CALCULUS 3

APPLICATIONS OF DOUBLE INTEGRALS

Equation you will need

Description	Equation
Mass of the lamina with density ρ(x, y) which occupies region D is given by	$m = \iint_D \rho(x, y)\, dA$
The co-ordinates of centre of mass $(\bar{x}, \bar{y})$ are	$\bar{x} = \frac{My}{m} = \frac{1}{m}\iint_D x\rho(x,y)dA$ $\bar{y} = \frac{Mx}{m} = \frac{1}{m}\iint_D y\rho(x,y)dA$
Surface area : The area of the surface with equation z=f(x, y), (x, y) ϵ D where fx and fy are continuous	$A(s) = \iint_D \sqrt{[f_x(x,y)]^2 + [f_y(x,y)]^2 + 1}\, dA$ $= \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$

Description	Equation
Moment of lamina about x-axis	$Mx = \iint_D y\rho(x,y)dA$
Moment of lamina about y-axis	$My = \iint_D x\rho(x,y)dA$
Moment of inertia about x-axis	$I_x = \iint_D y^2\rho(x,y)dA$
Moment of inertia about y-axis	$I_y = \iint_D x^2\rho(x,y)dA$
Moment of inertia about origin:	$I_0 = \iint_D (x^2+y^2)\rho(x,y)dA$
Radius of gyration $\bar{\bar{y}}$ with respect to x-axis	$m\bar{\bar{y}}^2 = I_x$
Radius of gyration $\bar{\bar{x}}$ with respect to y-axis	$m\bar{\bar{x}}^2 = I_y$

APPLICATIONS OF TRIPLE INTEGRALS

Equation you will need

Description	Equation
Volume of region E	$v(E) = \iiint_E dv$
Moment of inertia about x-axis :	$Ix = \iiint_E (y^2+z^2)\rho(x,y,z)dv$
Moment of inertia about y-axis :	$Iy = \iiint_E (x^2+z^2)\rho(x,y,z)dv$
Moment of inertia about z-axis :	$Iz = \iiint_E (x^2+y^2)\rho(x,y,z)dv$
Centre of mass $(\bar{x}, \bar{y}, \bar{z})$ is given by	$\bar{x} = \frac{Myz}{m}$, $\bar{y} = \frac{Mxz}{m}$, $\bar{z} = \frac{Mxy}{m}$

Description	Equation
If the density function of a solid object that occupies the region E is ρ(x, y, z) in units of mass per unit volume then at any given point (x,y,z), its mass is	$m = \iiint_E \rho(x,y,z)dv$
Moment about x-axis	$M_{yz} = \iiint_E x\rho(x,y,z)dv$
Moment about y-axis	$M_{xz} = \iiint_E y\rho(x,y,z)dv$
Moment about z-axis	$M_{xy} = \iiint_E z\rho(x,y,z)dv$

LINE INTEGRALS

Definitions to know

Line Integrals

If f is defined on a smooth curve C given by x=x(t), y=y(t), a ≤ t ≤ b then the line integral of f along c is:

$$\int_c f(x,y)ds = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*, y_i^*)\Delta s_i$$

if the limit exists. Or

$$\int_c f(x,y)ds = \int_a^b f(x(t),y(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

Gradient of a Scalar Field

Let f(x, y, z) be defined and differentiable at each point (x, y, z) in a certain region of space, then the gradient of f written as or grad f is defined as: $\vec{\nabla} f$

$$\vec{\nabla} f = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) f$$

$$= \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

Line Integral with Respect to Arc Length

$$\int_c f(x,y)dx = \int_a^b f(x(t),y(t))x^1(t)dt$$ and

$$\int_c f(x,y)dy = \int_a^b f(x(t),y(t))y^1(t)dt$$

Line Integrals of Vector Field

If $\vec{F}$ is a continuous vector field defined on a smooth curve c given by vector function $\vec{r}(t), a \le t \le b,$ then the line integral of F along C is:

$$\int_c \vec{F}.d\vec{r} = \int_a^b \vec{F}(\vec{r}(t).\vec{r})'(t)dt$$

Line Integral in Space

The line integral of f along C given by: x = x(t), y = y(t), z = z(t), a ≤ t ≤ b then

$$\int_c f(x,y,z)ds = \int_a^b f(x(t),y(t),z(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt$$

