Gage Repeatability and Reproducibility

Gage Repeatability and Reproducibility(Gage R&R)

- What is Gage R&R?
  - It is a statistical approach of determining if a gage or a gaging system is suitable for the process under measurement.
- Purpose
  - Measurement is an integral part of any manufacturing unit and is useful in predicting the quality of the manufacturing process .
  - The technique is very useful in predicting the inherent variation in the process, if any, as too much in-process variations can cause serious problems.
- Terms in Gage R&R:
  - Gage:

    A gage is any device that is used to obtain measurements.
  - Part:

    A part is an item that is subject to measurement. Typically a part should represent the entire operating range of the process under consideration.
  - Trial:

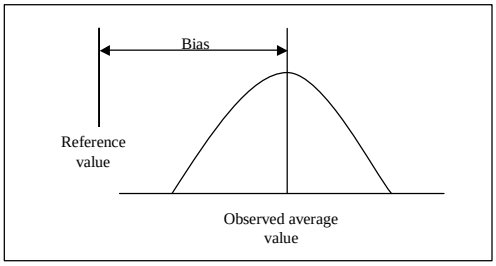
    A trial is a set of measurements on a part that is taken by an operator or a computer.
  - Measurement System:

    A measurement system is the complete process of obtaining measurements. This includes gages, people, procedures and operations.

Types of Measurement System Variation

Bias:

Bias is the difference between the observed average of measurement and the reference value. It is the systematic error that is an indication of a measuring instrument. The reference value is determined by averaging several measurements using standard measuring equipment.



Bias Example

Consider the following ten measurements by an appraiser. The reference value determined by layout inspection equipment is 0.80 mm.

X1 = 0.75	X6 = 0.80
X2 = 0.75	X7 = 0.75
X3 = 0.80	X8 = 0.75
X4 = 0.80	X9 = 0.75
X5 = 0.65	X10 = 0.70

The observed average is the sum of the measurements divided by 10.

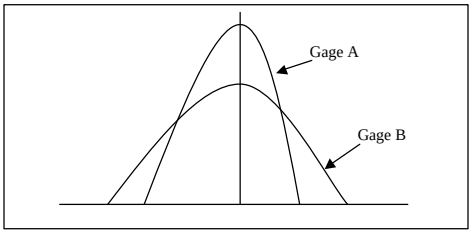
$$X = \frac{\sum X}{10} = \frac{7.5}{10} = 0.75$$

Bias = Observed Average – Reference Value

Bias = 0.75 – 0.80 = - 0.05

Repeatability:

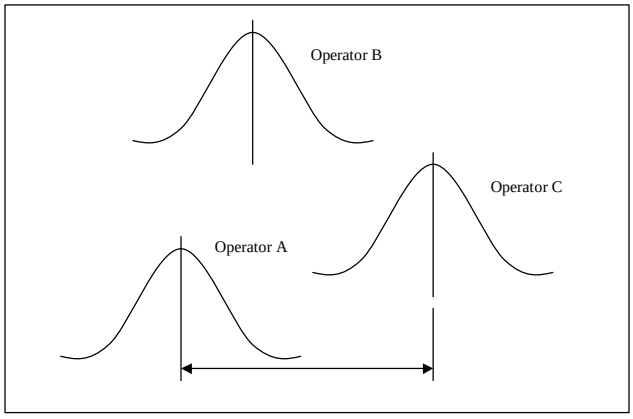
Repeatability is the variation in measurements obtained with **one measurement instrument** when used several times by an appraiser while measuring the identical characteristic on the **same part**. It is also commonly known as equipment variation.



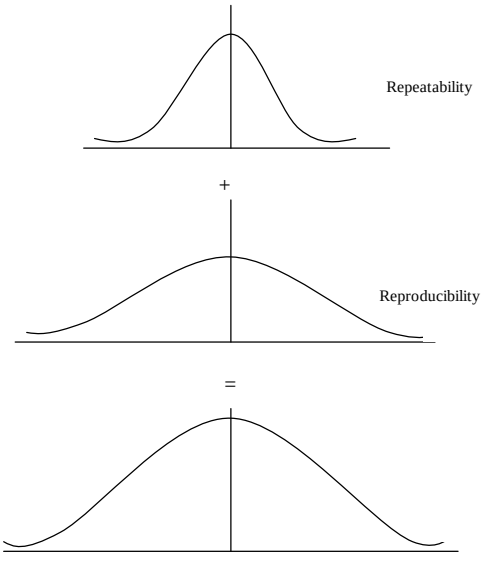
In the above figure, the repeatability of Gage A is more than that of Gage B as shown by their probability density functions.

