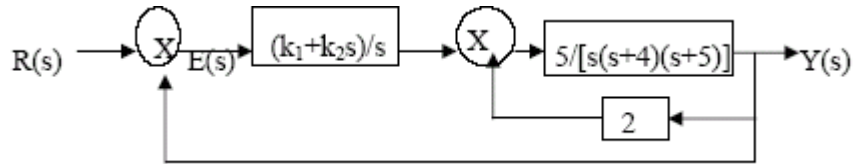


### Problem 1:

The block diagram of a feedback control system is shown

- Find forward path transfer fn  $Y(s)/E(s)$  & closed loop transfer fn  $Y(s)/R(s)$
- Find the steady state error of the system for unit step input



### Program Coding

```
k1=3;
k2=16;
sys1=5*tf(1,conv(conv([1 0],[1 4]),[1 5]));
sys2=feedback(sys1,2);
sys3=tf([k2 k1],[1 0]);
sys4=series(sys2,sys3)
sys=feedback(sys4,1)
t=0:0.1:10;
u=ones(size(t));
[y,t]=step(sys);
kp=dcgain(sys)
ess=(1/(1+kp))
```

### Output:

Transfer function: 
$$\frac{80s + 15}{s^4 + 9s^3 + 20s^2 + 10s}$$

Transfer function: 
$$\frac{80s + 15}{s^4 + 9s^3 + 20s^2 + 90s + 15}$$

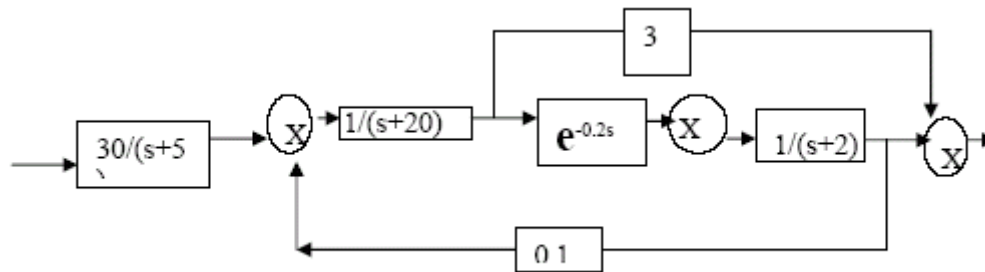
kp = 1  
ess = 0.5000

### Discussions:

- The system is a fourth order system
- Steady state error of the system is found to be 0.5 for unit step input.
- $E(s)$  in this particular circuit can be defined as error signal &  $E(s) = R(s) - Y(s)$ .

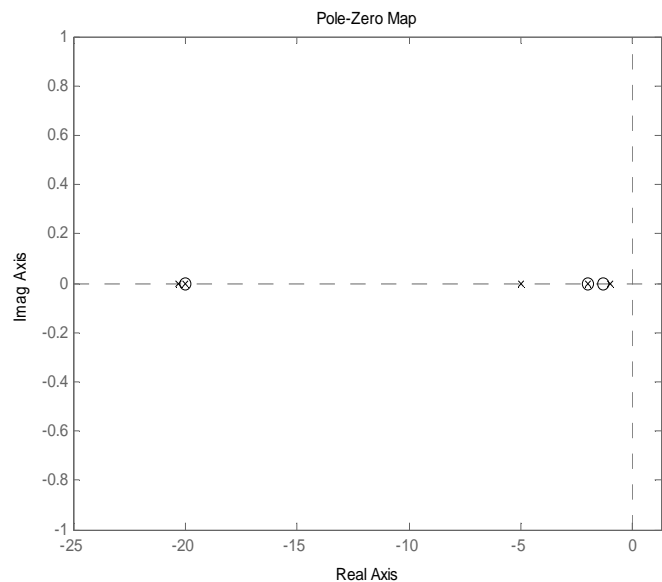
## Problem 2:

Reduce the block & find the poles & zeroes



### Program Coding

```
sys1=tf(1,[1 2]);
sys2=tf([1 2],[1 1]);
sys3=tf(1,[1 20]);
sys4=series(series(sys1,sys2),sys3);
sys5=3;
sys6=series(sys3,sys5);
sys7=parallel(sys4,sys6);
sys8=feedback(sys7,.1);
sys9=tf(30,[1 5]);
sys=series(sys8,sys9)
p=pole(sys)
z=zero(sys)
pzmap(sys,'k')
```



