

Bubble radius function - non isothermal film blowing

M.R.Pinheiro¹, Department of Mathematics, RMIT, Au,

msorfiap2@yahoo.com

Abstract

In this paper, we present some initial investigations on how to go analytically on solutions to the model for film blowing in a Nonisothermal situation [M.R04a]. The model used here bears some refinement on Han and Park results for nonisothermal film blowing, as explained in [M.R04a]. The methods we use here are asymptotic expansions, as presented in [Bat03] and Poincare series.

keywords: nonisothermal, film, blowing, newtonian, power-law, radius, solution, velocity, temperature.

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1 Introduction

Biodegradable “plastics”

The environmentally friendly packages are made of natures own

¹Correspondence to PO BOX 12396, A’Beckett st., Melbourne, AU, 8006

polymers by using starch, gluten etc. According to Helén ([Hel04]), biodegradable packages can be disposed by composting. As a result they degrade into CO₂, H₂O and biomass through the action of microorganisms, heat and moisture. If the conditions are anaerobic, methane is also formed. The main packaging materials are starch-based Mater-Bi, PHB/V- copolymer (with the trivial name of Biopol), polylactide, cellophane and biopolymer coated paper and board. For example, polylactides are made from polyesters by microbes. As reference materials the research uses synthetic polymers.

There are always interactions between the product and the environment. Packing controls this interaction. Factors affecting the quality of the product are moisture, O₂, CO₂, microbes, migrations, light and different aroma compounds. Therefore, in designing packages the critical quality properties of the product and the influences of different environmental factors concerning them must be known.

Many types of food products can be packed in biodegradable materials, but so far using them in dairy and meat products hasn't

been very promising. Some preliminary results of the research show that some of the tested materials provided reasonable shelf life for products like tomatoes and cucumbers. However, the barrier properties of biodegradable polymers are not sufficient for longer storage periods. The problem with the eatable plastic bags is that their structure is not yet sufficient enough to water. The major problem of biodegradable materials in general is the high water vapor permeability.

In the future, barrier and mechanical properties of decomposing packages will have to be developed to be able to compete with synthetic polymers. Another issue is to get the prices of these two material types into the same level. Potential markets for biodegradable packages are enormous beginning from beverage plates continuing to compost bags, yogurt cups etc.[Hel04]

Water vapor produces a thermal variation on the plastic. Therefore, studying things from a mathematical point of view might lead to success when dealing with situations like the one described above. Taking into account the hazard that plastics are for the environment, any progress in the field would be more than welcome.

Han and Park ([CDH75]) obtained some results on nonisothermal newtonian fluids as a restriction of power-law fluids. Tam ([D.C03]) took a completely newtonian approach, his results dealing with the isothermal situation. Alaie and Papanastasiou ([SA93]) considered the nonisothermal situation of film blowing and treated it with an integral constitutive model. Kanai and White ([Kan]) produced an experimental study on the stability of nonisothermal (temperature dependent viscosity) film blowing of viscoelastic newtonian melts. Yamane and White ([HY87]) researched on the significance of non-newtonian viscosity on non-isothermal film blowing. M.R. Pinheiro ([M.R04a]) proposed some corrections on the model presented by Han and Park in order to more adequately deal with temperature changes.

In this work, We depart from the corrected model ([M.R04a]) presented by M.R. Pinheiro, and we perform studies on the radius variations and its effects on the remaining functions involved in the system.

2 Notation

We tend to use capital letters for the non-scaled variables under study, which are also taken to vary independent from time, just dependent on height, that

is, we study the system from a steady-state perspective, rather than from a dynamic one:

$R=R(Z)$ =Generic radius of the bubble, which varies with height

R_f = Radius of the bubble from freeze line upwards

$E=E(Z)$ =Thickness of the bubble, which varies with height

$T=T(Z)$ =Temperature of the bubble, which varies with height

$U=U(Z)$ =Meridional velocity of the fluid, which varies with height

E_o = Initial thickness of the bubble

H =Final thickness of the bubble, final being at the height of the frost line

Z_1 =Frost line height

Z =Height

and small letters for the scaled variables under study:

$r=r(z)$ =radius

$w=w(z)$ =thickness

$s=s(z)$ =temperature

$u=u(z)$ =velocity

z =height

3 Definitions

1. $e_1 = \ln \frac{U}{U_o}$

Logarithmic growth of velocity, which can also be seen as a distance measure from initial to present velocity.