Reproducibility:

Reproducibility is the variation in the average of measurements made by **different** appraisers using the **same instrument** when measuring the identical characteristic on the **same part**. It is commonly known as appraiser variation.

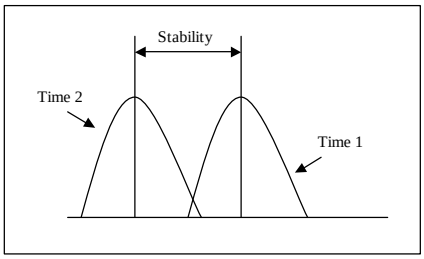


Repeatability and Reproducibility



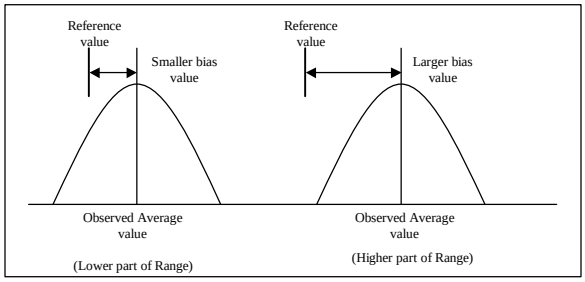
Stability:

Stability is the total variation in the measurements obtained with a measurement system on the same master or parts when measuring a single characteristic over an extended time period. Stability is sometimes referred to as drift.



Linearity:

Linearity is the difference in the bias values through the expected operating range of the gage.



Part Variation:

The Part Variation is essentially a measure of the variation of the process. If a large number of parts made by a process are measured, 99%(5.15 σ) of the parts would be within the variation limits. The Part Variation is always less than or equal to the total variation. In most industrial processes the part variation is large compared to the gage variation and so the assumption that the observed standard deviation is approximately equal to the total population standard deviation holds good.

Methods to determine Gage R&R

There are three basic and widely used methods for determining the Gage R&R. They are:

- Range method
- Average and Range method
- Analysis of Variance method (ANOVA)

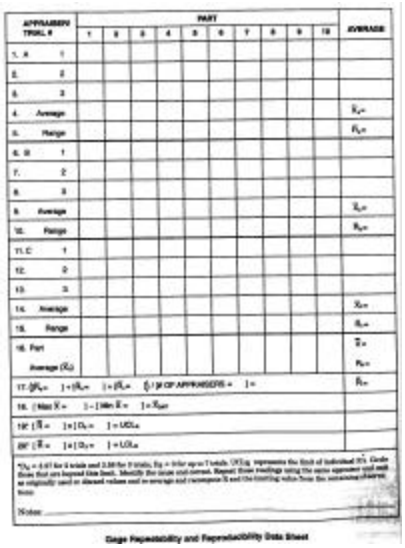
Let us see the Average and Range method in detail.

The Average and Range method is a statistical method that provides an estimate of the following components.

- Part Variation
- Repeatability
- Reproducibility
- R&R
- Total Variation

This method computes the total measurement system variability, which can be separated into components like repeatability, reproducibility and part variation.

The Average and Range method requires multiple parts, appraisers and trials to quantify the repeatability and reproducibility. The following is a typical Data sheet used in industries.



Average and Range Method(Example)

Consider the following example: (Taken from Measurement System Analysis Reference Manual)

Appraiser/ Trial #		Part 1	2	3	4	5	6	7	8	9	10	Average
A	1	217	220	217	214	216						216.8
	2	216	216	216	212	219						215.8
	3	216	218	216	212	220						216.4
	Average	216.3	218	216.3	212.7	218.3						216.3
	Range	1.0	4.0	1.0	2.0	4.0						2.4
B	1	216	216	216	216	220						216.8
	2	219	216	215	212	220						216.4
	3	220	220	216	212	220						217.6
	Average	218.3	217.3	215.7	213.3	220						216.9
	Range	4.0	4.0	1.0	4.0	0.0						2.6
C	1											
	2											
	3											
	Average											
	Range											
	Part Average ($\overline{X}_p$)	217.3	217.7	216	213	219.15						$\overline{\overline{X}} = 216.6$ $R_p = 6.15$
$\overline{\overline{R}} = (\overline{R}_A + \overline{R}_B)$ / No. of Appraisers = 2												2.5
$\overline{X}_{DIFF} = Max\,\overline{X} - Min\,\overline{X}$												0.6
$UCL_R = \overline{\overline{R}} * D_4$												6.4
$LCL_R = \overline{\overline{R}} * D_3$												0.00

