

Dominant Representation In The Understanding Of Basic Integrals Among Post Secondary Students

Munirah Ghazali*, Siti Aishah Sheikh Abdullah**, Zurida Ismail*
Mohd Irwan Idris*

*School of Educational Studies, Universiti Sains Malaysia

**Universiti Teknologi MARA Pulau Pinang

ABSTRACT: The concepts of integrals are basic to students' understanding of calculus particularly for those who intend to pursue higher education. The understanding of integrals require students to be able to translate and move between different integral representations such as symbolic, graphical and numeric with respect to basic definitions, integral operation, translation between representations and contextual applications. This paper reports the findings from a study with a focus of investigating students' understanding of the different integral representations. The analysis was based on four constructs: definitions, basic integral operations, translation of representation and application of integrals. The sample comprises of post-secondary students who has completed their Malaysian Certificate Examination (SPM) and are currently enrolled in one of the matriculation colleges. Preliminary analysis indicated that symbolic representation is dominant. The students experienced difficulties with translation and relating contexts with integrals. One of the implications of this study is that translations between representations need to be stressed in the teaching and learning of integrals.

1.0 Introduction

One of the topics in Form Four Additional Mathematics is calculus. Understanding the concepts of calculus is crucial since it serves as a foundation for higher learning in mathematics. However, students have considered calculus as a difficult subject. Learning of calculus is fraught with many difficulties. Various studies have shown that students have difficulties understanding the concepts in calculus (Munirah 1996, Munirah 2001). The difficulties were mainly related to such concepts as rates (Orton, 1984), limits (Cornu, 1981; Tall & Vinner, 1981), tangents (Vinner, 1982; Tall, 1987), and functions (Dreyfus & Eisenberg, 1982).

2.0 Multiple representations and understanding of integrals

Representation refers to the process and outcomes that can be observed either externally or inferred internally while involve in mathematical activities (Gray & Tall,1992). For example, a linear function can be represented either as a straight line in the Cartesian coordinates, linear equation, $y = mx + c$ or a mapping of two sets. According to the NCTM (2000) standards, representation refers to both process and production, that is, to the act of capturing a mathematical concept or relationship in some form and to the form itself. Furthermore, the term applies to processes and products that are observable externally as well as to those that occur 'internally,' in the minds of people doing mathematics.(NCTM, 2000, pg. 67) Representational activities involved either interpretation of graphical, equations or tables, or contextual problems. It can also be translational activities that focus on relation between representational modes. Reforms in the curriculum have emphasized on these activities. It is believed that representational activities do contribute towards better conceptual understanding of mathematics:

Different representations support different ways of thinking about and manipulating mathematical objects. An object can be better understood when viewed through multiple lenses.

(NCTM, 2000, pg. 360)

In the context of integration, understanding refers to the acquisition of basic definitions, basic operations and the ability to translate and interpret between the different representations, that is symbolic, graphical, numerical and applications. The understanding of integrals in this study is defined according to the following four constructs: basic definition of integral, basic operations of integral, translation between representations of integral and applications of integral.

Students' conceptual understanding in integral is crucial towards higher learning in mathematics. Orton (1983) showed that the majority of students in his study were able to apply the procedures and basic techniques of integration. However, they have a limited understanding of the underlying concepts. Orton's results also indicated that the procedure of breaking up an area or volume, making use of a limit process were not part of the students' understanding of integral. Rasslan & Tall (2001) reported that pre-university students' images of definite integral concepts are in conflict with the formal definitions and

they have difficulties in interpreting definite integrals in unfamiliar context. This aim of this paper is to discuss the findings from a study that was aimed at assessing students' understanding of basic integral concepts in multiple representations.

3.0 Methodology

3.1 Development of the diagnostic instrument

A diagnostic instrument was first developed to assess various aspect of students' understanding of integral. The choice of items took into consideration the content of the Additional Mathematics course as outlined in the syllabus, the modules for the matriculation program and a other text books as references. There were 10 items in all. The items combine both the objective and subjective format. Students were allocated one hour to attempt the test.

The items covered the three components of problems solving activities: '*Modeling*', '*Translating*' and '*Interpretations*'. Items 1 and 2 asked for basic definitions, items 5, 6, 8 and 9 were on basic integral operations while items 4, 7 and 10 were translation problems.

