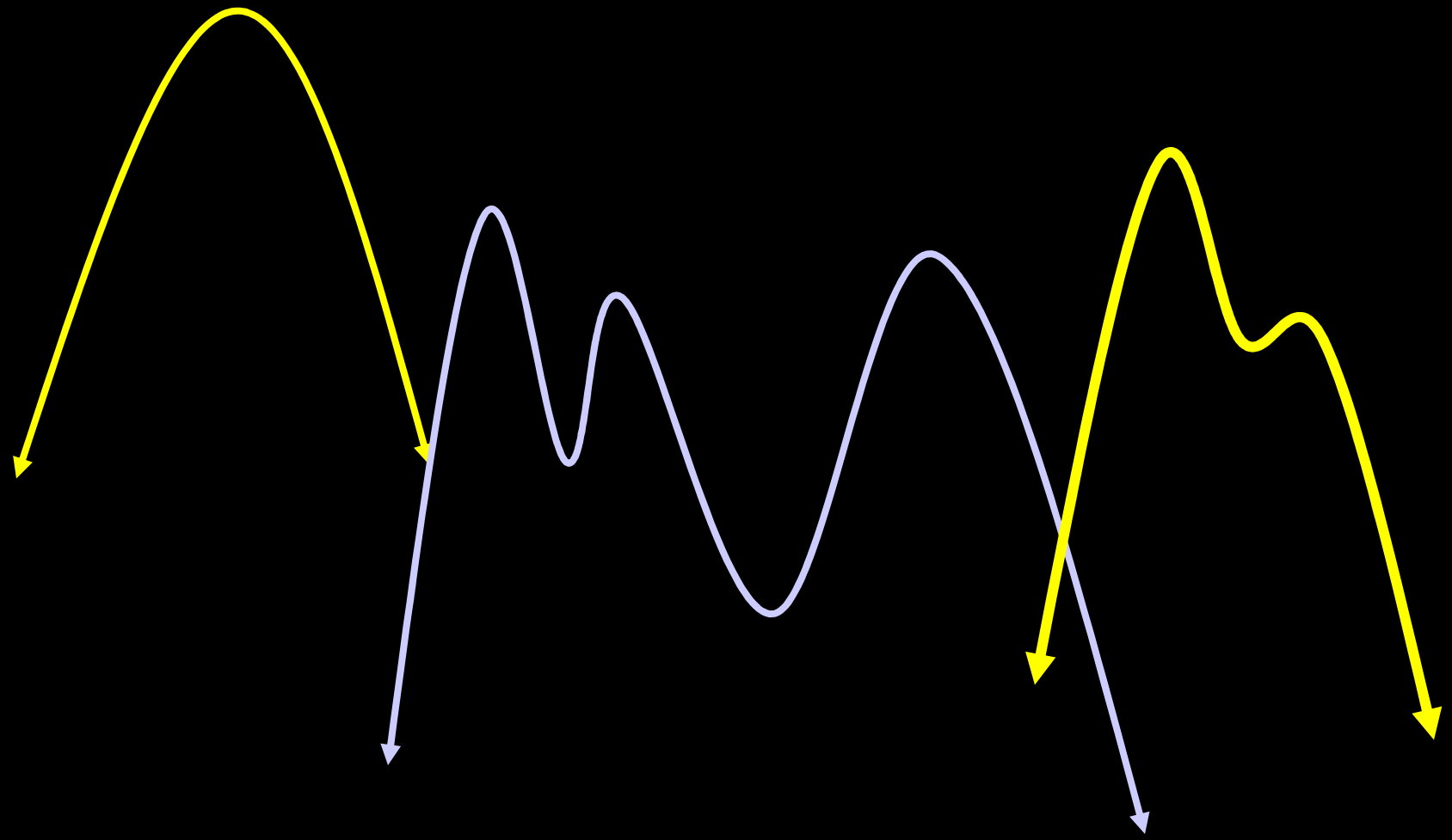
ODD power, NEG coefficient



EVEN power, POS coefficient



EVEN power, NEG coefficient



sketch

$$y = 3 + x^5 - 3x^4$$

without x and y axes

sketch

$$y = 5x^3 - x^2 + 3$$

without x and y axes

sketch

$$y = x^4 - 4x + 1$$

without x and y axes

sketch

$$y = -x^4 - 3x$$

without x and y axes

sketch

$$y = x^3 - 3x^5 + x$$

without x and y axes

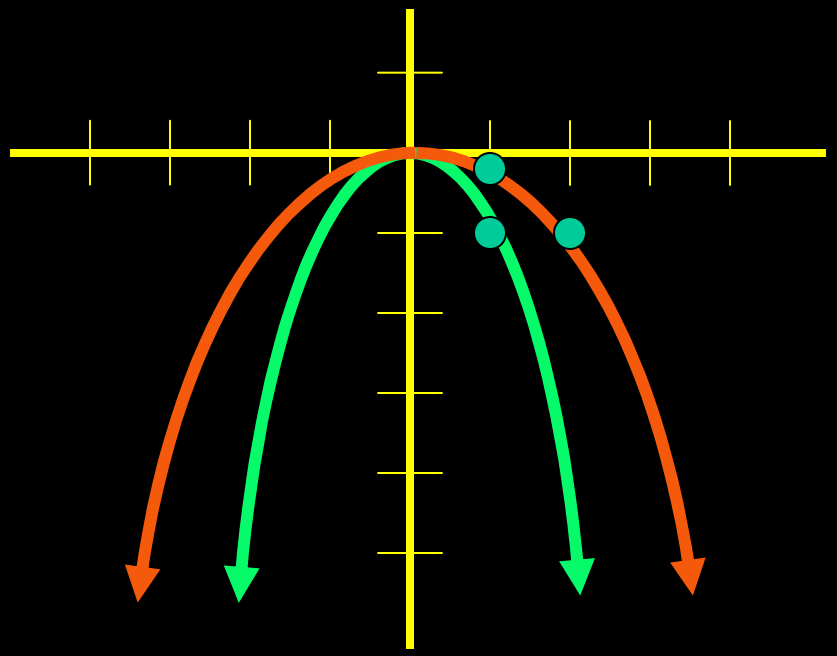
sketch

$$y = 9 - x^6$$

sketch

$$y = x^2 - x^3 - 9x + 9$$

Which of these parabolas is the graph of $y = -\frac{1}{4}x^2$



$$y = \frac{|x|}{x}$$