

# Starants II

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## Abstract

Digressing on the work of Comellas et al. into deterministic small-world networks, and extending the results of Pinheiro's work, we get to mathematically give reasons for the six degree of separation theory. In this paper we prove that, in considering people as mathematical objects, along with their circle of acquaintances, there might be, for  $n$  people, either a connection so strong as to give 3 degrees of separation, or so weak as to give  $2n + (n - 1)$  degrees of separation.

keywords: Small-world, communication networks, networks, combinatorial problems.

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## 1 Introduction

Deterministic small-world communication networks were introduced by Comellas, Ozon, and Peters in [COM00]. They are supposed to have strong local clustering (nodes have many neighbors in common), small diameter (largest of the shortest distances between nodes must be small), and would be located between Regular Lattices<sup>2</sup>, which are highly clustered, large worlds, where the diameter, or characteristic path length, grows linearly with the number of nodes, and Random Networks, which are poorly clustered, small worlds, where the diameter grows logarithmically with the number of nodes. We shall name them 'medium worlds'.

Circulant graphs are considered part of the deterministic small-world communication networks, once they have strong local clustering, but large average distance between pairs of nodes. They are included in the class of structured networks.

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<sup>2</sup>Obviously, this concept of Lattice must be a more graphical concept, having to do with something else rather than with algebra, once the algebraic concept, well described in [DAH06] does not have much to do with what Comellas writes about at all. On the other hand, [edi06] brings a broader sense, obviously allowed in language, which must be the one referred to by Comellas

In this paper, we want to contribute to Duncan et al. findings, by establishing a mathematical way of talking about their findings and mathematically proving results about their theory. Our previous results [PIN06c] brought constraints and mathematical symbology to Comellas et al. work.

## 1.1 Notation & some definitions

1.  $C_{n,\delta}$  - circulant graph of  $n$  nodes and  $\delta$  (degree) links per node such that each node  $i$  is adjacent to nodes  $(i \pm 1), (i \pm 2), \dots, (i \pm \frac{\delta}{2} \pmod{n})$ . This graph has got diameter  $D = \lceil \frac{n}{\delta} \rceil$  whenever  $\delta \neq 2$  and  $D = \lfloor \frac{n}{2} \rfloor$  otherwise.
2. Star graph - rooted tree containing  $n$  nodes with a central node (root) of degree  $(n - 1)$ .
3. Complete graph on  $n$  nodes - graph where every node has got degree  $(n - 1)$ .
4.  $S_{n,\delta}^C$  - string of  $n$  circulant graphs connected by means of  $K_2$  exactly  $\delta$  times for each circulant graph added (taking away the first and the last graph on the string which will use  $K_2$  exactly  $\frac{\delta}{2}$  times to make the connection): each vertex 'i' is connected to  $\left(\frac{\delta}{2} - \lambda \pm i\right), 1 \leq \lambda \leq \frac{\delta}{2}$ .
5.  $C_{n,\delta}^C$  - circle of circulant graphs connected by means of  $K_2$  exactly  $\delta$  times for each graph added: each vertex 'i' is connected to  $\left(\frac{\delta}{2} - \lambda \pm i\right), 1 \leq \lambda \leq \frac{\delta}{2}$ .
6.  $S_n^C$  - string of  $n$  circulant graphs connected by means of  $K_2$  as many times as we like for each circulant graph added.
7.  $C_n^C$  - circle of  $n$  circulant graphs connected by means of  $K_2$  as many times as we like for each circulant graph added.
8.  $SC_{n,\delta}$  - 'Starant' graph, that is, a circulant graph with degree  $\delta$  containing a star inside of it whose vertices coincide with the vertices of the circulant graph, a total of  $n$  vertices.