Divergence of a Vector Field

If $\vec{F} = P\hat{i} + Q\hat{j} + R\hat{k}$ is a vector field on R^3 and $\frac{\partial P}{\partial x}, \frac{\partial Q}{\partial y} and \frac{\partial R}{\partial z}$ exists then divergence of is defined

as: $div\vec{F} = \vec{\nabla}.\vec{F}$

$$= \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

Curl of a Vector Field

If $\vec{F} = P\hat{i} + Q\hat{j} + R\hat{k}$ is a vector field on R^3 and the partial derivatives of P, Q and R all exist, then Curl of $(u,v)\in D$ is a vector field defined as:

$$curl\vec{F} = \vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

Conservative Vector Field

A vector field $\vec{F}$ is said to be conservative if there exists a function f such that $\vec{\nabla} f = \vec{F}$

Flux

If $\vec{F}$ is a continuous vector field defined on surface S with unit normal vector $\hat{n}$, then the surface integral of $\vec{F}$ over S is called flux of $\vec{F}$.

Flux of $\vec{F} = \iint_S \vec{F}.d\vec{s} = \iint_S \vec{F}.\hat{n}ds$

If $\vec{F} = P(x,y,z)\hat{i} + Q(x,y,z)\hat{j} + R(x,y,z)\hat{k}$ And S is given by z=g(x, y) then $\iint_S \vec{F}.d\vec{s} = \iint_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right)dA$

LINE INTEGRALS

Surface Integrals

If the surface S is the graph of a function of two variables then it has equation of the form z=g(x, y), (x, y)εD and if f is continuous on S and g has continuous derivatives, then the surface integral of f over S is given by:

$$\iint_S f(x,y,z)ds = \int_D\int f(x,y,g(x,y))\sqrt{\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2+1}dA$$

Now if the surface S has a vector equation

$$\vec{r}(u,v) = x(u,v)\hat{i} + y(u,v)\hat{j} + z(u,v)\hat{k} \quad , \quad (u,v)\varepsilon D$$

Them

$$\int_S\int f(x,y,z)ds = \int_D\int f(\vec{r}(u,v))\,|\vec{r}_u \times \vec{r}_v|dA$$

Surface Area

If a smooth parametric surface S is given by the equation

$$\vec{r}(u,v) = x(u,v)\hat{i} + y(u.v)\hat{j} + z(u,v)\hat{k} \quad , \quad (u,v)\in D$$

Then the surface area of S is $A(S) = \iint_D |\vec{r}_u \times \vec{r}_v|\,dA$

where $\vec{r}u = \frac{\partial x}{\partial u}\hat{i} + \frac{\partial y}{\partial u}\hat{j} + \frac{\partial z}{\partial u}\hat{k}$

And $\vec{r}v = \frac{\partial x}{\partial v}\hat{i} + \frac{\partial y}{\partial v}\hat{j} + \frac{\partial z}{\partial v}\hat{k}$

Theorems to remember

Let C be a smooth curve given by the vector function $\vec{r}(t), a \le t \le b$. Let f be a differentiable function of two or three variables whose gradient vector $\vec{\nabla} f$ is continuous on C, then

$$\int_c \vec{\nabla} f.d\vec{r} = \int_a^b f(\vec{r}(b)) - f(\vec{r}(b))$$

$\int_c \vec{F}.d\vec{r}$ is independent of path in region D if and only if

$\int_c \vec{F}.d\vec{r} = 0$ for every closed path C and D.

If $\vec{F}(x,y) = P(x,y)\hat{i} + Q(x,y)\hat{j}$ is a conservative vector field, where P and Q have continuous first-order partial derivatives on a domain D then

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

Let $\vec{F} = P\hat{i} + Q\hat{j}$ be a vector field on an open simply connected region D. Suppose that P and Q have continuous first-order derivatives and

$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ throughout D, then $\vec{F}$ is conservative.