2. $e_2 = \ln \frac{E}{E_o}$

Logarithmic growth of thickness, which can also be seen as a distance measure from initial to present thickness.

3. $e_3 = \ln \frac{R}{R_o}$

Logarithmic growth of radius, which can also be seen as a distance measure from initial to present radius.

4 Force Balance Equation - Meridional direction

As explained in more detail in [M.R04a], our equation is:

$$F(Z) = F_o + \pi \Delta p (R^2 - R_o^2) + 2\pi \rho g \int_0^Z RE \frac{dZ}{\cos \theta}$$

But, it is also known that

$$F(Z) = T_{11}dA_1 \cos \theta = 2\pi RET_{11} \cos \theta$$

dA_1 being an infinitesimal part of the top of the fluid element under consideration.

5 Total composition of forces

Here, we proposed a small correction on the formula presented by Han and Park. For details, see [M.R04a].

Our pressure equation gets then translated into

$$\Delta P = \frac{P_L}{R_L} + \frac{P_H}{R_H} - \rho g E \sin \theta$$

where

$$\left\{ \begin{array}{l} P_L = T_{11}E \\ P_H = T_{33}E \\ R_L = \frac{1}{\frac{|R''|}{(\cos \theta)^{-3}}} \\ R_H = \frac{R}{\cos \theta} \end{array} \right.$$

6 Material Function

According to Han and Park ([CDH75]), one verifies the following viscosity relationship for a nonisothermal newtonian film blowing:

$$\eta_b = \eta_o e^{\frac{\left(\frac{1}{s(z)} - 1\right)^E}{RT_o}}$$

7 Temperature Equation

We accept Han and Park's temperature equation provided in [CDH75]:

$$s' = -Dr \sec \theta (s - s_a) - Yr \sec \theta (s^4 - s_a^4)$$

where

$$D = \frac{UR_o 2\pi Z_1}{Q\rho C_v}$$

and

$$Y = T_o^3 \frac{R_o 2\pi Z_1 \lambda \epsilon}{Q\rho C_v}$$

8 Scaling

We decide to disagree with the scaling for r , s , and u , chosen by Han and Park because we believe that ours will provide us with nicer initial conditions: $r(0) = 1$, $s(0) = 1$, $u(0) = 1$, $w(0) = 1$. Han and Park choose to scale their variables against a_o , the radius at the annular die. We chose to scale z against Z_1 , the freeze line height. With this, $z = 1$ at the freeze line height. Our process, therefore, takes place between $z = 0$ and $z = 1$, a much nicer interval. We propose:

$$\eta = \frac{\eta_b}{\eta_o}, z = \frac{Z}{Z_1}, r = \frac{R}{R_o}, u = \frac{U}{U_o}, w = \frac{E}{E_o}$$

9 Resulting System

As a result from the previous function and equations, one gets to describe thermal effects on a newtonian film blowing situation through the system presented in [M.R04b]:

for $z \in [0, a) \cup (d, 1]$, where a and c are the turning points of the bubble profile, we have, as an approximation, $r' = r'' = 0, r = k$:

$$\begin{cases} f_o + B(k^2 - 1) + \frac{A}{C} \int_0^Z (\lim_{u \rightarrow 0} \frac{1}{u}) dz = 0 \\ s' = k[D(s - s_a) + Y(s^4 - s_a^4)] \end{cases}$$

for $z \in (b, c)$, we have, as $r = c_3z + c_4$ for some c_3, c_4 in \mathfrak{R} , $r' = c_3$ and

$r'' = 0$. Therefore, our system is:

$$\begin{cases} 2B(c_3z + c_4)^3u(1 + (Cc_3^2)^2 + AC(c_3z + c_4)^2c_3(1 + (Cc_3^2)^2)^{\frac{3}{2}} \\ -C(1 + (Cc_3^2)^2)(2uc_3 + (c_3z + c_4)u')\eta = 0 \\ C(1 + C^2(c_3^2)^2)^{-1}(\frac{c_3u+2u'(c_3z+c_4)}{u(c_3z+c_4)}) \\ = f_o(\eta^{-1}) + B((c_3z + c_4)^2 - 1)(\eta^{-1}) + \frac{A}{C}(\eta^{-1}) \int_0^Z \frac{1}{u}(1 + C^2(c_3^2)^2)^{-\frac{1}{2}} dz \\ s' = D(c_3z + c_4)((1 + C^2(c_3^2)^2)^{\frac{1}{2}})(s - s_a) + Y(c_3z + c_4)((1 + C^2(c_3^2)^2)^{\frac{1}{2}})(s^4 - s_a^4) \end{cases}$$