9.  $S_{n,\delta}^S$  - string of  $n$  star-graphs connected by means of  $K_2$  exactly  $\delta$  times for each star-graph added (taking away the first and the last graph on the string which will use  $K_2$  exactly  $\frac{\delta}{2}$  times to make the connection): each vertex 'i' is connected to  $\left(\frac{\delta}{2} - \lambda \pm i\right), 1 \leq \lambda \leq \frac{\delta}{2}$ .
10.  $S_{n,\delta}^C$  - string of  $n$  circulant graphs connected by means of  $K_2$  exactly  $\delta$  times for each circulant graph added (taking away the first and the last graph on the string, which will use  $K_2$  exactly  $\frac{\delta}{2}$  times to make the connection): each vertex 'i' is connected to  $\left(\frac{\delta}{2} - \lambda \pm i\right), 1 \leq \lambda \leq \frac{\delta}{2}$ .
11.  $C_{n,\delta}^{SC}$  - circle of  $n$  Starant graphs connected by means of  $K_2$  exactly  $\delta$  times for each Starant graph added: each vertex 'i' is connected to  $\left(\frac{\delta}{2} - \lambda \pm i\right), 1 \leq \lambda \leq \frac{\delta}{2}$ .
12.  $C_{n,\delta}^S$  - circle of stars connected by means of  $K_2$  exactly  $\delta$  times for each star-graph added: each vertex 'i' will be connected to  $\left(\frac{\delta}{2} - \lambda \pm i\right), 1 \leq \lambda \leq \frac{\delta}{2}$ .
13.  $S_n^S$  - string of  $n$  star-graphs connected by means of  $K_2$  as many times as we like for each star-graph added.
14.  $C_n^S$  - string of  $n$  star-graphs connected by means of  $K_2$  as many times as we like for each star-graph added.

## 2 Connectivity, Accessibility, Spreadability

We must pay attention to the different concepts involved in all the work relating small worlds and our work plus Comellas' work. We wish to write a bit about it:

- Accessibility - this concept is only about how modern a society actually is, or how democratic. The trend is having top societies, such as the American one, where one believes the top concept of democracy was achieved, with more accessibility to its members than the others. Why? Majorly, a highly developed society knows time is valuable and would

not bother top authorities for nothing. With the absence of ignorance and shortage of knowledge in the members of society, the leaders are freer in the sense of not needing much protocol to be reached once protocol, in its intentions, was created to teach the ignorant not to bother those busy members of society with stupidities that could well be dealt with much lower in hierarchy level members. Accessibility also has to do with how telecommunication processes evolved with time, that is, the quicker and the more efficient the communication processes involved are, the more accessible all members of that society who owns those developed means of communication will be. Ideally, a highly developed society would present something like two degrees of separation between any member of the generalized public and their leaders, at most. Having six degrees would then be really outrageous for a highly developed society. In those regards, small world experimentations in real life, proposed and dealt with even on the internet, are really silly. That shows nothing about Mathematics and could never show also. Accessibility is way out of the scope of Mathematics because it is simply not interesting enough to deserve its attention. It is perhaps some matter for statistics and control of a society development/democracy measurements;

- Connectivity - this concept is about how intimate people actually are. Some member of society might have even a sister who has got connectivity zero with him/her whilst a mate has got connectivity 100%, for instance, and in that we refrain from mentioning any sexual factor. Connectivity has to do with disease spread and, therefore, is relevant to Health, Mathematics, and Statistics, as well as for the government which is highly worried about becoming a first World Country. A person who is highly connected to another is going to have constant personal contact with the other, via some means of communication. In that instance, how they communicate is also highly relevant. There are ways others than personal communication face to face to get intimate with a person. Say, for instance, Internet. Therefore, a person may be intimate with another and still not be relevant to disease spread, our major issue of concern. However, it is definitely true that if a person has got face to face contact that is relevant to disease spread and that any sort of constant communication of any sort increases the probability of face to face contact. In those regards, Statistics would be the