### Outputs:

Transfer function:

$$\frac{90 s^3 + 2100 s^2 + 6240 s + 4800}{s^5 + 48.3 s^4 + 745.5 s^3 + 3946 s^2 + 7320 s + 4080}$$

p =  
-20.2948  
-20.0000  
-5.0000  
-2.0000  
-1.0052

z =  
-20.0000  
-2.0000  
-1.3333

### Discussions:

1. Since there are no poles on the right hand of imaginary axis as shown by the pole-zero map the system is stable.
2. There are 5 poles of the system, all are real.
3. Because of the high order of the system, it is not very stable.
4. The closed loop poles are just in the left half of the s-plane.
5. This implies the system is stable but its relative stability is quite low.

### Problem 3:

Using Bode plot, determine maximum value of k for which the system will be stable

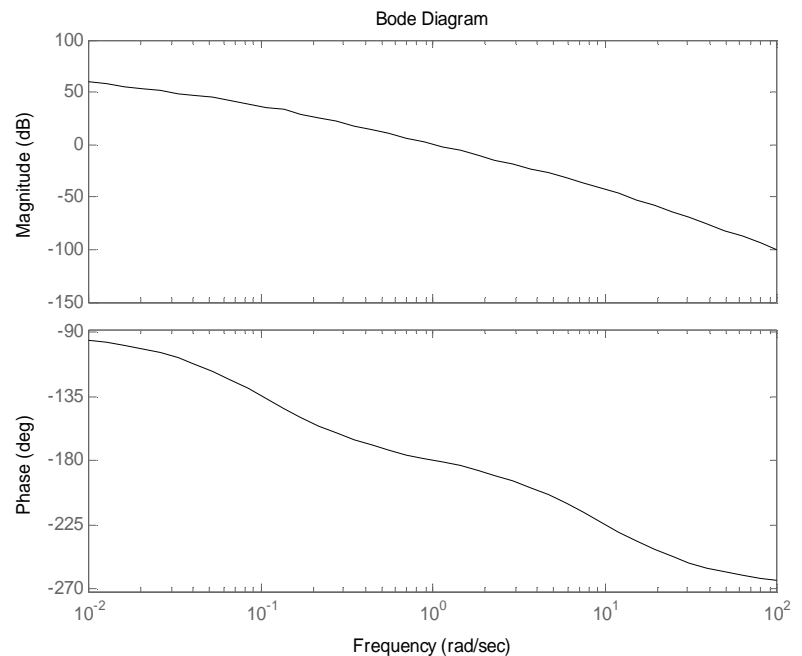
$$G(s) = k/(s^3 + 10s^2 + s)$$

#### Program Coding

```
sys=tf(1,[1 10 1 0]);  
[Kmax]=margin(sys)  
sys_full=Kmax*sys;  
[GM PM]=margin(sys_full)  
bode(sys_full,'k')
```

#### Output:

Kmax = 10  
GM = 1  
PM = 0.0020



#### Discussions:

1. The system will be stable if both gain margin & phase margin is positive.
2. So, the value of gain margin is also the maximum value of gain Kmax. That is why for unity gain system the maximum allowed gain for stability is calculated as the value of gain margin.
3. Here for maximum gain  $K_{max} = 10$ , both gain margin & phase margin is positive. If gain increases further, system will be unstable.