For $z \in [a, b]$ we have (since $r'' \geq 0$):

$$\begin{cases} 2Br^3u(1 + (Cr'^2)^2 + ACr^2r'(1 + (Cr')^2)^{\frac{3}{2}} \\ +C^3r''(2r^2u' + urr')\eta - C(1 + (Cr')^2)(2ur' + ru')\eta = 0 \\ C(1 + C^2(r')^2)^{-1}(\frac{r'u+2u'r}{ur}) \\ = f_o(\eta^{-1}) + B(r^2 - 1)(\eta^{-1}) + \frac{A}{C}(\eta^{-1}) \int_0^Z \frac{1}{u}(1 + C^2(r')^2)^{-\frac{1}{2}} dz \\ s' = Dr((1 + C^2(r')^2)^{\frac{1}{2}})(s - s_a) + Yr((1 + C^2(r')^2)^{\frac{1}{2}})(s^4 - s_a^4) \end{cases}$$

Whilst for $z \in [c, d]$ we have (since $r'' \leq 0$):

$$\left\{ \begin{array}{l} 2Br^3u(1 + (Cr'^2)^2 + ACr^2r'(1 + (Cr')^2)^{\frac{3}{2}} \\ -C^3r''(2r^2u' + urr')\eta - C(1 + (Cr')^2)(2ur' + ru')\eta = 0 \\ C(1 + C^2(r')^2)^{-1}\left(\frac{r'u+2u'r}{ur}\right) \\ = f_o(\eta^{-1}) + B(r^2 - 1)(\eta^{-1}) + \frac{A}{C}(\eta^{-1}) \int_0^Z \frac{1}{u}(1 + C^2(r')^2)^{-\frac{1}{2}} dz \\ s' = Dr((1 + C^2(r')^2)^{\frac{1}{2}})(s - s_a) + Yr((1 + C^2(r')^2)^{\frac{1}{2}})(s^4 - s_a^4) \end{array} \right.$$

all with the following parameters, material function, and scaling:

$$A = \frac{\rho g R_o^2}{\eta_o U_o}, \quad B = \frac{\pi \Delta p R_o^3}{\eta_o Q}, \quad C = \frac{R_o}{Z_1}, \quad f_o = \frac{F_o R_o}{\eta_o Q}, \quad \beta = \frac{E}{RT_o}$$

$$D = \frac{UR_o 2\pi Z_1}{Q\rho C_v}, \quad Y = T_o^3 \frac{R_o 2\pi Z_1 \lambda \epsilon}{Q\rho C_v}$$

$$\eta_b = \eta_o e^{\beta(\frac{1}{s(z)} - 1)}$$

$$\eta = \frac{\eta_b}{\eta_o}, \quad z = \frac{Z}{Z_1}, \quad r = \frac{R}{R_o}, \quad u = \frac{U}{U_o}, \quad w = \frac{E}{E_o}$$

And with the following initial conditions:

$$r(0) = w(0) = s(0) = u(0) = 1$$

10 Possible analytical solution

In comparing the system obtained from freeze line height to inflexion point, shown above, let's call it S_2 , with the system previously obtained, we notice

that the difference between the two of them relies on the term with r^3 for the first equation and on the term with f_o for the second, the difference just regarding signs.

We approach the problem in a way similar to the way Tam approached it: we start with the removal of gravity effects in our equations, that is, we make $A = 0$.

Besides, we are well aware of the fact that $u = r'$.

Consequently, we achieve the following set of equations:

for $z \in [0, a) \cup (b, c] \cup (d, 1]$, where a and c are the turning points of the bubble profile, we have, as an approximation, $r' = r'' = 0, r = k$:

$$\begin{cases} f_o + B(k^2 - 1) + \frac{A}{C} \int_0^Z (\lim_{r' \rightarrow 0} \frac{1}{r'}) dz = 0 \\ s' = k[D(s - s_a) + Y(s^4 - s_a^4)] \end{cases}$$