Green's theorem: Let D be a closed bounded region in the xy-plane boundary C consists of finitely many smooth curves. Let P and Q be continuous function of x and y having continuous partial derivatives

$\frac{\partial P}{\partial y}$ and $\frac{\partial Q}{\partial x}$ in D. Then $\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dA = \oint_C Pdx + Qdy$

the line integral being taken along the entire boundary C of D that C is positively oriented, piecewise- smooth and simple closed curve.

Area of region D using Green's theorem : $A = \oint_C xdy = -\oint_C udx = \frac{1}{2}\oint_C xdy - ydx$

Stoke's theorem: Let S be an oriented piecewise-smooth surface that is bounded by a simple, closed, piecewise-smooth boundary curve c with positive orientation. Let $\vec{F}(x,y,z)$ be a continuous vector function which has continuous first partial derivatives in a region of space which contains S, then

$$\oint_C \vec{F}.d\vec{r} = \iint_S (\vec{\nabla}\times\vec{F}).\hat{n}ds = \iint_S (curl\vec{F}).d\vec{s}$$

Divergence Theorem: Suppose V is the volume bounded by a closed piecewise smooth surface S. Suppose $F(x,y,z)$ is a vector field which is continuous and has continuous first order partial derivatives in V, then

$$\iiint_v \nabla.\vec{F}dv = \iint_S \vec{F}.\hat{n}ds$$ where $\hat{n}$ is the outwards drawn unit normal vector to S.

LINE INTEGRALS

Important Identities

* $div(curl\vec{F}) = 0$	* curl (grad f) = $\vec{0}$	* $div(\vec{F}+\vec{G}) = div\vec{F} + div\vec{G}$
* $curl(\vec{F}+\vec{G}) = curl\vec{F} + curl\vec{G}$	* $div(f\vec{F}) = f div\vec{F} + \vec{F}.\vec{\nabla} f$	* $curl(f\vec{F}) = f curl\vec{F} + (\vec{\nabla} f)\times\vec{F}$
* $div(\vec{F}\times\vec{G}) = \vec{G}.curl\vec{F} - \vec{F}.curl\vec{G}$	* $div(\vec{\nabla} f\times\vec{\nabla} g) = 0$	* $curl(curl\vec{F}) = grad(div\vec{F}) - \nabla^2\vec{F}$
* $grad(\vec{F}.\vec{G}) = (\vec{G}.\vec{\nabla})\vec{F} + (\vec{F}.\vec{\nabla})\vec{G} + \vec{G}\times curl\vec{F} + \vec{F}\times curl\vec{G}$		

DIFFERENTIAL EQUATIONS

Classification of Differential Equations

Ordinary differential equation: If only ordinary derivatives appear in the differential equation, it is said to be ordinary differential equation.

Partial differential equation: If the differential equation involves only partial derivatives it is said to be a partial differential equation.

Linear and non-linear differential equation: An ordinary differential equation F(t, y, y', ----, $y^{(n)}$) = 0 is said to be Linear if F is a linear function of the variables y, y', ----, $y^{(n)}$. A similar definition applies to partial differential equations.

The differential equation which is not of the above form is said to be non-linear.

Separable equation: An equation is said to be separable if it can be written in the differential form:

$$M(x)dx + N(y)dy = 0$$

Exact equation: A differential equation $M(x,y) + N(x,y)\frac{dy}{dx} = 0$ is said to be exact if there exists a function $\psi(x,y)$ such that $\frac{\partial\psi}{\partial x} = M(x,y)$ and $\frac{\partial\psi}{\partial y} = N(x,y)$

Second order linear equation: A second order differential equation $\frac{d^2y}{dt^2} = f\left(t, y, \frac{dy}{dt}\right)$ is said to be linear if the function f has the form

$$f\left(t, y, \frac{dy}{dt}\right) = g(t) - p(t)\frac{dy}{dt} - q(t)y$$

(A second order linear differential equation can also be written as $y'' + p(t)y' + q(t)y = g(t)$)

Second order homogeneous/non – homogeneous equation: A second order differential equation $y'' + p(t)y' + q(t)y = g(t)$ is said to be homogeneous if g(t) = 0 for all t, otherwise it is called non – homogeneous.

Linear Independence and Wronskian

Two functions f and g are said to be linearly dependent on an interval I if there exist two constants k_1 and k_2, not both zero such that, $k_1 f(t) + k_2 g(t) = 0$ for all t in I.

