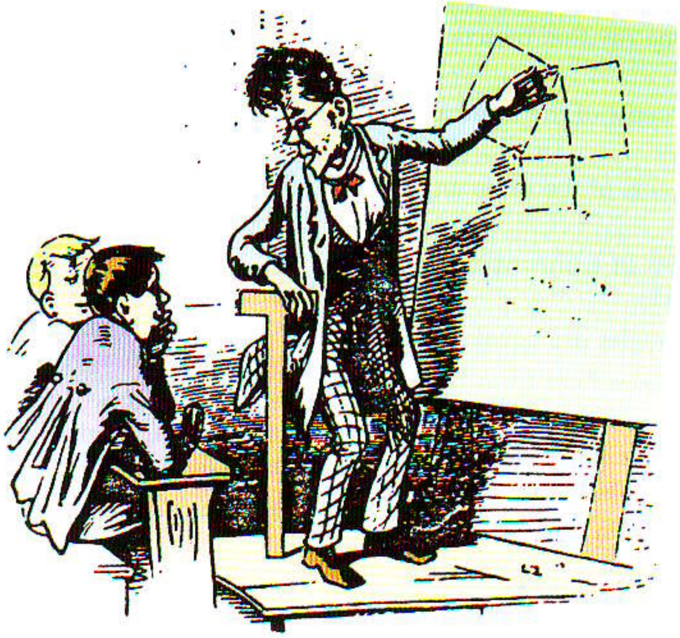

Teacher: ". . . and now I want to prove this theorem."
Pupil: "Why bother to prove it, teacher? I take your word for it."

# Postulates and Theorems

## Chapter 2

**Postulate 1** Two points determine a line. 61

**Postulate 2** Three noncollinear points determine a plane. 61

**Theorem** *The Pythagorean Theorem.* The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. 65

**Theorem** *The Triangle Angle Sum Theorem.* The sum of the angles of a triangle is 180°. 66

**Theorem** If the diameter of a circle is $d$, its circumference is $\pi d$. 66

**Theorem** If the radius of a circle is $r$, its area is $\pi r^2$. 66

## Chapter 3

**Postulate 3** *The Ruler Postulate.* The points on a line can be numbered so that positive number differences measure distances. 85

**Postulate 4** *The Protractor Postulate.* The rays in a half-rotation can be numbered from 0 to 180 so that positive number differences measure angles. 92

**Theorem 1** *The Betweenness of Points Theorem.* If A-B-C, then AB + BC = AC. 86

**Theorem 2** *The Betweenness of Rays Theorem.* If OA-OB-OC, then $\angle AOB + \angle BOC = \angle AOC$. 93

***Corollary to the Ruler Postulate*** A line segment has exactly one midpoint. 99

***Corollary to the Protractor Postulate*** An angle has exactly one ray that bisects it. 100

**Theorem 3** Complements of the same angle are equal. 106

**Theorem 4** Supplements of the same angle are equal. 106

**Theorem 5** The angles in a linear pair are supplementary. 111

**Theorem 6** Vertical angles are equal. 112

**Theorem 7** Perpendicular lines form right angles. 118

***Corollary to the definition of a right angle*** All right angles are equal. 118

**Theorem 8** If the angles in a linear pair are equal, then their sides are perpendicular. 118

## Chapter 4

**The Distance Formula** The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. 134

**Postulate 5** *The ASA Postulate.* If two angles and the included side of one triangle are equal to two angles and the included side of another triangle, the triangles are congruent. 147

**Postulate 6** *The SAS Postulate.* If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, the triangles are congruent. 147

***Corollary to the definition of congruent triangles*** Two triangles congruent to a third triangle are congruent to each other. 141

**Theorem 9** If two sides of a triangle are equal, the angles opposite them are equal. 158

***Corollary*** An equilateral triangle is equiangular. 159

**Theorem 10** If two angles of a triangle are equal, the sides opposite them are equal. 159

***Corollary*** An equiangular triangle is equilateral. 159

**Theorem 11** *The SSS Theorem.* If the three sides of one triangle are equal to the three sides of another triangle, the triangles are congruent. 164

## Chapter 5

**Theorem 12** *The Exterior Angle Theorem.* An exterior angle of a triangle is greater than either remote interior angle. 191

**Theorem 13** If two sides of a triangle are unequal, the angles opposite them are unequal in the same order. 196

**Theorem 14** If two angles of a triangle are unequal, the sides opposite them are unequal in the same order. 196

**Theorem 15** *The Triangle Inequality Theorem.* The sum of any two sides of a triangle is greater than the third side. 201

## Chapter 6

**Postulate 7** *The Parallel Postulate.* Through a point not on a line, there is exactly one line parallel to the given line. 226

**Theorem 16** In a plane, two points each equidistant from the endpoints of a line segment determine the perpendicular bisector of the line segment. 212

