

CS502 Fall 2005
Homework 4

Due Feb 18, 2006

Problem 1. A certain country uses coins with integer values v_0, v_1, \dots, v_n , where $v_0 = 1$ and $v_0 < v_1 < \dots < v_n$. Let $f(i, j)$ be the minimum number of coins needed to produce a value of exactly i using only v_0, v_1, \dots, v_j (there is no limit to how many coins of the same value may be used).

(a) (10 points) Prove that for $i \geq 0$ and $0 \leq j \leq n$,

$$f(i, j) = \begin{cases} i & \text{if } j = 0 \\ f(i, j-1) & \text{if } j > 0 \text{ and } v_j > i \\ \min(f(i, j-1), f(i-v_j, j) + 1) & \text{otherwise.} \end{cases}$$

(b) (10 points) Give an $O(nk)$ algorithm $\text{Min-Change}(v[1..n], k)$ that returns $f(k, n)$.

Problem 2. (10 points) A knapsack has a capacity of 5. There are $n = 4$ objects with integer weights $w[1..4] = \{2, 3, 4, 5\}$, and values $v[1..4] = \{3, 4, 5, 6\}$. Use the 0/1 knapsack algorithm to compute the to achieve maximum total value of packed items and the actual items placed in the knapsack.

Problem 3. (10 points) Compute a Huffman encoding of the string “accctggccccccg”. Show the binary tree and provide the codes for the letters a, c, g and t.