There were four objective items. Item number 5 had four sub-questions and students needed to choose either '*right*' or '*wrong*.' Item 1 had five sub-items. Item 2 had four sub-items and students were required to choose either '*yes*' or '*no*'. Item 4 was also an objective question where students had to chose the correct forms of graphical representations of integrals. Item 9, another objective question and for this item, students were asked to select either '*true*' or '*false*'. Items 3, 5, 6, 7, 8 and 10 are open ended. Figure 1 summarized the constructs and items used in the instrument to measure students' understanding of integral while Table 2 illustrates some of the tasks used.

Table 1

Construct of basic integral concepts

Items Construct	Items	Type of question
Basic definition	1,2,3	Open/objective
Basic integration operations	5, 6, 8, 9	Open/objective
Translation: Symbol → Graphical	4	Open
Word problems → Symbolic	7	Open
Graphical → Symbolic	10	Open

3.2 Respondents

A total of 219 secondary and matriculation students comprising of 137 male and 154 female students were involved in the study. They were given about an hour to complete the test. The diagnostic test was administered to the students in the beginning of the academic year. Students' responses were assessed and selected respondents will be interviewed later as the second part of the study (not discussed in this paper). For the purposes of this paper, only the results of the diagnostic part will be discussed.

3.3 Assessment of students work

The students' responses were assessed based on the final answers as well as the process of arriving at the final answer. The scoring for each item are as follows:

- Right concept, procedures and answers (3 marks)
- Right concepts, wrong procedure and wrong answers (2 marks)
- Some concepts but no working shown (1 marks)
- Wrong concepts, wrong procedures and wrong answers (0 mark)

Each objective item was given 3 marks for each correct answer. Item 1 has 5 sub-items; hence the total score for the item is 15 with 3 marks for each correct response. Item 2 has 4 sub-items giving a total score of 12. The total marks for all the items are 60.

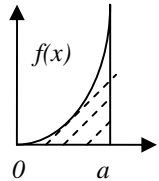
Table 2

Sample of tasks used

Task 7: The numbers of bacteria culture at time t (t in hours) is N . Rate of growth of bacteria after time t is $20t$. Express N in terms of t .

Task 1(c): If $\frac{d[F(x)]}{dx} = f(x)$; then $\int f(x)dx = F(x) + c$. (T/F)

Task 2(a): If f is a function, integrals of $f(x)$ from a to b is the shaded area,



(T/F)

Task 2(d): Integral of $f(x)$ from a to b is $\frac{df}{dx}$ (T/F)

4.0 Results and discussion

The papers were graded and analyzed statistically using SPSS 12.0. The results of the analysis are discussed according to the constructs used in the diagnostic test. Table 3 shows the mean values for each construct. The mean score for basic definition is 0.40 and for basic operations is 0.42. The scores for translations from symbolic to graphic representations is 0.77, from word problems to symbolic representations is 0.32 and from graphic to symbolic form is 0.65.

Table 3

Mean scores according to item constructs.

	Basic definition	Basic operations	Translations Symbol to graphic	Translation word problem to symbol	Translation Graphic to Symbol
N	286	290	291	291	291
Mean	.403	.425	.770	.316	.650
Std. Deviation	.064	.175	.421	.861	1.224

It was observed that the above average students faced difficulties in understanding the basic definitions and operations of integrals. The analysis also showed that students faced difficulties in translating between word problems to symbolic representations of integrals. For this task only 7 % of the students answered correctly and hence obtained 3 marks while almost 49% did not get any marks because they could not come up with the correct response. Task 1(c) for instance assess students' understanding of basic definitions and almost 33% thought the statement was wrong, while 4% did not attempt the question at all. Surprisingly in task 2(d) almost 77% thought that the statement was true and only 18% chose false. In other open-ended task, it was noted that students were confused between integral and differential operations. They applied the wrong operations while some conveniently chose to 'mix' the two operations.

5.0 Conclusion

The preliminary result of this study indicated that students did not seem to have difficulties in translating integrals from graphical to symbolic forms and similarly otherwise. However, more attention should be given to the translation of word problems to symbolic representations of integrals as well as their understanding of the basic concepts and operations in integrals. The students' performance in the test clearly showed the areas of weaknesses. Hence, teachers should make attempts to address these areas so that students could develop a better understanding of the relevant concepts. More examples as well as practice exercises involving different representations and manipulating mathematical objects should be given to enhance students' understanding and mastery of skills.

6.0 References

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