area concerned. However, if we put things in a static point of time, as Comellas and us did, what matter is the actual static picture of how people interact, which might be interpreted as being face to face. In those regards, our interpretation of things does describe what is necessary as tool to analyze disease spread with a perfection not seen before. There is a start of that in our first paper, named Starants. This second paper brings already a small club of friends who are in constant contact face to face or not depending on what the intentions of using the graphs are, that is, it is suitable for disease spread as much as it is for traffic control, as we have already mentioned in Medium Worlds, yet to be published. We then provide tools for the computer scientist to feed machines with information relevant to disease spread control once the person would easily mention those he/she has as face to face contact easily to the doctor or, if necessary, by means of force and use of a lie detector. With that, by computer insertion of data, the computer scientist and the doctor are able to predict how long it is going to take them to fully isolate those people who might already be infected and by analogy to the patient's chain of face to face contact, once people usually get friends with those with same chaining power, one gets to know how quickly they can stop the spread if the disease started at time X in the patient and he/she is the first to take notice of the disease. It is all reasonable and workable in terms of both Mathematics and the computers. It might be simple Mathematics, but is still highly useful. Were it not the human being, like us, devising a better or more accurate way of representing people's circles of acquaintances, the computer scientists could never do the trick for the immunologist or the person worried about stopping the disease spread. In that sense, our finding is a really major finding, not to say striking. Towards that direction, any small improvement is a great improvement, a great step into human's problems solution. Just like the first guy in the moon said.

- Spreadability - Spreadability is definitely a geometric feature of diseases. If diseases ever spread linearly, stopping them would be quite easy. Therefore, linear spread is not a concern, but epidemia, exponential upwards spread is. For those ends, one may check on [NEW02]. Having that into mind makes us realize that it is necessary that we act quickly and that is when things all converge, that is, that is when accessibility becomes a relevant factor. One can easily prove, in a Phi-

losophy Essay, how ‘being highly developed’ in the sense we exposed before, is then highly relevant for stopping disease spread, that is, at the end of the day, it is all really connected. If in our graphs it is 6, 100, 1000 steps until the last person is infected, it might be the case that democracy and telecommunications have evolved in such a speed that with a simple telephone call we reach the physician of the last infected person, that is, several people calling, the same number of infected or possibly infected, might stop disease spread in five minutes if all telephones/means of quickest communication are answered. In that sense, the Maths is really not relevant, but our graphs are. Why? To take note and calculate, to measure and update the control and the spread.

We now believe to have explained the relevance of our work in terms of disease spread control and display. However, our work is also suitable to study past disease spread and how it could be or could not be controlled. In that sense, all our calculations are useful for some specific purpose, yet to be defined.

So is future disease spread and prediction is not only highly relevant in weather, but should become a major concern in health, if it is not already. With precision in calculation we can predict easily how a population is going to be infected and how we can stop it if it ever happens again or for the first time.

One can easily create more scientific methods than the ones in place for the study of diseases. Even knowing as little as we know about Health, we certainly can tell there is not much Mathematics involved so far and it is obviously that, contrary to what we stated in [PIN06b], in the case of the Sorites, disease spread is totally suitable to the World of Mathematics once it is obviously computer-friendly or has to be anyway. Yet to be proven if all we state is feasible in terms of computers.

History of humanity basically works the same way we devise disease spread control working. Humans make use of history to predict and choose our future. For instance, everyone now knows that Marxism, as such, is indeed only a dream. Socialism, as applied, is harmful to society, etc.

Based on past experiences well documented in terms of Mathematics relating disease spread, almost all World problems that are health-related in terms of disease spread are on the way to be well controlled in their factors. That is a major finding once it is severely common these days that general practitioners mention virus but have not idea of what the disease actually is and do

not bother either. Any virus is all the same. Spreads same way, they think. Well, if they all think so, it must be the case that they do. And if they do, I do not want to even catch a flu, do you?