For $z \in [a, b]$ we have (since $r'' \geq 0$):

$$\left\{ \begin{array}{l} 2Br^3r'(1 + (Cr'^2)^2 + ACr^2r'(1 + (Cr')^2)^{\frac{3}{2}}) \\ + C^3r''(2r^2r'' + r(r')^2)\eta - C(1 + (Cr')^2)(2(r')^2 + rr'')\eta = 0 \\ C(1 + C^2(r')^2)^{-1}\left(\frac{(r')^2 + 2r''r}{r'r}\right) \\ = f_o(\eta^{-1}) + B(r^2 - 1)(\eta^{-1}) + \frac{A}{C}(\eta^{-1}) \int_0^Z \frac{1}{r'}(1 + C^2(r')^2)^{-\frac{1}{2}} dz \\ s' = Dr((1 + C^2(r')^2)^{\frac{1}{2}})(s - s_a) + Yr((1 + C^2(r')^2)^{\frac{1}{2}})(s^4 - s_a^4) \end{array} \right.$$

for $z \in (b, c)$:

$$\left\{ \begin{array}{l} 2B(c_3z + c_4)^3c_3(1 + (Cc_3^2)^2 + AC(c_3z + c_4)^2c_3(1 + (Cc_3)^2)^{\frac{3}{2}}) \\ - C(1 + (Cc_3)^2)(2c_3^2)\eta = 0 \\ C(1 + C^2(c_3)^2)^{-1}\left(\frac{c_3^2}{c_3(c_3z + c_4)}\right) \\ = f_o(\eta^{-1}) + B((c_3z + c_4)^2 - 1)(\eta^{-1}) + \frac{A}{C}(\eta^{-1}) \int_0^Z \frac{1}{c_3}(1 + C^2(c_3)^2)^{-\frac{1}{2}} dz \\ s' = D(c_3z + c_4)((1 + C^2(c_3)^2)^{\frac{1}{2}})(s - s_a) + Y(c_3z + c_4)((1 + C^2(c_3)^2)^{\frac{1}{2}})(s^4 - s_a^4) \end{array} \right.$$

Whilst for $z \in [c, d]$ we have (since $r'' \leq 0$):

$$\left\{ \begin{array}{l} 2Br^3r'(1 + (Cr'^2)^2 + ACr^2r'(1 + (Cr'^2)^2)^{\frac{3}{2}} \\ -C^3r''(2r^2r'' + r(r')^2)\eta - C(1 + (Cr'^2)^2)(2(r')^2 + rr'')\eta = 0 \\ C(1 + C^2(r')^2)^{-1}\left(\frac{(r')^2 + 2r''r}{r'r}\right) \\ = f_o(\eta^{-1}) + B(r^2 - 1)(\eta^{-1}) + \frac{A}{C}(\eta^{-1}) \int_0^Z \frac{1}{r'}(1 + C^2(r')^2)^{-\frac{1}{2}} dz \\ s' = Dr((1 + C^2(r')^2)^{\frac{1}{2}})(s - s_a) + Yr((1 + C^2(r')^2)^{\frac{1}{2}})(s^4 - s_a^4) \end{array} \right.$$

all with the following parameters, material function, and scaling:

$$A = \frac{\rho g R_o^2}{\eta_o U_o}, \quad B = \frac{\pi \Delta p R_o^3}{\eta_o Q}, \quad C = \frac{R_o}{Z_1}, \quad f_o = \frac{F_o R_o}{\eta_o Q}, \quad \beta = \frac{E}{RT_o}$$

$$D = \frac{UR_o 2\pi Z_1}{Q\rho C_v}, \quad Y = T_o^3 \frac{R_o 2\pi Z_1 \lambda \epsilon}{Q\rho C_v}$$

$$\eta_b = \eta_o e^{\beta\left(\frac{1}{s(z)} - 1\right)}$$

$$\eta = \frac{\eta_b}{\eta_o}, \quad z = \frac{Z}{Z_1}, \quad r = \frac{R}{R_o}, \quad u = \frac{U}{U_o}, \quad w = \frac{E}{E_o}$$

And with the following initial conditions:

$$r(0) = s(0) = 1$$

To get a single equation in r departing from the second equations above, one just needs to isolate $Cr'r''$ on each one of them, use the result on the first equations, and multiply the result by $\frac{2}{r'}$. This leads us to the following results:

$$2C^2r''r^2(f_o + B(r^2 - 1)) - r(f_o + B(3r^2 - 1))(1 + C^2(r')^2) - 5Cr'e^{\beta\left(\frac{1}{s(z)} - 1\right)} = 0$$

for $z \in [a, b)$ and

$$-2C^2 r'' r^2 (-f_o + B(r^2 - 1)) - r(-f_o + B(3r^2 - 1))(1 + C^2 (r')^2) - 5Cr' e^{\beta(\frac{1}{s(z)} - 1)} = 0$$

for $z \in (c, d]$

and we can, for perturbation purposes, consider that our ϵ is $e^{-\beta(\frac{1}{s(z)} - 1)}$ once the temperature will decrease to close to zero, very close.