**Theorem 17** Equal corresponding angles mean that lines are parallel. 220

***Corollary 1*** Equal alternate interior angles mean that lines are parallel. 220

***Corollary 2*** Supplementary interior angles on the same side of a transversal mean that lines are parallel. 220

***Corollary 3*** In a plane, two lines perpendicular to a third line are parallel. 220

**Theorem 18** In a plane, two lines parallel to a third line are parallel to each other. 226

**Theorem 19** Parallel lines form equal corresponding angles. 231

***Corollary 1*** Parallel lines form equal alternate interior angles. 231

***Corollary 2*** Parallel lines form supplementary interior angles on the same side of a transversal. 231

***Corollary 3*** In a plane, a line perpendicular to one of two parallel lines is also perpendicular to the other. 231

**Theorem 20** *The Angle Sum Theorem.* The sum of the angles of a triangle is 180°. 237

***Corollary 1*** If two angles of one triangle are equal to two angles of another triangle, the third angles are equal. 237

***Corollary 2*** The acute angles of a right triangle are complementary. 237

***Corollary 3*** Each angle of an equilateral triangle is 60°. 237

**Theorem 21** An exterior angle of a triangle is equal to the sum of the remote interior angles. 237

**Theorem 22** *The AAS Theorem.* If two angles and the side opposite one of them in one triangle are equal to the corresponding parts of another triangle, the triangles are congruent. 243

**Theorem 23** *The HL Theorem.* If the hypotenuse and a leg of one right triangle are equal to the corresponding parts of another right triangle, the triangles are congruent. 244

## Chapter 7

**Theorem 24** The sum of the angles of a quadrilateral is 360°. 259

***Corollary*** A quadrilateral is equiangular iff it is a rectangle. 259

**Theorem 25** The opposite sides and angles of a parallelogram are equal. 266

**Theorem 26** The diagonals of a parallelogram bisect each other. 266

**Theorem 27** A quadrilateral is a parallelogram if its opposite sides are equal. 271

**Theorem 28** A quadrilateral is a parallelogram if its opposite angles are equal. 271

**Theorem 29** A quadrilateral is a parallelogram if two opposite sides are both parallel and equal. 271

**Theorem 30** A quadrilateral is a parallelogram if its diagonals bisect each other. 271

**Theorem 31** All rectangles are parallelograms. 276

**Theorem 32** All rhombuses are parallelograms. 276

**Theorem 33** The diagonals of a rectangle are equal. 277

**Theorem 34** The diagonals of a rhombus are perpendicular. 277

**Theorem 35** The base angles of an isosceles trapezoid are equal. 282

**Theorem 36** The diagonals of an isosceles trapezoid are equal. 282

**Theorem 37** *The Midsegment Theorem.* A midsegment of a triangle is parallel to the third side and half as long. 287

## Chapter 9

**Postulate 8** *The Area Postulate.* Every polygonal region has a positive number called its area such that (1) congruent triangles have equal areas and (2) the area of a polygonal region is equal to the sum of the areas of its nonoverlapping parts. 339

**Postulate 9** The area of a rectangle is the product of its base and altitude. 345

***Corollary to Postulate 9*** The area of a square is the square of its side. 345

**Theorem 38** The area of a right triangle is half the product of its legs. 352

**Theorem 39** The area of a triangle is half the product of any base and corresponding altitude. 353

***Corollary*** Triangles with equal bases and equal altitudes have equal areas. 353

**Theorem 40** The area of a parallelogram is the product of any base and corresponding altitude. 359

**Theorem 41** The area of a trapezoid is half the product of its altitude and the sum of its bases. 359

**Theorem 42** *The Pythagorean Theorem.* The square of the hypotenuse of a right triangle is equal to the sum of the squares of its legs. 366

**Theorem 43** *Converse of the Pythagorean Theorem.* If the square of one side of a triangle is equal to the sum of the squares of the other two sides, the triangle is a right triangle. 366

## Chapter 10

**Theorem 44** *The Side-Splitter Theorem.* If a line parallel to one side of a triangle intersects the other two sides in different points, it divides the sides in the same ratio. 393

***Corollary*** If a line parallel to one side of a triangle intersects the other two sides in different points, it cuts off segments proportional to the sides. 394

**Theorem 45** *The AA Theorem.* If two angles of one triangle are equal to two angles of another triangle, the triangles are similar. 400

***Corollary*** Two triangles similar to a third triangle are similar to each other. 401

**Theorem 46** Corresponding altitudes of similar triangles have the same ratio as that of the corresponding sides. 408

**Theorem** *The SAS Similarity Theorem.* If an angle of one triangle is equal to an angle of another triangle and the sides including these angles are proportional, then the triangles are similar. 412

**Theorem** *The SSS Similarity Theorem.* If the sides of one triangle are proportional to the sides of another triangle, then the triangles are similar. 412

**Theorem 47** The ratio of the perimeters of two similar polygons is equal to the ratio of the corresponding sides. 415

**Theorem 48** The ratio of the areas of two similar polygons is equal to the square of the ratio of the corresponding sides. 416