#### Problem 4:

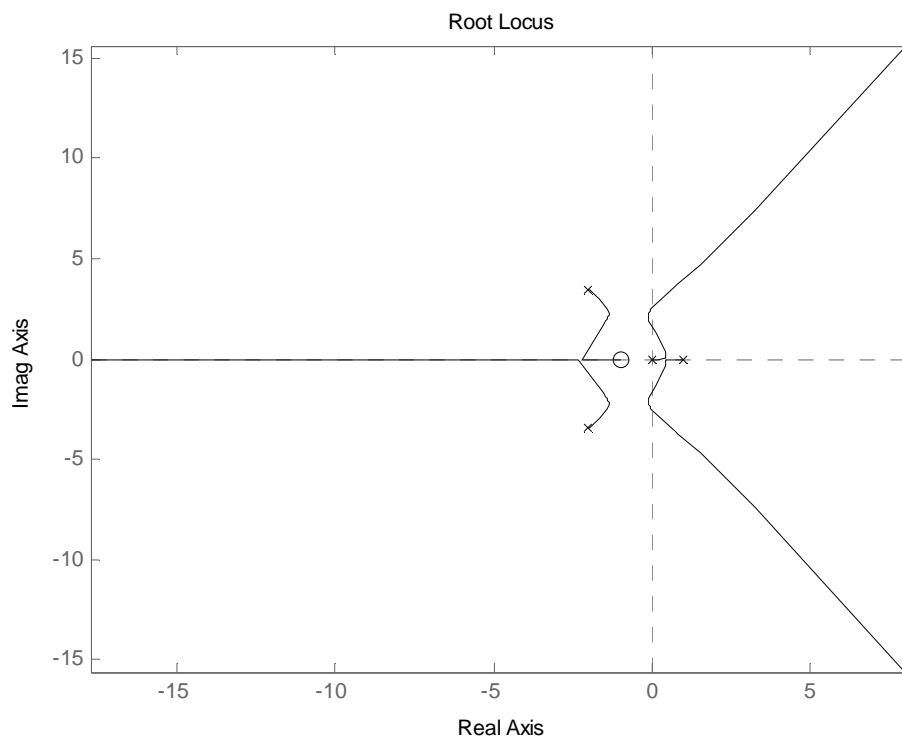
The open loop transfer fn of the system is  $K(s+1)/[s(s-1)(s^2+4s+16)]$ . Determine the root locus of the system

#### Program Coding

```
K=5;  
num=[1 1];  
den=conv(conv([1 0],[1 -1]),[1 4 16]);  
sys=K*tf(num,den)  
rlocus(sys,'k')
```

#### Outputs:

Transfer function:

$$\frac{5s + 5}{s^4 + 3s^3 + 12s^2 - 16s}$$


### Problem 5:

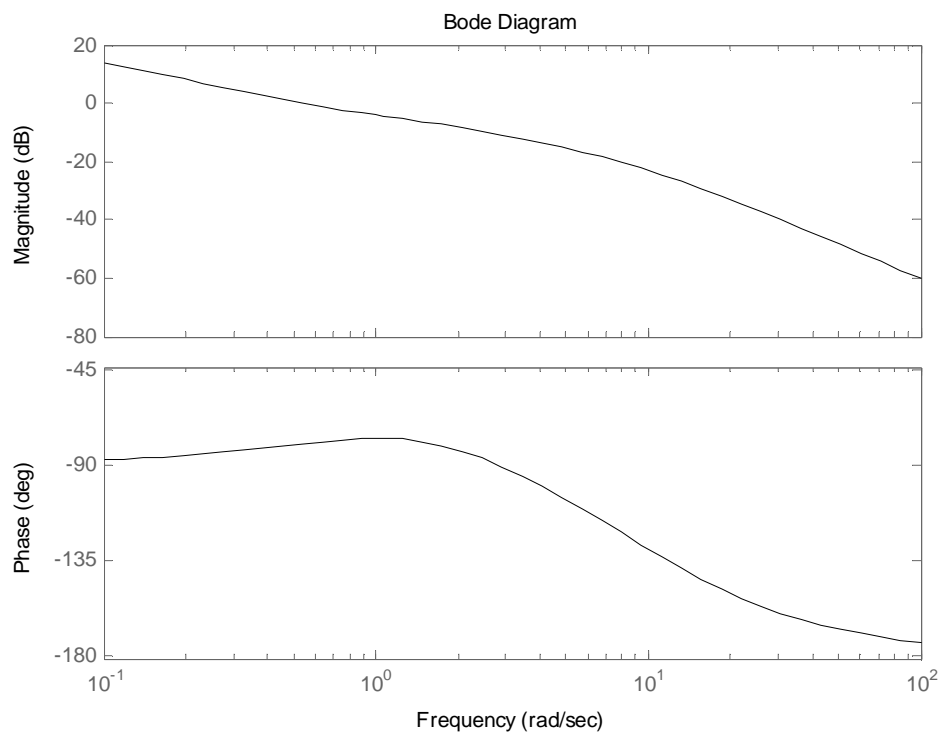
Draw the Bode plot of the open loop system  $10(s+1)/[s(s+2)(s+10)]$ . & determine the gain & phase margin.