What comes into play now is the bubble profile. From [J.S03] we notice that the bubble profile has to be determined in different ways. There are two almost straight lines easily identifiable from the simulations - they are located at the beginning and at the end of the curve - respectively, that is, both towards $z = 0$ and towards the end of the profile. On the other hand, there is a region of quick change in the radius, this more suitable to the Poincare expansion than to the asymptotic approach. And, right after the first region of quick change in the profile, comes a stable region again, again almost a straight line. After this almost straight line we have another region of quick change and the curve gets finalized by another sort of straight line again. We now wish to nominate these different stages of the curve by splitting it into intervals. We will then have:

1. A straight line segment, which we call C_1 , with function that expresses it being called S_1 - for $z \in [0, a)$;
2. A region of quick change, which we call C_2 , with a function that expresses it being called S_2 - for $z \in (a, b)$;
3. Another region of stability, again an almost straight line, let's call it C_3 , with function S_3 to express it - for $z \in (b, c)$;
4. Another region of quick change, we call it C_4 and its curve will be named S_4 - for $z \in (c, d)$;
5. Another region of stability, C_5 , with function S_5 to express it - for $z \in (d, 1]$.

This way, our solution should look more or less like:

$$\begin{aligned}
r(z) = & \text{Heaviside}(a - z)S_1 + \text{Heaviside}(b - z)(\text{Heaviside}(z - a))S_2 + \\
& \text{Heaviside}(c - z)(\text{Heaviside}(z - b))S_3 + \text{Heaviside}(d - z)(\text{Heaviside}(z - c))S_4 \\
& + \text{Heaviside}(1 - z)(\text{Heaviside}(z - d))S_5
\end{aligned}$$

where $0, a, b, c, d, 1$ are the delimiters of each interval of interest where the curves lie.

In order to work out an approximate precise solution curve expression we must perform computer simulations and, through these computer simulations, we found out that:

1. The value for C that gives the best curve fit was 0.16 (for all 30 different simulations performed);
2. The value for A that gives the best curve fit was 0.009332488386 (for the majority of the simulations);
3. The value for B that gives the best curve fit was 0.1871228447 for a single simulation where we think the best fit was achieved;
4. The value for DD and Y that gives the best curve fit was 0.007945372818 and 0.05.

And with the above values we get (approximately) that $a = 0.2$, $b = 0.57$, $c = 0.97$. We are sure that $r(0) = r(a) = 1$ and $dr(0) = dr(b) = 0$ since $C1$ is a straight line. We emphasize that the plots we were aiming were those where the end of the curve is actually flat, the beginning is flat, to give us slope zero, so that we actually get 4 pieces, as exposed above. We performed simulations around the values used in the literature researched.

11 $S_1 = S_5 - z \in [0, a) \cup (d, 1)$

For the first segment of our profile, the curve goes almost as in a straight line. Our equation is

$$\begin{cases} f_o + B(k^2 - 1) + \frac{A}{C} \int_0^Z (\lim_{r' \rightarrow 0} \frac{1}{r'}) dz = 0 \\ s' = k[D(s - s_a) + Y(s^4 - s_a^4)] \end{cases}$$

If we had not used the fact that $r' = r'' = 0$ then we would end up thinking of applying the method of the matched asymptotic expansions ([J.S03]) for C_1 . In doing so, we get that our outer solution is $r^0(z) = 1$, and this is the way the profile behaves away from the boundary layer. From the exact solution we see that when $z = O(\epsilon)$, $r' = O(1)$ and $r'' = O(1)$ as well. In order to examine the “layer” region adjacent to $z = 0$, we “blow it up” by defining a local or BL variable θ defined by:

$$\theta = \frac{z}{\epsilon^\lambda}$$

aiming to find out the right ‘stretching’ transformation. The choice implies that we create a function