No. of Appraisers = 2
No. of Trials = 3
No. of parts = 5

Repeatability – Equipment Variation (EV):

$$EV = \overline{\overline{R}} * K_1$$

= 2.5 x 3.00

= 7.5

Note:

• All calculations are based upon predicting 5.15σ (99% area under the normal curve)

• $K_1 = 5.15/d_2$ where d_2 depends on the no. of trials (m) and the number of parts times the no. of appraisers (g). The value of d_2 is obtained from Table 1.

In our case m = 3 and g = 2 x 5 = 10. Looking up Table 1 we get $d_2 = 1.72$.

Therefore $K_1 = 5.15/1.72 = 3.00$.

Reproducibility – Appraiser Variation (AV):

$$AV = \sqrt{(\overline{X}_{DIFF} * K_2)^2 - (EV^2 / nr)}$$

$$= \sqrt{(0.6 * 3.65)^2 - (7.5^2 / 5 * 3)}$$

$$= 1.0461$$

Note:

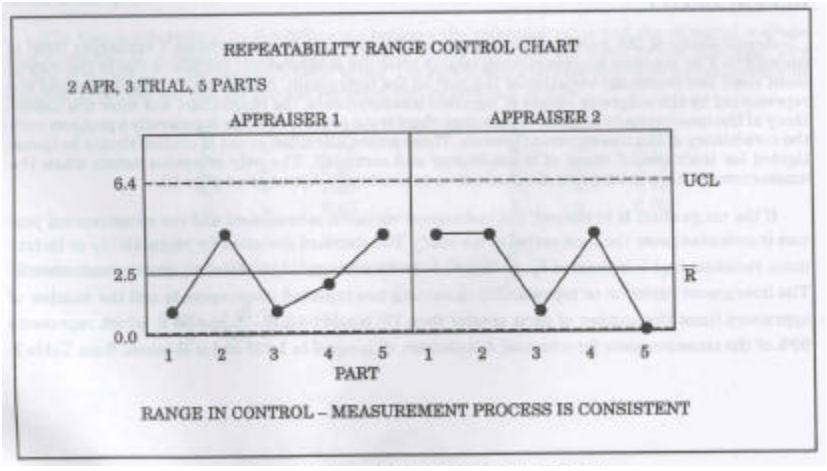
• If a negative value is calculated under the square root sign, the value AV defaults to zero.

• n = No.of parts and r = No.of Trials

• $K_2 = 5.15/d_2$ where d_2 depends on the no. of appraisers (m) and g is 1, since there is only one range calculation.

In our case m = 2. Looking up Table 1 we get $d_2 = 1.41$

Therefore $K_2 = 5.15/1.41 = 3.65$.



All the points are within limits(UCL_R and LCL_R) and so the measurement process is under control and is said to be consistent.

Repeatability and Reproducibility (R&R):

$$R\,\&\,R = \sqrt{EV^2 + AV^2}$$
$$= \sqrt{7.5^2 + 1.0461^2}$$
$$= 7.57$$

Part Variation (PV):

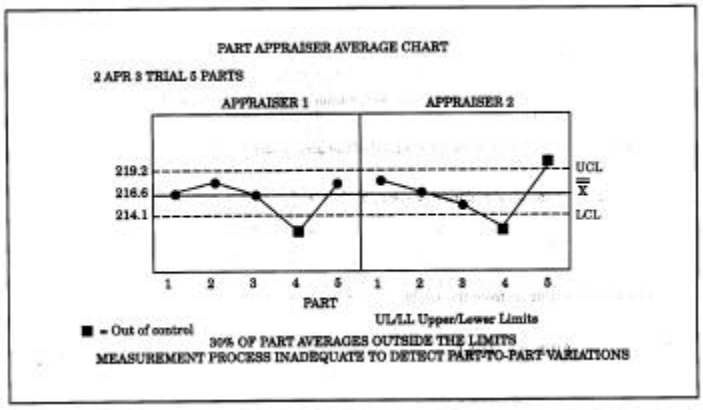
$$PV = R_p * K_3$$
$$= 6.15 * 2.08$$
$$= 12.79$$

Note:

• K3 = 5.15/d_2 where d_2 is dependent on the no.of parts (m) and g = 1, since there is only one range calculation.