#### Program Coding

```
num=[1 1];  
den=conv(conv([1 0],[1 2]),[1 10]);  
sys=10*tf(num,den)  
bode(sys,'k')  
[Gm,Pm,Wcg,Wcp]=margin(sys)
```

#### Outputs:

```
Gm =    Inf  
Pm = 100.2782  
Wcg =    Inf  
Wcp =    0.5493
```



#### Discussions:

1. Since GM & Pm both are positive, the system is stable.
2. Since GM is infinite, system will be stable for any positive gain.

## Problem 6:

A unity feedback system has open loop transfer fn

$$G(s) = k/[s(s+1)(s+5)]$$

Relative stability calls for a peak overshoot of 14%. Find the value of k to meet this specification & the resulting closed loop poles. Find  $\omega_n$ ,  $M_p$ ,  $t_s$  for closed loop response.

### Program Coding

```
den=conv(conv([1 1],[1 5]),[1 0]);
Gs=tf(1,den);
Mp=0.14;
a=(log(Mp)/pi)^2;
b=1+((log(Mp)/pi)^2);
Zeta=sqrt(a/b)
subplot(2,1,1),rlocus(Gs,'k')
sgrid([0.1,Zeta],[1,5,10,15])
[k,poles_openloop]=rlocfind(Gs)
sys=feedback(k*Gs,1);
[Wn,Z,P]=damp(sys);
Wn=Wn(1)
Zeta=Z(1)
Poles_closetloop=P
disp('Peak overshoot, time to peak, Tr, Ts, Td For the step response are as
following :')
t=0:0.01:10;
y=step(sys,t);
u=ones(size(t));
subplot(2,1,2),plot(t,y,'k',t,u,':k')
title('Input dotted, output solid')
[mp,i]=max(y);
Maximum_Overshoot=(mp-1)*100
Tp=t(i)
j=max(find((y<0.98)|(y>1.02)));
Ts=t(j)
x1=max(find((y>=0.09)&(y<=0.11)));
x2=max(find((y>=0.89)&(y<=0.91)));
Tr=t(x2-x1)
d=max(find((y<=0.51)&(y>=0.49)));
Td=t(d)
```

### Outputs:

Zeta = 0.5305

Select a point in the graphics window

selected\_point =  
0.0016 + 0.5852i

k = 3.4185

poles\_openloop =  
-5.1593  
-0.4203 + 0.6971i  
-0.4203 - 0.6971i

$$W_n = 0.8140$$

$$\text{Zeta} = 0.5164$$

$$\begin{aligned} \text{Poles\_closeloop} = \\ -0.4203 + 0.6971i \\ -0.4203 - 0.6971i \\ -5.1593 \end{aligned}$$

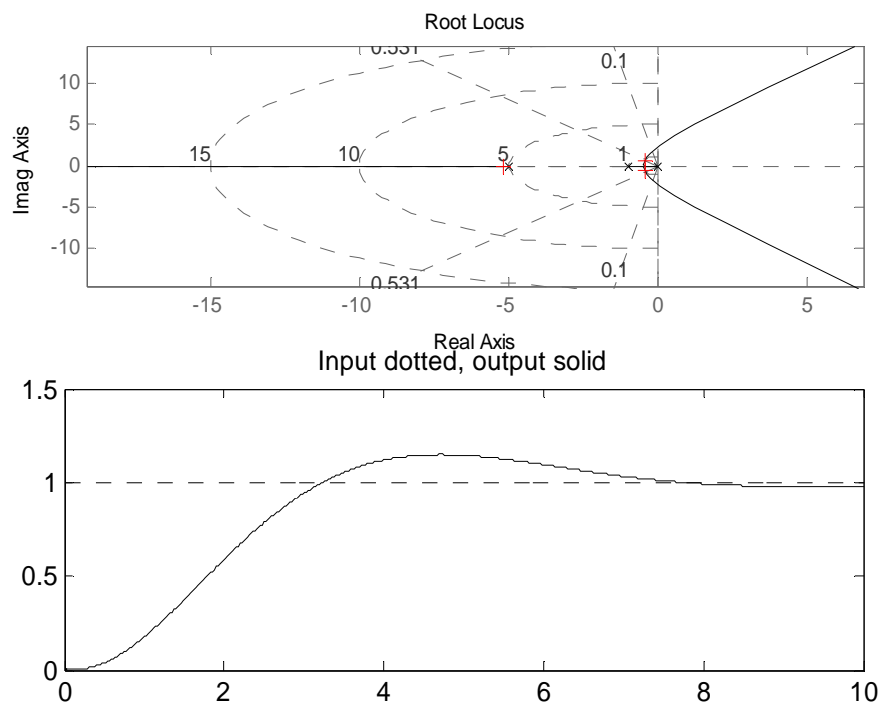
Peak overshoot, time to peak,  $T_r$ ,  $T_s$ ,  $T_d$  For the step response are as following :  
Maximum\_Overshoot = 14.8337