Two functions f and g are called linearly independent if they are not linearly dependent.

The wronskian of the function $f_1, f_2.f_3, ---- f_n$ is defined as:

$$W(f_1, f_2.f_3, ---- f_n)(t) = \begin{vmatrix} f_1 & f_2 & -------- & f_n \\ f_1^{(1)} & f_2^{(1)} & ------- & f_n^{(1)} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ f_1^{(n-1)} & f_2^{(n-1)} & & f_n^{(n-1)} \end{vmatrix}$$

Abel's Theorem

If y_1 and y_2 are solutions of the differential equation $L[y] = y'' + p(t)y' + q(t)y = 0$ where p and q are continuous on an open interval I then the wronskian $W(y_1, y_2)(t)$ is given by $W(y_1,y_2)(t) = c\exp\left[-\int p(t)dt\right]$

where c is a certain constant that depends on y_1 and y_2 but not on t. Further $W(y_1, y_2)(t)$ either is zero for all t in I (if c = 0) or else is never zero in I (if $c \neq 0$)

Series Solution of an Ordinary Differential Equation

To find the series solution of the equation $ay'' + by' + cy = 0$, $(a \neq 0)$, take

$$y = f(x) = \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + ----$$

Find y' and y''. Substitute for y, y' and y'' in the given equation and determine the values of the coefficients c_0, c_1, c_2, - - - - -

DIFFERENTIAL EQUATIONS

Solution of Homogeneous Equation

- Characteristic equation: For the differential equation $ay''+by'+cy=0$ $(a\neq 0)$ the characteristic equation is given by $ar^2+br+c=0$ (obtained by taking $y=e^{rx}$ as a solution of differential equation)
- Solutions of homogeneous equation $ay''+by'+cy=0$

Roots of characteristic equation: $ar^2+br+c=0$	General solution
r_1, r_2 real and distinct	$y=c_1e^{r_1t}+c_2e^{r_2t}$
$r_1=r_2=r$	$y=c_1e^{rt}+c_2te^{rt}$
r , r_2 complex: $\alpha\pm i\beta$	$y=e^{\alpha x}\left(c_1\cos\beta x+c_2 Sin\beta x\right)$

Solution of Non-homogeneous Equation

- The solution of non-homogeneous equation $ay''+by'+cy=g_i(t)$ is:
  y(t) = Solution of corresponding homogeneous equation + particular solution.
- The particular solution of $ay''+by'+cy=g_i(t)$ by the method of undetermined coefficients:

$g_i(t)$	$y_i(t)$
$P_n(t)=a_0t^n+a_1t^n+-----+a_n$	$t^s\left(A_1t^n+A_1t^{n-1}+------+A_n\right)$
$P_n(t)=e^{\alpha t}$	$t^s\left(A_1t^n+A_1t^{n-1}+------+A_n\right)e^{\alpha t}$
$P_n(t)=e^{\alpha t}\begin{cases}Sin\beta t\\ Cos\beta t\end{cases}$	$t^s\begin{bmatrix}\left(A_1t^n+A_1t^{n-1}+------+A_n\right)e^{\alpha t}Cos\beta t+\\ \left(B_0t^n+B_1t^{n-1}+-----+B_n\right)e^{\alpha t}Sin\beta t\end{bmatrix}$

Note: Here s is the smallest non-negative integer that will ensure that no term in $y_i(t)$ is a solution of corresponding homogeneous equation.

Euler Equation

The equation of the type $L[y]x=x^2y''+\alpha xy'+\beta y=0$ where α and β are real constants, are called Euler equations.

Solutions of Euler equation: For the Euler equation $x^2y''+\alpha xy'+\beta y=0$ take $y=x^r$ as a solution and obtain the quadratic equation $r(r-1)+\alpha r+\beta=0$

Roots of quadratic equation: $r(r-1)+\alpha r+\beta=0$
r_1 , r_2 are real and distinct
r_1 , r_2 complex: $\alpha\pm i\beta$
$r_1=r_2=r$

DIFFERENTIAL EQUATIONS

Laplace Transforms

Definition to know

The Laplace transform of f which is denoted by $\mathrm{L}\{f(t)\}$ or $F(s)$ is defined by:

$L\{f(t)\} = F(s) = \int_0^{\infty} e^{-st} f(t)\,dt$ whenever this improper integral converges.