In our case m = 5 and g = 1. Looking up Table 1 we get d_2 = 2.48.

Therefore K_3 = 5.15/2.48 = 2.08.



There are three points that fall outside the limits and so the measurement process is not adequate to detect part-to-part variations.

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R} \qquad LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R}$$

For constant A_2 look up Table 2.

Total Variation (TV):

$$TV = \sqrt{(R \,\&\, R)^2 + (PV)^2}$$

$$= \sqrt{7.57^2 + 12.79^2}$$

$$= 14.86$$

Summary:

GA File: FITTING

GAGE R & R
AVERAGE AND RANGE METHOD

STUDY #: 1 STUDY DATE: MM-DD-YY
GAGE - #: xxxxxx DESC: xxxxxx TYPE: xxxxxx
CHAR - #: 1 DESC: xxxxxx NAME: char #1
2 APR, 5 PARTS, 3 TRIALS Comment:

	5.15 STD. DEV.	%VAR. STUDY	PERCENT CONTRIBUTION
Repeatability	7.5	50.3	25.3
Reproducibility	1.0	6.9	0.5
GAGE R & R	7.6	50.8	25.8
Part-to-Part	12.8	86.1	74.2

BASED ON DATA CATEGORIES, GAGE SYSTEM O.K. FOR ATTRIBUTE DATA
Note:
Tolerance = N.A. Study variation = 14.9
Number of distinct data categories = 2

Table 4. R&R Average and Range Method

Table 1

g \ m	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1.41	1.91	2.24	2.48	2.67	2.83	2.96	3.08	3.18	3.27	3.35	3.42	3.49	3.55
2	1.28	1.81	2.15	2.40	2.60	2.77	2.91	3.02	3.13	3.22	3.30	3.38	3.45	3.51
3	1.23	1.77	2.12	2.38	2.58	2.75	2.89	3.01	3.11	3.21	3.29	3.37	3.43	3.50
4	1.21	1.75	2.11	2.37	2.57	2.74	2.88	3.00	3.10	3.20	3.28	3.36	3.43	3.49
5	1.19	1.74	2.10	2.36	2.56	2.73	2.87	2.99	3.10	3.19	3.28	3.35	3.42	3.49
6	1.18	1.73	2.09	2.35	2.56	2.73	2.87	2.99	3.10	3.19	3.27	3.35	3.42	3.49
7	1.17	1.73	2.09	2.35	2.55	2.72	2.87	2.99	3.10	3.19	3.27	3.35	3.42	3.48
8	1.17	1.72	2.08	2.35	2.55	2.72	2.87	2.98	3.09	3.19	3.27	3.35	3.42	3.48
9	1.16	1.72	2.08	2.34	2.55	2.72	2.86	2.98	3.09	3.18	3.27	3.35	3.42	3.48
10	1.16	1.72	2.08	2.34	2.55	2.72	2.86	2.98	3.09	3.18	3.27	3.34	3.42	3.48
11	1.16	1.71	2.08	2.34	2.55	2.72	2.86	2.98	3.09	3.18	3.27	3.34	3.41	3.48
12	1.15	1.71	2.07	2.34	2.55	2.72	2.86	2.98	3.09	3.18	3.27	3.34	3.41	3.48
13	1.15	1.71	2.07	2.34	2.55	2.71	2.86	2.98	3.09	3.18	3.27	3.34	3.41	3.48
14	1.15	1.71	2.07	2.34	2.54	2.71	2.86	2.98	3.08	3.18	3.27	3.34	3.41	3.48
15	1.15	1.71	2.07	2.34	2.54	2.71	2.86	2.98	3.08	3.18	3.26	3.34	3.41	3.48
>15	1.128	1.693	2.059	2.326	2.534	2.704	2.847	2.970	3.078	3.173	3.258	3.336	3.407	3.472

Table 2. d_2^* Values For The Distribution of the Average Range[1]

(d_2^* values for g >15)

Table 2. Control Chart constants

Number of Observations in Subgroup	A_2	D_3	D_4
2	1.880	0	3.267
3	1.023	0	2.575
4	0.729	0	2.282
5	0.577	0	2.115
6	0.483	0	2.004
7	0.419	0.076	1.924
8	0.373	0.136	1.864
9	0.337	0.184	1.816
10	0.308	0.223	1.777
11	0.285	0.256	1.744
12	0.266	0.284	1.716
13	0.249	0.308	1.692
14	0.235	0.329	1.671
15	0.223	0.348	1.652