## Chapter 11

**Theorem 49** The altitude to the hypotenuse of a right triangle forms two triangles similar to it and to each other. 429

***Corollary 1*** The altitude to the hypotenuse of a right triangle is the geometric mean between the segments into which it divides the hypotenuse. 429

***Corollary 2*** Each leg of a right triangle is the geometric mean between the hypotenuse and its projection on the hypotenuse. 429

**Theorem 50** *The Isosceles Right Triangle Theorem.* In an isosceles right triangle, the hypotenuse is $\sqrt{2}$ times the length of a leg. 442

***Corollary*** Each diagonal of a square is $\sqrt{2}$ times the length of one side. 442

**Theorem 51** *The 30°-60° Right Triangle Theorem.* In a 30°-60° right triangle, the hypotenuse is twice the shorter leg and the longer leg is $\sqrt{3}$ times the shorter leg. 442

***Corollary*** An altitude of an equilateral triangle having side $s$ is $\frac{\sqrt{3}}{2}s$ and its area is $\frac{\sqrt{3}}{4}s^2$. 443

**Theorem 52** Two nonvertical lines are parallel iff their slopes are equal. 463

**Theorem 53** Two nonvertical lines are perpendicular iff the product of their slopes is $-1$. 463

**Theorem 54** *The Law of Sines.* If the sides opposite $\angle A$, $\angle B$, and $\angle C$ of $\triangle ABC$ have lengths $a$, $b$, and $c$, then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. 469

**Theorem 55** *The Law of Cosines.* If the sides opposite $\angle A$, $\angle B$, and $\angle C$ of $\triangle ABC$ have lengths $a$, $b$, and $c$, then $c^2 = a^2 + b^2 - 2ab \cos C$. 469

## Chapter 12

**Postulate 10** *The Arc Addition Postulate.* If C is on $\overset{\frown}{AB}$, then $m\overset{\frown}{AC} + m\overset{\frown}{CB} = m\overset{\frown}{ACB}$. 499

***Corollary to the definition of a circle*** All radii of a circle are equal. 485

**Theorem 56** If a line through the center of a circle is perpendicular to a chord, it also bisects the chord. 485

**Theorem 57** If a line through the center of a circle bisects a chord that is not a diameter, it is also perpendicular to the chord. 485

**Theorem 58** The perpendicular bisector of a chord of a circle contains the center of the circle. 485

**Theorem 59** If a line is tangent to a circle, it is perpendicular to the radius drawn to the point of contact. 492

**Theorem 60** If a line is perpendicular to a radius at its outer endpoint, it is tangent to the circle. 492

**Theorem 61** In a circle, equal chords have equal arcs. 499

**Theorem 62** In a circle, equal arcs have equal chords. 499

**Theorem 63** An inscribed angle is equal in measure to half its intercepted arc. 505

***Corollary 1*** Inscribed angles that intercept the same arc are equal. 505

***Corollary 2*** An angle inscribed in a semicircle is a right angle. 505

**Theorem 64** A secant angle whose vertex is inside a circle is equal in measure to half the sum of the arcs intercepted by it and its vertical angle. 511

**Theorem 65** A secant angle whose vertex is outside a circle is equal in measure to half the difference of its larger and smaller intercepted arcs. 511

**Theorem 66** *The Tangent Segments Theorem.* The tangent segments to a circle from an external point are equal. 517

**Theorem 67** *The Intersecting Chords Theorem.* If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord. 517

## Chapter 13

**Theorem 68** Every triangle is cyclic. 531

***Corollary*** The perpendicular bisectors of the sides of a triangle are concurrent. 531

**Theorem 69** A quadrilateral is cyclic iff a pair of its opposite angles are supplementary. 536

**Theorem 70** Every triangle has an incircle. 543

***Corollary*** The angle bisectors of a triangle are concurrent. 543

**Theorem 71** The medians of a triangle are concurrent. 549

**Theorem 72** The lines containing the altitudes of a triangle are concurrent. 549

**Theorem 73** *Ceva's Theorem.* Three cevians, AY, BZ, and CX, of △ABC are concurrent iff $\frac{AX}{XB} \cdot \frac{BY}{YC} \cdot \frac{CZ}{ZA}$. 555

## Chapter 14

**Theorem 74** Every regular polygon is cyclic. 573

**Theorem 75** The perimeter of a regular polygon having $n$ sides is $2Nr$, in which $N = n \sin \frac{180}{n}$ and $r$ is its radius. 581

**Theorem 76** The area of a regular polygon having $n$ sides is $Mr^2$, in which $M = n \sin \frac{180}{n} \cos \frac{180}{n}$ and $r$ is its radius. 587

**Theorem 77** If the radius of a circle is $r$, its circumference is $2\pi r$. 592

***Corollary*** If the diameter of a circle is $d$, its circumference is $\pi d$. 592

**Theorem 78** If the radius of a circle is $r$, its area is $\pi r^2$. 599