Some elementary Laplace transforms

$f(t) = \mathrm{L}^{-1}\{f(s)\}$	$F(s) = \mathrm{L}\{f(t)\}$	$f(t) = \mathrm{L}^{-1}\{f(s)\}$	$F(s) = \mathrm{L}\{f(t)\}$
I. 1	1/s, s > 0	II. e^{at}	$1/s-a$, s > a
III. t^n, n = positive integer	$\frac{n!}{s^{n+1}}$, s > 0	IV. t^p, p > -1	$\frac{\Gamma(p+1)}{s^{p+1}}$, s > 0
V. Sin at	$\frac{a}{s^2+a^2}$, s > 0	VI. Cos at	$\frac{s}{s^2+a^2}$, s > 0
VII. Sin h at	$\frac{a}{s^2-a^2}, s > \lvert a \rvert$	VIII. Cos h at	$\frac{s}{s^2-a^2}, s > \lvert a \rvert$
IX. $e^{at} Sinbt$	$\frac{b}{(s-a)^2+b^2}, s > a$	X. $e^{at} Cosbt$	$\frac{s-a}{(s-a)^2+b^2}, s > a$
XI. $t^n e^{at}$, n positive integer	$\frac{n!}{(s-a)^{n+1}}, s > a$	XII. $u_c(t)$	$\frac{e^{-cs}}{s}, s > 0$
XIII. $u_c(t) f(t-c)$	$e^{-cs} F(s)$	XIV. $e^{ct} f(t)$	$f(s-c)$
XV. f(ct)	$\frac{1}{c} F\left(\frac{s}{c}\right), c > 0$	XVI. $\int_0^t f(t-\tau) g(\tau)\,d\tau$	$F(s)G(s)$
XVII. $\delta(t-c)$	e^{-cs}	XVIII. $(-t)^n f(t)$	$F^{(n)}(s)$
XIX. $f^{(n)}(t)$	$8^n F(s) - s^{n-1} f(0) - .... - f^{(n-1)}(0)$		

MATRICES

Definition to know

A system of mn numbers arranged in the form of an ordered set of m – rows, each row consisting of an ordered set of n numbers, is called a matrix. A matrix is denoted by capital letters A, B, C, ----- and the element in the i^{th} row and j^{th} column of A is denoted by a_{ij} . Then if A is m×n, then

$$A = \begin{pmatrix} a_{11} & a_{12} & \text{---------} & a_{1n} \\ a_{21} & a_{22} & \text{---------} & a_{2n} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \text{---------} & a_{mn} \end{pmatrix}$$

MATRICES

Type of Matrices

(a). Row matrix: An m×n matrix is called a row matrix if m = 1. For example:

$$\begin{pmatrix} 1 & 0 & 2+3i \end{pmatrix}$$

(b). Column matrix: An m×n matrix is called a column of n = 1. For example:

$$\begin{pmatrix} 2 \\ 9 \\ 7 \\ 5 \end{pmatrix}$$

(c). Square matrix: An m×n matrix is called a square matrix if m = n. For example

$$\begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$$

(d). Rectangular matrix: A matrix which is not square.

(e). Zero matrix: A matrix of whose each of the element is zero. It is denoted by O

(f). Diagonal Matrix: A square matrix with all its non-diagonal elements as zero. For example

$$A=\begin{pmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{pmatrix}$$

(g). Scalar matrix: A diagonal matrix all of whose diagonal elements are equal.