$$R(\theta, \epsilon) = r(\epsilon^{-\lambda} z, \epsilon)$$

and our new equation looks like

$$2C^2 \frac{d^2 r}{\epsilon^{2\lambda} d\theta^2} r^2 (f_o + B(r^2 - 1)) \epsilon^{-r} (f_o + B(3r^2 - 1)) (1 + C^2 \left(\frac{dr}{\epsilon^\lambda d\theta}\right)^2) \epsilon^{-5} C \frac{dr}{\epsilon^\lambda d\theta} = 0$$

that is

$$2C^2 \frac{d^2 r}{d\theta^2} r^2 (f_o + B(r^2 - 1)) \epsilon^{1-\lambda} - r (f_o + B(3r^2 - 1)) \left(\epsilon^{2\lambda-1} + C^2 \left(\frac{dr}{d\theta}\right)^2 \right) \epsilon^{1-\lambda} - 5C \frac{dr}{d\theta} = 0$$

For $\lambda < 1$, $\epsilon^- > 0$ and $\lambda = 1$, $\epsilon^- > 0$ we get $R^i = K$. For $\lambda > 1$, $\epsilon^- > 0$ we

have to change our equation into

$$2C^2 \frac{d^2 r}{d\theta^2} r^2 (f_o + B(r^2 - 1)) - r (f_o + B(3r^2 - 1)) \left(\epsilon^{2\lambda-1} + C^2 \left(\frac{dr}{d\theta}\right)^2 \right) - 5C \epsilon^{\lambda-1} \frac{dr}{d\theta} = 0$$

This equation does not look very ‘friendly’, but may be easily compared to

the result we get from the equation containing $r' = r'' = 0$. The latter is

$$\begin{cases} f_o + B(k^2 - 1) + \frac{A}{C} \int_0^Z (\lim_{r' \rightarrow 0} \frac{1}{r'}) dz = 0 \\ s' = k[D(s - s_a) + Y(s^4 - s_a^4)] \end{cases}$$

Here, one may assume that $\lim_{r' \rightarrow 0} \frac{1}{r'}$ is actually ∞ but our infinity is confined to the space where the bubble profile lives. With this, the integral might actually be workable. Once the value of the integral is found, we also determine k . But, for our solution purposes, it suffices stating that $r(z) = c_1, c_5$ in C_1 and C_5 .

12 $S_2 - z \in (a, b)$

S_2 is the expression of C_2 , a region of quick change. Therefore, the best way to deal with it is using a Poincare series (see [J.S03]). We also have to work with the equation for that sector (let's consider C_2 goes from a to point b) of the profile:

$$2C^2 r'' r^2 \epsilon (f_o + B(r^2 - 1)) - r \epsilon (f_o + B(3r^2 - 1))(1 + C^2 (r')^2) - 5Cr' = 0$$

$$\text{for } z \in (a, b).$$

By making use of a Poincare type expansion on r , we get that, for $z \in (a, b)$, our S_2 will be:

- Since the only term without ϵ is $-5Cr'_0$, and changing the scaling of z to division by a does not change our equation in r , we get that $r'_0(z) = r''_0(z) = 0$ and $r_0(z) = 1$.
- From the terms in ϵ , we get that $r'_1(z) = \frac{-f_0 - 2B}{5C}$, what therefore implies that $r_1(z) = \frac{-f_0 - 2B}{5C}z + k_1$.
- From the terms in ϵ^2 , we get that $r'_2(z) = \frac{-f_0 - 2B}{25C^2}z(-f_0 - 8B) + \frac{k_1}{5C}(-f_0 - 8B) - \frac{3BC}{5} \left(\frac{-f_0 - 2B}{5C} \right)^2$, what therefore implies that

$$r_2(z) = \frac{f_0+2B}{50C^2}z^2(f_0+8B) - \frac{k_1z}{5C}(f_0+8B) - \frac{3B(f_0+2B)^2}{125C}z + k_2$$

With this, our S_2 can be better approximated by the following function:

$$S_2(z, \epsilon) = 1 + \left[\frac{-f_0 - 2B}{5C}z + k_1 \right] \epsilon + \left[\frac{f_0 + 2B}{50C^2}z^2(f_0 + 8B) - \frac{k_1z}{5C}(f_0 + 8B) - \frac{3B(f_0 + 2B)^2}{125C}z + k_2 \right] \epsilon^2$$

And the same sort of reasoning, the same solution, applies to C_4 , that is,

S_4 .