One must understand, however, that there are obviously several mathematical studies on disease spread and none should be disregarded. However, normal people do not usually understand those graphs very well and, at the end of the day, those who will invest money, worry about disease spread, are definitely not the mathematicians. Plots of disease spread with variations per race, per age, per time, etc, might be useful for a statistician, a mathematician, and even give something to the practitioners to think of for a time. However, if we are able to simulate in a computer with actual names of members of society the own disease spread, it is like a soap opera, a movie, and there is no member of society who will not understand it. With more people able to understand it, there will be more people who really matter, in terms of solving the problem, able to produce theories on how to stop disease spread. If telecommunications development is a major issue, then our language should be as suitable to those in the telecommunications field as it is to mathematicians and governmental people. In this sense, the technical plots involved in [NEW02] are useless.

Therefore, in a sense, our work complements and supplements what already exists.

On the other hand it is true novelty once it makes six degree theory, which only became famous due to its simplicity in exhibition, mathematically provable in the same sort of popular way it started, or, at least, we take the first steps into that.

For the sadness of many other scientists, mathematicians, statisticians, we should state that there is no more basic Philosophy<sup>3</sup> principle than that of always choosing the simplest way of proving, the more direct, the one that is more easily understood. In those regards, our work is sensible difference from the others.

### 3 New results

In [PIN06c] we argue that a person's circle of acquaintances might be mathematically represented by means of a Starant graph, which is basically a

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<sup>3</sup>As it should be well known, the Science involved in Mathematics is part of Philosophy and had its roots precisely there.

circulant graph with a star inside of it, making its diameter as short as 2. We now want to consider a situation where we label each vertex of a Starant with a number and treat the connections as cycles. Bearing in mind that no matter how many people a person knows one will always get the same diameter in a Starant, 2, makes us think that it is easy to mathematically prove Duncan's theory. One must be very attentive, though, for the fact that a person's circle of acquaintances might be an 'incomplete' Starant, that is, it is still a star, that is for sure, but the vertices of the star might not have edges between them, that is, a person might know  $n$  people who actually do not know each other.

In this sense, the theory we present here is still not complete. But it is already a good start.

Another issue that should deserve our careful observation is that in considering people's circle of acquaintances to be our Starants, it does not make any sense connecting the root of the star graph, that is, the person who originated that circle of acquaintances to another root. It is very logical to understand that if I have  $m$  people in my circle of acquaintances, all of them are going to be part of my Starant. Since we are not working with the dynamics of the problem, just the static, it only makes sense to think of possible connections from the people that I know to other people that someone else's know, never from root to root. That will always establish one degree of separation inside of each Starant. Therefore, for a square of  $n$  people, there are at least  $n$  degrees of separation.

Right here, one starts doubting Duncan's statements immediately. But if one thinks of the folding of the universe, similar to the Mobius band [gC06], and squeezing, then one thinks that is possible, that is, in an infinite grid of people and considering one person on the top and another on the bottom part of Mobius band then it is all possible. What we are actually stating is obviously that his theory would be a dream only achievable eventually if the right people in the universe are picked in an infinite number of participants in what regards connectivity, not accessibility. It is also obvious that as much as the concept used by Comellas on Lattices differs from the mathematical one, our concept of connectivity differs from the usual media concept. Given the limitation of Language in comparison to the speed of thought [PIN06a], that is the best term we could come up with, so that we beg the reader to read our definition of connectivity recursively each time the word is applied in our papers.

For the good understander, we have just stated and proved that it is humanly

impossible that Duncan's theory works for any amount of people, even infinite, considering the way static chains work. Well, in Language it is all acceptable, natural Language, as we explain in [PIN06a]. But in Science, that is, Philosophy, which encompasses Mathematics, it is not possible to accept his assertion as true, that is, just like in our Sorites solution from [PIN06b], we are forced to say 'no' to the veracity of his 'findings'.

Once any probabilistic chain may be photographed from a static point of view, his theory makes no sense at all in terms of connectivity. May still make sense in terms of accessibility, however.