$$T_p = 4.7200$$

$$T_s = 9.8300$$

$$T_r = 2.0800$$

$$T_d = 1.8100$$



### Discussions:

1. To find the value of  $k$  from rootlocus, the cutting point of rootlocus & Zeta line is treated for selection of the point.

### Problem 7:

$$G(s) = k/[s(s+2)(s+6)]$$

Find the root locus of the system. Also find the value of k for which the system will be stable.

### Program Coding

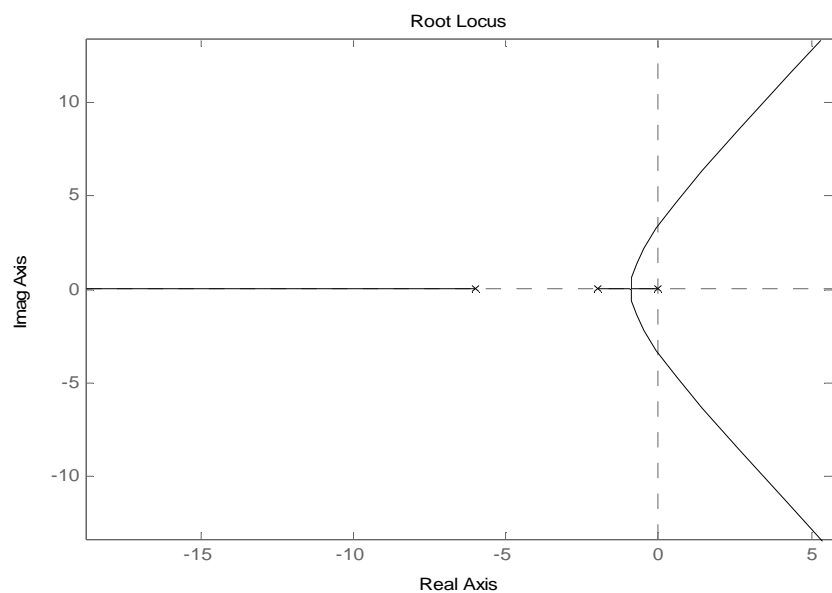
```
sys=tf(1,conv(conv([1 0],[1 2]),[1 6]))
rlocus(sys,'k')
[Kmax]=margin(sys)
```

### Outputs:

Transfer function:

$$\frac{1}{s^3 + 8s^2 + 12s}$$

Kmax = 96.0000



### Discussions:

1. The system TF is deduced from the given block diagram using MATLAB coding. The TF is given by  $M(s) = k/[s(s+2)(s+6)]$
2. As has been obtained by the use of functions, the value of the k for which the system is just stable is around 96 & the value of k for which it is unstable is 96. Thus approximately, the system performs sustained oscillations for a value of k around 96. Hence the range of k for a stable system is  $0 < k < 96$ .
3. The value of k for marginal stability obtained from the point of insertion of the root loci branches with the imaginary axis.
4. From the answer sheet the value of k for which the system is stable is found  $0 < k \leq 96$ .



### Problem 8:

A unity feedback system has a plant transfer fn of  $G(s) = [k(s+4)]/[(s-1)(s-2)]$ .