13 $S_3 - z \in (b, c)$

14 Analytical Solution

$$\begin{aligned} r(z) = & \text{Heaviside}(a-z)c_1 + \text{Heaviside}(b-z)(\text{Heaviside}(z-a)) \left[1 + \left[\frac{-f_0 - 2B}{5C}z + k_1 \right] \epsilon \right. \\ & \left. + \left[\frac{f_0 + 2B}{50C^2}z^2(f_0 + 8B) - \frac{k_1z}{5C}(f_0 + 8B) - \frac{3B(f_0 + 2B)^2}{125C}z + k_2 \right] \epsilon^2 \right] + \\ & \text{Heaviside}(c-z)(\text{Heaviside}(z-b))[c_3z + c_4] + \text{Heaviside}(d-z)(\text{Heaviside}(z-c)) \left[1 + \left[\frac{-f_0 - 2B}{5C}z \right. \right. \\ & \left. \left. + \left[\frac{f_0 + 2B}{50C^2}z^2(f_0 + 8B) - \frac{k_1z}{5C}(f_0 + 8B) - \frac{3B(f_0 + 2B)^2}{125C}z + k_2 \right] \epsilon^2 \right] \right. \\ & \left. + \text{Heaviside}(1-z)(\text{Heaviside}(z-d))c_5 \right] \end{aligned}$$

15 Another resolution to the problem

$$2C^2r''r^2(f_o - B(r^2 - 1)) + r(f_o - B(3r^2 - 1))(1 + C^2(r')^2) - 5Cr'e^{\beta(\frac{1}{s(z)}-1)} = 0$$

Another approach to the problem would be considering that the minimum the function $r(z)$ reaches is $r(0) = 1$ and the maximum is $r(1) = A$. Therefore, one could say that

$$\begin{aligned} & 2C^2r''f_o + (f_o - 2B)(1 + C^2(r')^2) - 5Cr'e^{\beta(\frac{1}{s(z)}-1)} \\ & < -2C^2r''r^2(-f_o + B(r^2 - 1)) - r(-f_o + B(3r^2 - 1))(1 + C^2(r')^2) - 5Cr'e^{\beta(\frac{1}{s(z)}-1)} \\ & < -2C^2r''A^2(-f_o + B(A^2 - 1)) - A(-f_o + B(3A^2 - 1))(1 + C^2(r')^2) - 5Cr'e^{\beta(\frac{1}{s(z)}-1)} \end{aligned}$$

We also know that $s(0) = 1$ is the maximum temperature reached in the whole process, where the temperature varies between 0 and 1. Therefore, we can also say that:

$$\begin{aligned} & 2C^2r''f_o + (f_o - 2B)(1 + C^2(r')^2) - 5Cr'e^\beta \\ & < -2C^2r''r^2(-f_o + B(r^2 - 1)) - r(-f_o + B(3r^2 - 1))(1 + C^2(r')^2) - 5Cr'e^{\beta(\frac{1}{s(z)}-1)} \\ & < -2C^2r''A^2(-f_o + B(A^2 - 1)) - A(-f_o + B(3A^2 - 1))(1 + C^2(r')^2) - 5Cr'e^{\beta(\frac{1}{s(z)}-1)} \end{aligned}$$

If we are then able to find a solution for the equations

$$r'' = \frac{5Cr'e^\beta - (f_o - 2B)(1 + C^2(r')^2)}{2C^2f_o}$$

and

$$r'' = \frac{5Cr'e^{\beta(\frac{1}{s(z)}-1)} + A(-f_o + B(3A^2 - 1))(1 + C^2(r')^2)}{-2C^2A^2(-f_o + B(A^2 - 1))}$$

we then are going to be able to say that our solution is between those two.

16 Conclusions

We presented an initial analytical investigation into the bubble radius profile of a nonisothermal film blowing process. The present work must have a follow-up with trials to obtain analytical solutions departing from the solutions we presented in this paper, where we took away the gravity term and left part of the solution as a question mark. Because of the value of A , our solution might be very close to reality. So far, however, we have not written about our empirical notes, obtained from [Bat03], in order to check on proximity to real life situations. Future work must bring this sort of results along with analytical trials on getting more of our solutions, even, possibly, revealing the value of our constants. Due to the length of the present paper, though, we left some equations to be solved in the near future.

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