The reason behind that is obviously that despite the length involved in the chain, whatever it is, even supposing the Mobius band fitted the picture of the chain as illustration, to make it easier, telecommunication could always create regular lines through which anyone could be reached via two steps (place an ideal, abstract, communication hub in the middle of the Mobius band, as we did with the middle of our circulant in order to create our Starant, and you can easily see how there could be a star generated by that hub, reaching every single person in our obviously finite chain of World people - only infinite in terms of human counting: perspective, referential, like in Physics, infinite for the human eyes, human life time in terms of counting one by one. But not even so, given the statistics already in place which by a single internet click gives us the result today of the World population counting).

Suppose then that we have a labeled Starant,  $SC_{5,1}$ , with the vertices from the circulant labeled with numbers from one to five and the root of the star labeled with the number 6. For this graph, it is true that the following cycles exist:

$$(12)(23)(34)(45)(15)(64)(65)(61)(62)(63).$$

Suppose now that we have another Starant, also  $SC_{5,1}$ , which we connect with the previous one, let's rename it to  $SC2_{5,1}$ . We now label its vertices from 7 to 12, 12 being the root of the star. For  $SC2_{5,1}$  we then get:

$$(78)(89)(9\ 10)(10\ 11)(7\ 11)(12\ 9)(12\ 8)(12\ 7)(12\ 10)(12\ 11)$$

Suppose now that  $SC_{5,1}$  connects to  $SC2_{5,1}$  via  $K_2$ , originating a new cycle: (10 1). Therefore, the longest of the shortest paths between them (diameter) is now the diameter of  $SC_{5,1}$ , 2, plus the diameter of  $SC2_{5,1}$ , 2, plus the edge connecting them, that is 5. One must bear in mind that this actually means that there is no intersection between the two circle of acquaintances, and we

might represent this by means of the following symbology:

$$SC_{5,1} \cap SC_{2,5,1} = \emptyset.$$

We know that one of the ways of decreasing this diameter is decreasing one of the Starants diameter, that is, for instance, making their degree 1, instead of 2. Notice that this is possible in real life because it is possible that all our acquaintances know each other, even in the same level of connectivity. Therefore, if we do this to one of them, we then get diameter 4 to our set of two Starants. If we do this to both of them, we get diameter 3. This proves that the connection of a Starant to another takes something between 3 and 5 steps, that is, the total diameter of an ‘Australian square’, let’s call it  $D_{A_n,\delta}$  may be expressed as

$$3 \leq D_{A_n,\delta} \leq 5.$$

With this, we write our first theorem:

**Theorem 3.1.** *An Australian square has got diameter between 3 and 5.*

*Proof.* As written above. □

which follows our first new definition:

**Definition 4.** *We call an Australian square the static picture of two Starant graphs with no intersection between their set of vertices connected to each other via an obligatory  $K_2$ .*

The importance of giving a name to our set of starants glued this way is to be able to refer to precisely that static picture with no mistake. Once there is no similar graph theory object like that, given Starants did not exist before either, we must create a new name. Having into sight that Australia is first World but not as first World as the States, being a few degrees away in terms of development of democracy and everything else, we actually think it is all real life related coherently. The best intentions of giving popular names to mathematical or computer objects is making them easy to remember in the heads of non-mathematicians. We believe that this is the best choice we could possibly have. Once we consider that there is regularity, also expected in development, balance of things, it should be a highly developed society, not a messy one where relationships are highly uneven when one member of

society is compared to another. If indices are correctly built, and it is not our area of study so we just assume they are, first World nations should have their name originating from the fact that they are much more developed than the others and because we have associated development with accessibility and we are interested in reaching Duncan's work, which we already stated to be only possibly valid in the scope of accessibility, that is severely relevant. With this in mind, one can see that if we duplicate the amount of Australian squares, that is, we make them two, in getting at least 3 and at most 5 as diameter for each Australian square, and supposing they only connect either horizontally or vertically, our new diameter would be between 5 and 11.