(h). Unit matrix: A scalar matrix all of whose diagonal elements are equal to unity. (It is also called Identity Matrix)

(i). Triangular matrix: If every element above or below the diagonal is zero. For example:

$$A=\begin{pmatrix} 2 & 3 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$ (Upper triangular matrix)

$$B=\begin{pmatrix} 0 & 0 & 0 \\ 1 & 3 & 0 \\ 5 & 4 & 7 \end{pmatrix}$$ (Lower triangular matrix)

Addition / Subtraction of Matrix

If $A=(a_{ij})$ and $B=(b_{ij})$ are two matrices of same order m×n then

$$A \pm B = (a_{ij} \pm b_{ij})_{m\times n}$$

Multiplication

(i). Scalar multiplication: If $A=(a_{ij})$ is a matrix of order m×n and k is a scalar then

$$kA = (ka_{ij})_{m\times n}$$

(ii). Matrix multiplication: If $A=(a_{ij})$, $B=(b_{ik})$ are two matrices of order m×n and n×p respectively then their multiplication, $AB = C = (c_{ik})_{m\times p}$

where $c_{ik} == \sum_{j=1}^{n} a_{ij} b_{jk}$

Note :- AB is not necessarily equal to BA.

Transpose of a Matrix

The matrix obtained from a given matrix A by interchanging its rows and columns is called the transpose of A and is generally denoted by A' or A^T

Symmetric Matrix

Any square matrix $A=(a_{ij})$ is said to be symmetric if $a_{ij} = a_{ji}$ or if $A = A^T$

Skew Symmetric Matrix

Any square matrix $A=(a_{ij})$ is said to be skew symmetric if $a_{ij} = -a_{ji}$ i.e. if $A = -A^T$

Note: If A and B are symmetric, then AB is symmetric if and only if AB = BA.

Minors and Co-factors

The minor of an element of a determinant Δ is the determinant obtained by omitting from Δ, the row and column containing the element.

The co-factor of the element occurring in the m^{th} row and n^{th} column is $(-1)^{m+n}$ times its minor.

MATRICES

Determinant of a Matrix

- The symbol $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ in which any four numbers a_1, a_2, b_1 and b_2 are arranged in square array consisting of two horizontal lines and two vertical lines, and bounded by two vertical bars is called a determinant of second order. It is denoted by Δ.

  Δ is denoted as: $\Delta = a_1 b_2 - a_2 b_1$

- The symbol $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ in which nine numbers $a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$ are arranged in a square array, consisting of three rows and three columns, bounded by two vertical bars is called s determinant of third order. It is denoted by Δ and is defined as

  $$\Delta = a_1\left(b_2c_3 - b_3c_2\right) + b_1\left(c_2a_3 - c_3a_2\right) + c_1\left(a_2b_3 - a_3b_2\right)$$

- The determinant of a square matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & \text{------} & a_{1n} \\ a_{21} & a_{22} & \text{------} & a_{2n} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \text{------} & a_{nn} \end{pmatrix}$$

is given by:

$$|A|(or \det A) = \begin{vmatrix} a_{11} & a_{12} & \text{------} & a_{1n} \\ a_{21} & a_{22} & \text{------} & a_{2n} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \text{------} & a_{nn} \end{vmatrix}$$

Eigen values and Eigen vectors

The equation AX = Y can be viewed as a linear transformation that maps a given vector into a new vector Y.

The values of λ that satisfy the equation det(A – λI) = 0

Are called Eigen values of the matrix A and the non-zero solution of equation (A – λI)X = O that are obtained by using such a value of λ are called the Eigen vectors.

Adjoint of a Square Matrix

Let $A = \left(a_{ij}\right)$ be any square matrix. Then the matrix $\left(A_{ij}\right)$ where A_{ij} denotes the co-factor of a_{ij} in |A| is called the adjoint of A and is denoted by adjA or A*.

In other words adjA is a matrix obtained after replacing every $(i, j)^{th}$ element of A by the co-factor of $(j, i)^{th}$ element in |A|.

Inverse of a Matrix

For the matrices A and B, if AB = BA = I (identity matrix) then B is called the inverse of A and it is denoted by A^{-1}

Note: If det A ≠ 0 then $A^{-1} = \dfrac{adjA}{\det A}$

System of Linear Algebraic Equation

A set of n simultaneous linear algebraic equations in n Variables

$$a_{11}x_1 + a_{12}x_2 + \text{-----} a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \text{-----} a_{2n}x_n = b_2$$
$$\vdots \qquad \vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \text{-----} a_{nn}x_n = b_n$$

can be written as: AX = B

where $A = \begin{pmatrix} a_{11} & a_{12} & \text{------} & a_{1n} \\ a_{21} & a_{22} & \text{------} & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \text{------} & a_{nn} \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$

and $B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$