For  $k=8$ , draw the bode diagram & also find the gain & the phase margin

#### Program Coding

```
k=8;
num=k*poly(-4);
den=poly([1 2]);
sys=tf(num,den)
cl_sys=feedback(sys,1)
bode(cl_sys,'k')
[GM PM GCF ECF]=margin(cl_sys)
```

#### Outputs:

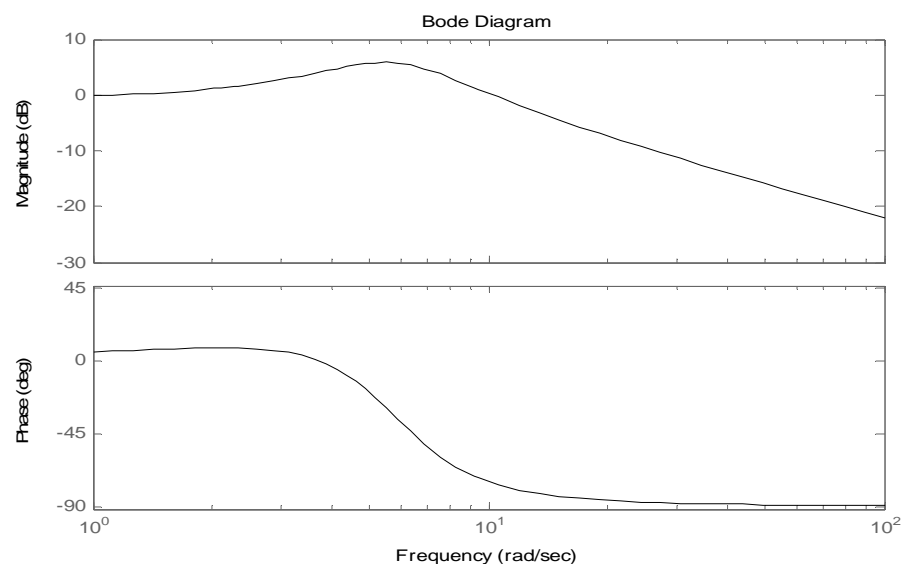
Transfer function:

```
8 s + 32
-----
s^2 - 3 s + 2
```

Transfer function:

```
8 s + 32
-----
s^2 + 5 s + 34
```

```
GM = Inf
PM = 104.3658
GCF = NaN
ECF = 10.2852
```



#### Discussions:

1. The system will be stable if both gain margin & phase margin is positive.
2. So, the value of gain margin is also the maximum value of gain  $K_{max}$ . That is why for unity gain system the maximum allowed gain for stability is calculated as the value of gain margin.
3. Since GM is infinite, system will be stable for any positive gain.

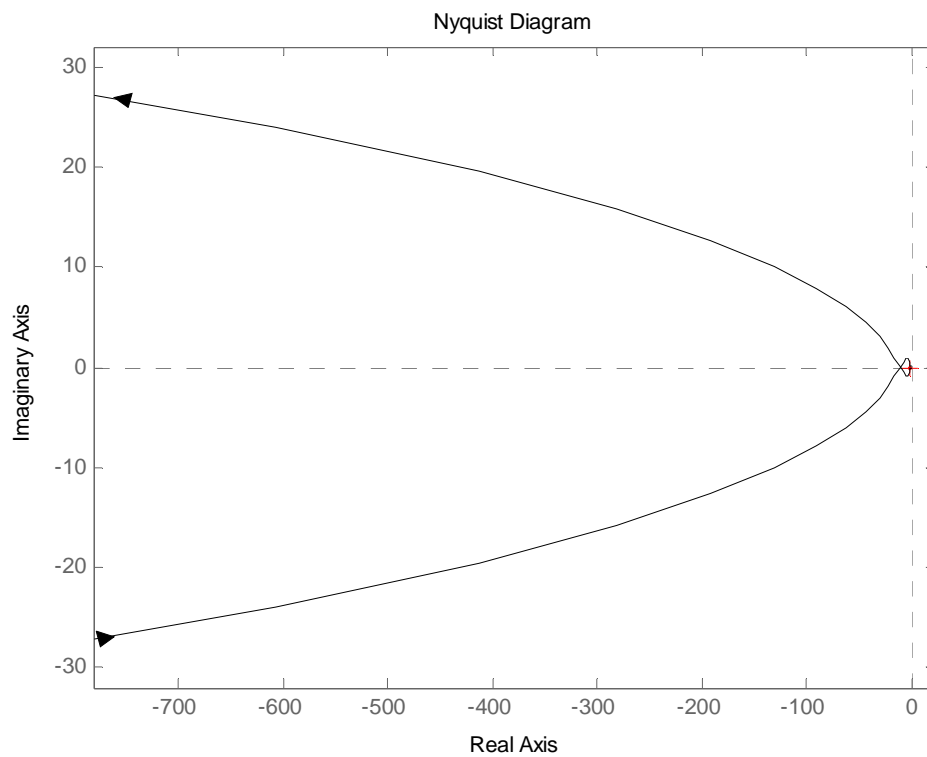
### Problem 9:

Consider a system of  $G(s)H(s) = (4s+1)/[s^2(s+1)(2s+1)]$  Draw the Nyquist plot of the system

#### Program Coding

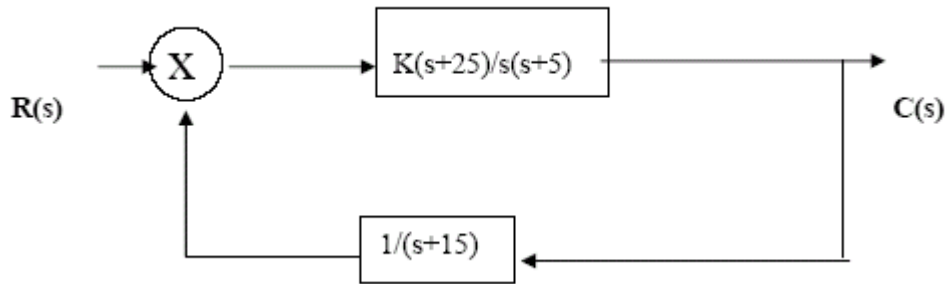
```
sys=tf([4 1],conv(conv(conv([1 0],[1 0]),[1 1]),[2 1]));  
nyquist(sys)
```

#### Output:



**Problem 10:**

Find the value of  $k$  for which the system will be stable.

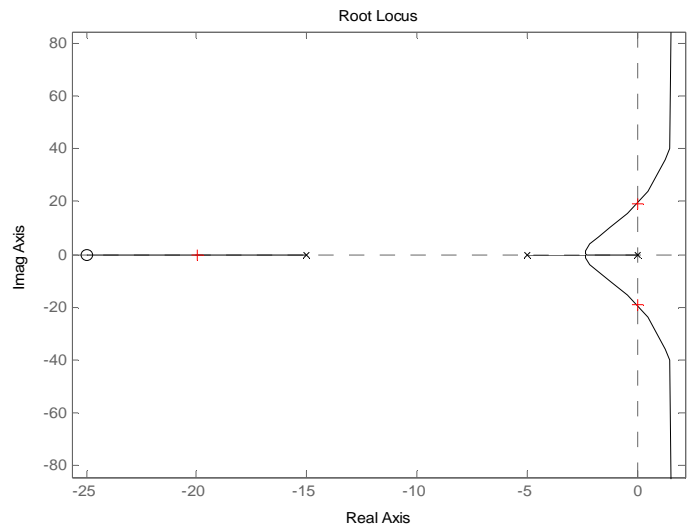
**Program Coding**

```
sys1=tf([1 25],conv([1 0],[1 5]));
sys2=tf(1,[1 15]);
sys=series(sys1,sys2)
[kmax]=margin(sys)
rlocus(sys,'k')
[k,poles_openloop]=rlocfind(sys)
```

**Outputs:**

```
Transfer function:
      s + 25
-----
s^3 + 20 s^2 + 75 s
kmax = 300.0570
Select a point in the graphics window
selected_point =
    -1.3826 -19.1926i

k = 298.6056
poles_openloop =
    -19.9384
    -0.0308 +19.0887i
    -0.0308 -19.0887i
```

**Discussions:**

1. As has been obtained by the use of functions `rlocus(sys)` & `[k,poles]=rlocus(sys)`, the value of the  $k$  for which the system is stable for value of  $k$  less than 300. Hence the range of  $k$  for a stable system is  $0 < k < 300$
2. The value of  $k$  for marginal stability obtained from the point of intersection of the root loci branches with the imaginary axis.
4. From the answer sheet the value of  $k$  for which the system is stable is found  $k = 298.6056$
5. The function `rlocfind` is used to determine the value of  $k$ . This function gives us the option of selecting any point on the root locus plot after the execution of the program.
6.  $K$  is the point at which the system is still stable. This value was taken since a better approximation could not be obtained.