So far, notice the fact that we only allow our starants to connect from end of star to end of star via a  $K_2$  graph, with no exception. We seriously believe this would be the minimum that would happen in any society or randomly chosen group of people in a static picture.

If we allow them to connect diagonally, though, say that only  $SC2_{5,1}$  and  $SC4_{5,1}$  do it, we still get no reduction. However, if we also diagonally connect  $SC1_{5,1}$  and  $SC3_{5,1}$ , we then get a reduction of diameter from the bounds 5 and 11 to 3 and 5. With this, we have proved that:

**Theorem 4.1.** *An American square has got diameter between 3 and 11.*

And we now introduce a new definition:

**Definition 5.** *We call American square the set of two Australian squares with no intersection between their sets of vertices connected by means of at most one  $K_2$  sort of connection between each couple.*

Notice the danger here. If this sort of reasoning spreads, then Duncan not only is correct but it would actually be 3, half of his predictions, rather than 6, once it all is passive of reduction to 3, sufficing that diagonal connections can always be made by at least one member of each person's circle of acquaintances. However, here, we must notice that this regularity would have to be possible to be kept for an extenuating amount of people, that is, the World population, what is, again, impossible. Notice that it is apparently always necessary that each acquaintance's circle has got at least one member which knows someone from each other circle belonging to the description. Considering the dimension of things, one can see that it would be easily achievable in small populations but not in large ones, the opposite of his statements. In this case, the hub would be replaced by diagonal connections transferred to

the already existing graphs.

One may notice that the higher bound is always growing and the lower bound is always the same. We already spoke in detail about the lower bound and do think we have exhausted its possibilities. On the other hand, the higher bound is never interesting once for disease spread, we are always worried about the lower bound, and it could be useful for communication theories. In this case, it is easy to predict already by means of mapping the territory with our American or Australian squares.

We can then state a more general theorem:

**Theorem 5.1.** *For a group of  $n$  people, it takes something between 3 and  $2n + (n - 1)$  steps for them to communicate amongst themselves<sup>4</sup> if and only if the actual graph connections correspond to all possibilities of connections involved, that is, if telecommunication is ever considered, it has to be confined to the boundaries of our graphical display.*

Therefore, if we have 100 people, it might take 299 steps to get effective communication/transmission of disease.

## 6 What are Australian/American squares good for?

Notice that an Australian square holds more or less the same results for steps in communication than the six degree separation theory. That must mean there is a way of connecting or corresponding their theory with our graphs in order to explain it mathematically. If not precisely to our graphs to something very similar containing small alterations. What that means is that we can also prove it geometrically, which makes the explanation not only scientific but popular.

Any mathematical theory is proven wrong by a single example where it does not work. Duncan's theory is in the scope of probability, however, and that is when it becomes really hard to prove that it is wrong once it works in terms

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<sup>4</sup>Considering they must go through a possible path in the graph representing their connectivity. This reasoning actually fits precisely the scope of disease spread and that is where we disagree with Duncan's choice about communication. We are talking about face to face contact that is actually possible for connection reasons of any sort.

of average so that there might be a case or two that fail but in average it all works. However, if we find a single person from whom it is impossible to find any connection with the Namibia or Afheganistan president, for instance, there is then a distance of infinity to that president and there is no average of 6 made ever possible. So that it is obviously possible to find that example, even with a retard, a person who never goes out and only knows their nurse who knows only their father or mother, for instance, who are humble people and never traveled. This way, it is infinity and his theory can only be wrong. With the Mobius band argumentation, we took the assertion that the degree was 6 not the average into consideration. And we are now talking about the average, which is the actual proposal of Duncan's work. In any hypothesis, it is not true.

We have proved, however, that with highly developed communication, any network could have 3 degree diameter. In this hypothesis, stopping disease in the quickest time as possible would mean one step from everyone else's first contact. That would be the only way of guaranteeing effective disease spread stop for any disease.

We believe we have settled the matter in terms of showing to everyone else that six degree theories could only refer to accessibility because it is mathematically, statistically, and sociologically unacceptable otherwise for human reasoning.

Therefore, the importance of our work is also the same as in the Sorites paradox, defining things precisely, their scope. Not only that is a basic step for further progress towards the right direction of research, but helps everyone else to see the value of our/their research into small/medium worlds.

In that sense, if there is any doubt about the use of our terminology, we hope to have finally sorted those doubts out.

Apart from that, it seems that in real life it is quite possible that someone we know is part of someone else's circle of acquaintances. This situation would originate, for instance, an Australian square with acquaintances' circle intersection different from the empty set. This is a case where the vertices of the Starants will overlap. With this, there is one less step to go from one Starant to the other and we are stuck with  $2n$  distance in the string situation but 3 as minimum as well. Easy to see that this is the only way we could possibly get 6, that is, Duncan assumes there is always or in majority or in average one acquaintance in common between two different people's circle of acquaintances.

We can say that it is very likely that someone that we know is also part of

someone else's circle of acquaintances and we would expect them to be part of a certain number of circles in average, what could then reinforce their result. But the interesting information here is that as the number of circles increase, that is, as we go in a World scale, that probability actually decreases, that is, it is precisely the opposite of the message that has been conveyed by Duncan. That is easy to see if one considers something as simple as Brazil and Europe and all the circles of acquaintances of Brazilians having nothing in common with the circles of acquaintances in Europe in its vast majority. Since Europe is enormous, and so is Brazil, that will not give us an average that favors Duncan's observation at all.

Because this trick is easily done with other continents, the average cannot be what he states. It is obviously the case that we have worked in the positive direction, but one can easily go backwards to state that the following would give us a definite no in statistical terms to Duncan's statements:

- We consider all continents in the World as basis;
- We get half of a continent who does not know anyone in any other continent;
- We get half who know less than one tenth of each chosen half in the other continents.

In order not to have 6, that is, twice the minimum degree after we draw diagonal links, for  $n$  people, it suffices having, for instance,  $\frac{n}{2}$  people that know less than  $\frac{n}{5}$  circles of circles of acquaintances others than their own. That actually seems very likely. Once more, we cannot provide mathematical evidence to destroy a theory born in statistics, please check [PIN06a] for full argumentation in those regards understanding that Statistics makes use of Mathematics but is more than Mathematics because it involves human factors and at the same time does not contain it in full. Unfortunately, the assertion made by Duncan was in the complementary set of the intersection between Mathematics and Statistics so that there is no room for a mathematical proof there at all. If it is all about probability, suffices stating that it is also likely that it does not happen and showing a situation which makes that statistics result hardly attainable. Because it is all probabilistic, however, there is no possible other negative evidence than this. It cannot be conclusive.

Another interesting point is that if we depart from static small pictures, like

we have done, it is usually the case that total strangers never have common acquaintances and we actually believe this is the point in Duncan's theory presentation. As failure as the Sorites paradox, its presentation illudes us to think Duncan intends a message that he never did, perhaps, once if he had ever intended the message we get immediately, given the illusion, he would always be incorrect. If someone tells the theory wrong, probably, actually, what happened to the Sorites, more than likely, like this:

- Presenter: Someone in the audience, please, raise your hand... What is your name?
- Blah, blah, blah.
- P: Mrs. Blah, I want you to contact the President of the USA now. Can you think of how?
- Mrs. Blah goes: well, I know someone who knows someone...
- P: Call!
- Mrs Blah calls that someone and wow, in 5 calls we get to the President!
- P: You see, anyone, anywhere may get to anyone else anywhere in at most 6 communication steps!

Just like our Sorites paradox, we get it all wrong, please check on [PIN06b]. First of all, Duncan never said that. He said that in average, that means, not everyone, not at any time...it might even be the case that Mrs Blah never get to speak to the President at all. And that is another interesting point. Even if I know someone in the President's office, it might be the case that the telephone, even having his personal number, is never answered by the time we need in the presentation or test. If that happens with everyone else, once it is obviously impossible that it all is about personal contact, where it obviously fails, or the majority of people involved, his communication theory demands a time extension to be correct. But that time makes the problem not being solved by the time which is necessary to prove his assertion so that it might not be true even if accessibility is of 2 steps type, that is, best as possible, top democracy in the World. It suffices having a problem with the telephone company at that time and day and it will never be true. It will never be true in average, it will never be true in any possible sense.

Boring enough, the static picture of Duncan's problem might as well be the one where the telephone lines all fail. In this case, none of our theories apply. Basically, we write about possible connections as well, however. And we did mention face to face contact. That is why we worried about reducing the scope of the population involved, even because if we can control every small village in terms of disease spread quickly via their local medical center, the epidemic is indeed stopped.

With all these conclusions, we actually notice that the only interesting contribution that Mathematics could provide the World with using Duncan's work would be in the area of very small graphs, that is, describing determined finite number of people involved. We obviously suggested community health centers to be the societies involved and their frequent patients. Once more, for it all work and be suitable for Mathematics contribution, it would be necessary the frequency and the obligatory attachment of all that population to their local community health center. However, the Mathematics, as usual, may be there ready to be applied in ideal situations that never happen in real life or only by luck do. We have then tried to convey the message that his theory seems to be quite attractive in the same sense as the Sorites paradox, something with wrong presentation. However, the Sorites made us come up with really useful insights for Logic and other Sciences. The same way, we may start thinking of these other concepts brought up by Duncan's theory trying to fit some Maths where there is no room for it, human communication. Statistics is suitable, because it is about average, actual facts, and may be measured against time. However, cannot ever be of mathematical use. We truly think the only possible contribution of Duncan's theory for something passive of Maths is what we have done, the graphs so that computers might be used to control, predict, register, disease spread in a more accurate way. Anything else falls inside of the same scope reported in the Sorites problem and it is useless. The creation of a new graph, however, gives us other paths to pursue in terms of pure mathematical theories. And we shall as well do it.

## 7 Conclusion

In this paper, we have worked with new mathematical objects using the concepts of cycles and Starants. We have introduced both Australian squares and American squares. Our work seems to be, in a sense, on the back of Dun-

can's et al. reasoning, but he never stated so or hinted on that. Therefore, we take it as ours, 100%. Our conclusions are suitable to prove, control, measure, and study disease spread scientifically/computer-friendly in a way to provide actual solutions, practical solutions, rather than just useless studies as the ones presented in [NEW02], which might be totally accurate but represent no practical contribution to the computer environment so that they are not applicable in terms of providing governments with actual tools to monitor, control, stop, disease spread. Our work gives the computer scientists a way of inserting data in the computers accurately so that any amateur may make use of it and study/solve their problems of disease spread besides cataloguing. And we also believe to have achieved the best model for such, some totally perfect model of reality via graph theory. With the improvement of our mathematical tools, we wish to make our graphs computer friendly in a way to make it easy for the Statistics to forecast immediately in real time. Obviously, this work complements any studies of the sort presented in [NEW02]. However, if one takes the spread to be the quickest as possible, disregarding their actual spreading function/mathematical model, diseases will always be stopped the quickest way as possible so that the modeling work loses its practical value once all we want in the modern world is stopping any disease spread in the quickest time as possible. For that end, it suffices having a model for the worst and quickest one.

We also believe we have progressed further in terms of providing restrictions for the situations in which Duncan's theories apply. We have proved that Duncan may only be referring to accessibility, as defined in our terms. We have also proved that there is the same likelihood that he is wrong as there is that he is right, all using the same sort of statistical inferences.

We have proved that his theory must be very well restricted. We also have empowered people with prediction 'static power' over a certain period time of precise figures in terms of connectivity of people if we are right about the way they connect, that is, given hour X of day Y, we are able to mathematically measure connectivity or predict it in terms of what is possible to have in human's face to face eventual frequent contact.

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